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MATH

STUDENT BOOK

6th Grade | Unit 7

Alpha Omega
PUBLICATIONS

MATH 607
Probability and Geometry

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Alpha Omega
PUBLICATIONS

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Probability & Geometry

Introduction

In this unit, you will be introduced to the topic of probability. You will learn how to find the probability of simple events, and their complements. You will also find the probability of independent events using lists, tables, and tree diagrams to show the sample space. All of this will be a solid foundation for exploring more complex events in probability.

You will also be introduced to geometry, its terminology, and some basic shapes. You will learn to classify polygons based on their attributes. You will specifically look at triangles and quadrilaterals and find several types of each. You will also compare polygons and find out about similar and congruent figures. These basic tools will be useful in your future explorations in geometry.

Objectives

Read these objectives. The objectives tell you what you should be able to do when you have successfully completed this LIFEPAC©. Each section will list according to the numbers below what objectives will be met in that section. When you have finished the following LIFEPAC, you should be able to:

- Find the theoretical probability of a simple event and its complement.
- Display the sample space of an event on a tree diagram, list, or table and find the probability of independent events.
- Use correct geometric terminology and notation.
- Classify acute, obtuse, right, and straight angles.
- Use angle relationships (vertical, complementary, and supplementary) to solve problems.
- Classify triangles, quadrilaterals, and other polygons based on their attributes.
- Find a missing angle measure of a triangle or a quadrilateral.
- Determine if two figures are congruent, similar, or neither.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. PROBABILITY

If you reached into this bag of marbles (without looking), what are the chances you would draw a blue marble? How likely would it be to draw a green marble?

It's easy to see which color marble would have a better chance of being drawn, but can we measure the chance of something happening? Yes we can, and this area of mathematics is called **probability**. In this lesson, you will learn how to find the probability, or likelihood, of different events.

Objectives

Review these objectives. When you have completed this section, you should be able to:

- Find the theoretical probability of a simple event and its complement.
- Display the sample space of an event on a tree diagram, list, or table.
- Find the probability of independent events.
- Find the experiment probability of an event.

Materials

pencil paper calculator

Vocabulary

complementary event. two events with no outcomes in common, where one or the other must occur.

compound event. An event where two or more events happen one after the other, or at the same time.

digit. One of the numerals from 0 to 9.

estimate. An approximate value close to the actual value.

experimental probability. Probability based on results of trials.

independent events. A compound event where the likelihood of one event does not affect the other(s).

place value. The position of a digit in a number, which determines its value.

sample space. An organized listing of all possible outcomes for an experiment.

tree diagram. An organizing tool used to find the sample space for compound events.

Note: *All vocabulary words in this LIFEPAC appear in* **boldface** *print the first time they are used. If you are not sure of the meaning when you are reading, study the definitions given.*

INTRODUCTION TO PROBABILITY

In mathematics, a situation where we want to find the probability, like drawing a certain color marble from a bag, is called an experiment. Each result of the experiment is called an outcome.

For this bag of marbles, there are 10 possible outcomes, because there are 10 different marbles that can be drawn.

When we do an experiment, we are looking for a specific outcome, called an event. For the bag of marbles, one event might be drawing a green marble out of the bag. Each result where the event occurs is called a favorable outcome.

For the bag of marbles below, there are 7 favorable outcomes for the event of drawing a green marble because there are 7 green marbles in the bag.

Probability is a measure of how likely an event is to occur, when all outcomes are equally likely. For instance, each marble in the bag is an outcome and is equally likely to be drawn as any other marble. When all outcomes are equally likely, the probability (P) of the event is expressed as a ratio:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

This is more properly called the theoretical probability because it is a measure of what we *think* will happen, but not necessarily what will happen in the experiment.

This might help!

"*P*" stands for probability, and whatever is in parentheses is the event that we want to find the probability of. If we were discussing the probability of drawing a red marble, we would call it *P*(red).

Example:

What is the probability of drawing a green marble from the bag?

Solution:

We will find the ratio of the number of favorable outcomes to the total number of outcomes for the event.

Favorable outcomes – 7

Counting the green marbles, we can see that there are 7 in the bag.

Total outcomes – 10

Counting all of the marbles, we can see that there are 10 marbles in the bag.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{7}{10}$$

Because probability is expressed as a ratio, we can also write it as a percent. Remember, to change a fraction to a percent, rewrite the fraction with a denominator of 100 since percent is a ratio out of 100.

To get a denominator of 100, multiply both the numerator and denominator by 10.

$$\frac{7 \times 10}{10 \times 10} = \frac{70}{100} = 70\%$$

Therefore, there is a $\frac{7}{10}$, or 70% chance of drawing a green marble from the bag.

Probability will always be a number from 0 to 1, or a percent from 0 to 100. The closer the probability is to 1, the more likely the event will occur. We can show this relationship on a number line:

0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
P = 0% Impossible	P = 25% Unlikely	P = 50% Equally likely/unlikely	P = 75% Likely	P = 100% Certain

Events that have less than a 50% probability are less likely to occur. Events that have more than a 50% probability are more likely to occur.

Because we have a 70% chance of drawing a green marble from the bag, we could say that it is likely that we would draw a green marble from the bag. How likely is it that we would draw a blue marble?

S-T-R-E-T-C-H

Can you think of an event where the likelihood would be *certain*? Can you think of an event where the likelihood would be *impossible*? If a bag held only red marbles, you would be certain to draw a red marble. It would be impossible to draw a green marble.

Example:

What is the probability of drawing a blue marble from the bag?

Solution:

We will find the ratio of the number of favorable outcomes to the total number of outcomes for the event.

Favorable outcomes – 1

We can see that there is 1 blue marble in the bag.

Total outcomes – 10

There are 10 marbles in the bag, as there were before.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{10}$$

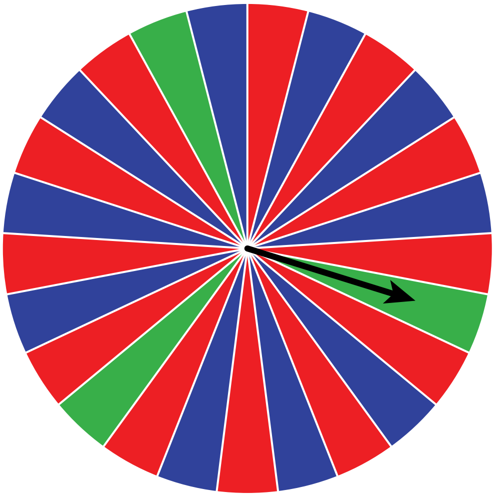
We can write the ratio as a percent.

Multiply by $\frac{10}{10}$ to get a denominator of 100.

$$\frac{1 \times 10}{10 \times 10} = \frac{10}{100} = 10\%$$

Therefore, there is a $\frac{1}{10}$, or 10% chance of drawing a blue marble from the bag. So, we would say that is very *unlikely* that a blue marble would be drawn from the bag.

Let's take a look at a spinner, it has 25 equal sections. What is the probability of the spinner landing on blue? Red? Green?



We will find the ratio of the number of favorable outcomes to the total number of outcomes for each event.

We know that there are 25 sections in the spinner, so there will be 25 total outcomes for each event.

Blue

Favorable outcomes– 10

Counting the blue sections, we can see that there are 10.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{10}{25}$$

We can write the ratio as a percent.

Multiply by $\frac{4}{4}$ to get a denominator of 100.

$$\frac{10 \times 4}{25 \times 4} = \frac{40}{100} = 40\%$$

There is a 40% chance of the spinner landing on blue. So, we would say that is unlikely that the spinner will land on blue.

Red

Favorable outcomes – 12

Counting the red sections, we can see that there are 12.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{12}{25}$$

We can write the ratio as a percent.

Multiply by $\frac{4}{4}$ to get a denominator of 100.

$$\frac{12 \times 4}{25 \times 4} = \frac{48}{100} = 48\%$$

There is a 48% chance of the spinner landing on red, almost 50%. So, we could say that is almost equally likely as unlikely that the spinner will land on red.

Green

Favorable outcomes – 3

Counting the green sections, we can see that there are 3.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{3}{25}$$

We can write the ratio as a percent.

Multiply by $\frac{4}{4}$ to get a denominator of 100.

$$\frac{3 \times 4}{25 \times 4} = \frac{12}{100} = 12\%$$

There is a 12% chance of the spinner landing on green. So, we could say that is very unlikely that the spinner will land on green.

Let's Review!

Before going on to the practice problems, make sure you understand the main points of this lesson.

- ✓ Theoretical probability is a ratio of the favorable outcomes of an event to the total number of outcomes.
- ✓ Probability will always be between 0 and 1, or 0% and 100 %.
- ✓ The closer the probability is to 1, or 100%, the more likely the event is to occur.

Match the following items.

1.1 ________ one of the numerals from 0 to 9 — a. digit

________ an approximate value close to the actual value — b. estimate

________ the position of a digit in a number, which determines its value — c. place value

Circle each correct answer.

Use this bag of marbles for questions 1.2 - 1.5.

1.2 What is the probability of drawing a red marble?

a. $\frac{5}{5}$ b. $\frac{3}{5}$

c. 50% d. 30%

1.3 What is the probability of drawing a blue marble?

a. $\frac{1}{4}$ b. 20%

c. $\frac{2}{5}$ d. 30%

1.4 What is the probability of drawing a green marble?

a. 40% b. $\frac{3}{8}$ c. 38% d. $\frac{3}{10}$

1.5 What is the likelihood of drawing a green marble?

a. likely b. equally likely as unlikely

c. unlikely d. very unlikely

1.6 A classroom teacher randomly draws the name of a student. If the teacher has between 20 and 30 students in her class, what is a good estimate of the probability of a specific student's name being drawn?

a. 50% b. 5% c. 75% d. 0%

1.7 There are 40 outcomes for an event. If it is very likely the event will occur, how many favorable outcomes would there be?

a. 37 b. 5 c. 40 d. 21

Check each correct answer (you may select more than one answer).

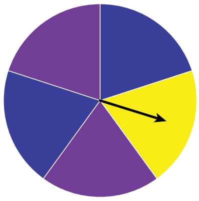
1.8 If this spinner is spun, what is the probability the spinner will land on yellow?

☐ 15% ☐ 20% ☐ $\frac{1}{4}$ ☐ $\frac{1}{5}$

1.9 Which color has a 40% probability of being spun?

☐ purple ☐ yellow

☐ blue ☐ none of the colors



COMPLEMENTARY EVENTS

Here is a bag of marbles that contains red marbles and blue marbles. If you know that the probability of drawing a blue marble is $\frac{5}{12}$, can you find the probability of drawing a red marble?

In this lesson, you will see how the events of drawing a red marble and drawing a blue marble are related. You'll also learn how to find the probability of events when one or the other event must occur.

Let's take a look inside the bag of marbles and see if we can find the probability of drawing a red marble.

Remember, to find the probability of an event, we need to find the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

Counting the marbles in the bag, there are 24. So, there will be 24 total outcomes. Now, we can find the probability of each event.

Blue

There are 10 blue marbles in the bag, so there are 10 favorable outcomes.

Divide by $\frac{2}{2}$ to simplify the ratio.

$$\frac{10 \div 2}{24 \div 2} = \frac{5}{12}$$

Red

There are 14 red marbles in the bag, so there are 14 favorable outcomes.

Divide by $\frac{2}{2}$ to simplify the ratio.

$$\frac{14 \div 2}{24 \div 2} = \frac{7}{12}$$

So, the probability of drawing a blue marble is

$$\frac{5}{12}$$

and the probability of drawing a red marble is

$$\frac{7}{12}$$

How does knowing the probability for drawing a blue marble help us find the probability for drawing a red marble?

Notice that if we add the two probabilities, they add to 1:

$$\frac{5}{12} + \frac{7}{12} = \frac{12}{12} = 1$$

Key point!

When it was stated that the probability of drawing a blue marble was $\frac{5}{12}$, this does not necessarily mean that there are 12 marbles in the bag and that 5 are blue. It means that the ratio of blue marbles to all marbles is $\frac{5}{12}$. For every 12 marbles, 5 are blue.

When we have two events, where one or the other must occur (we can only draw blue *or* red), and they have no outcomes in common (none of the blue marbles are also red, and vice versa), their probabilities will always add to 1, or 100%. These are called **complementary events**. The events are complements of each other.

So, if we know two events are complementary, and we know the probability of one event, we can find the probability of its complement. Since we know what the probability of drawing a blue marble is, we can use this information to find the probability of drawing a red marble.

$P(\text{blue}) + P(\text{red}) = 1$

To find the probability of $P(\text{red})$, subtract $P(\text{blue})$ from both sides of the equation.

$P(\text{red}) - P(\text{blue}) = 1 - P(\text{blue})$

Substitute $\frac{5}{12}$ for the probability of drawing a blue marble.

$P(\text{red}) = 1 - \frac{5}{12}$

$P(\text{red}) = \frac{7}{12}$

The probability of drawing a red marble is $\frac{7}{12}$.

We could say that the probability of drawing a red marble is the same as the probability of *not* drawing a blue marble, since we can only draw blue or red.

$P(\text{blue}) + P(\text{not blue}) = 1$

In general, we write the sum of complementary events as:

$P(\text{event}) + P(\text{not event}) = 1$

The probabilities add to 1 because they account for all of the possible outcomes. Every outcome is either the event, or not the event.

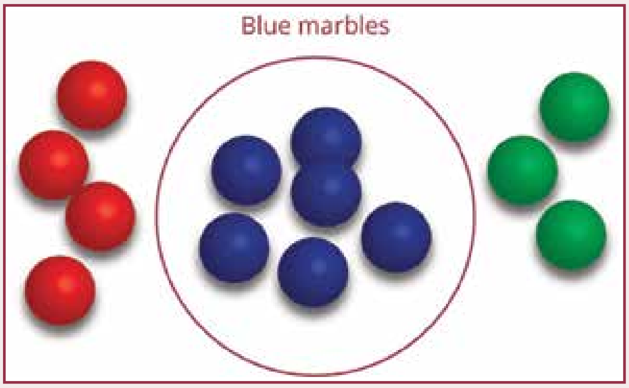
Let's look at a few examples.

Vocabulary

"Complimentary" is when someone says something kind. Outside of math, "complementary" means that two things go together. Our probability definition is similar, but the events go together to form one whole.

Connections

Think of a Venn diagram to picture complementary events:



The marbles in the circle are blue. The marbles outside of the circle are not blue.

Example:

Which of the following events are complementary?

1. Rolling a 2 or rolling a 5 on a 6-sided number cube.
2. Rolling an odd number or rolling an even number on a 6-sided number cube.
3. Randomly choosing a vowel from the alphabet, or randomly choosing a consonant from the alphabet.
4. Choosing heads for a coin flip, or choosing tails for a coin flip.

Solution:

For each pair of events, we will check to see if all outcomes are either the first event or the second event (not the first event).

1. The outcomes for rolling a 6-sided number cube are 1, 2, 3, 4, 5, or 6. Rolling a 2 or a 5 does not account for all of the outcomes. Rolling a 2, or *not* rolling a 2, would be complementary.
2. The outcomes for rolling a 6-sided number cube are 1, 2, 3, 4, 5, or 6. All of these outcomes are odd or even (not odd), so the events are complementary.
3. The outcomes are the letters of the alphabet. All letters are either vowels or consonants (not vowels), so the events are complementary.
4. A coin can be heads or tails only, so the events are complementary.

Example:

In a school, 12% of the students are first graders. What is the probability that a randomly chosen student will not be a first grader?

Solution:

The students are either first graders or not first graders, so the events are complementary. We know that the two probabilities add to 1, or 100%.

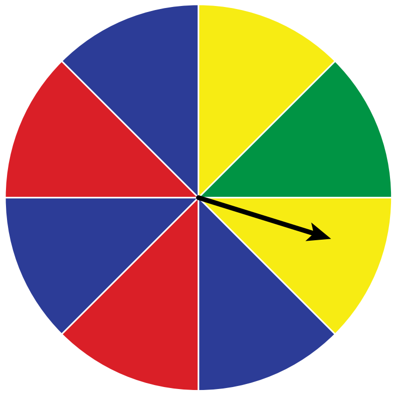
P(event) + P(not event) = 100%

P(1st) + P(not 1st) = 100%	To find the probability of *P*(red), subtract *P*(blue) from both sides of the equation.
P(not 1st) = 100% - P(1st)	Subtract *P*(1st) from each side of the equation.
P(not 1st) = 100% - 12%	
P(not 1st) = 88%	Subtract.

So, the there is an 88% probability that a randomly chosen student will not be a first grader.

Example:

Find the probability that the spinner will land on red, green, or yellow, but not blue.



Solution:

We can look at the experiment as complementary events. All of the outcomes that are *not* red, green, or yellow, are blue. So, we can find the probability of the spinner landing on blue, and then find its complement.

Blue

Favorable outcomes - 3

There are three blue sections on the spinner.

Total outcomes – 8

We were told that the spinner has eight sections, so there are eight total outcomes.

$P(\text{event}) = \dfrac{\text{Favorable outcomes}}{\text{Total outcomes}}$

Not Blue (red, yellow, green)

$P(\text{event}) + P(\text{not event}) = 1$

$P(\text{blue}) + P(\text{not blue}) = 1$

$P(\text{not blue}) = 1 - P(\text{blue})$ Subtract $P(\text{1st})$ from each side of the equation.

$P(\text{not blue}) = 1 - \frac{3}{8}$

$P(\text{not blue}) = \frac{5}{8}$ Subtract.

So, the probability that the spinner will land on red, yellow, or green (not blue) is $\frac{5}{8}$.

Let's Review!

Before going on to the practice problems, make sure you understand the main points of this lesson.

- ✓ The probabilities of complementary events will always add to 1 or 100%.
- ✓ $P(\text{event}) + P(\text{not event}) = 1$

Circle each correct answer.

1.10 How many outcomes are there in common in complementary events?

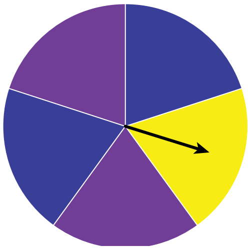
a. 0 b. 1 c. 2 d. all outcomes

1.11 If this spinner is spun, what is the probability that the spinner will *not* land on yellow?

a. 45% b. 80% c. $\frac{1}{4}$ d. $\frac{1}{5}$

1.12 A spinner has many equal sections of different colors. The probability that the spinner will not land on a blue section is 34%. What is the probability that the spinner will land on a blue section?

a. 68% b. 34%
c. 66% d. can't be determined



Use this bag of marbles for questions 1.13 - 1.15.

1.13 What is the probability of *not* drawing a red marble from this bag?

a. $\frac{5}{5}$ b. $\frac{3}{5}$

c. 50% d. 30%

1.14 What is the probability of *not* drawing a blue marble?

a. $\frac{1}{5}$ b. 20%

c. $\frac{2}{5}$ d. $\frac{4}{5}$

1.15 What is the probability of drawing a red or blue marble?

a. $\frac{3}{7}$ b. 70% c. 40% d. $\frac{3}{10}$

Place a check mark next to each correct answer (check all that apply).

1.16 What is the complementary event to drawing a blue marble from the bag?

- ☐ drawing a red marble
- ☐ drawing a green marble
- ☐ drawing a red or green marble
- ☐ not drawing a blue marble

1.17 A spinner has 7 equal sections which are numbered from 1-7. Which of the following are complementary events?

- ☐ Spinning an odd number, or spinning an even number
- ☐ Spinning 4, or not spinning 4
- ☐ Spinning 1, 2, 3, or spinning 4, 5, 6
- ☐ Spinning 1, 2, 4, or spinning 3, 5, 6, 7

1.18 In a school, the probability is 44% that a student chosen at random will be a boy. What is the probability that the student will be a girl?

- ☐ 56%
- ☐ 44%
- ☐ $\frac{14}{25}$
- ☐ $\frac{33}{50}$

Answer true or false.

1.19 ______________ Two events are complementary if they have the same probability.

SAMPLE SPACE

If we flip a coin, it can only land on heads or tails, so the probability is, $\frac{1}{2}$ or 50% that the coin will land on heads. But, what if 2 coins are flipped? What is the probability that both coins will land on heads? Is it still $\frac{1}{2}$?

As more events are involved in an experiment, finding the probability gets complicated. In this lesson, we will explore what happens when two events occur at the same time or happen one after the other. These are called **compound events**. We will learn how to find the probability of these events.

We know that to find the probability of an event, we need to find the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

This idea still applies even if two or more events happen at the same time (like flipping coins). If the outcome of one event does not affect the outcome of the other event, they are called independent events. Flipping two coins are independent events because they are happening at the same time, and the results of each coin flip do not affect the other coin.

If we flip two coins at once, we can find the probability of both coins landing on heads if we can find all of the outcomes, because then we can compare the number of favorable outcomes to total outcomes. We will make a systematic list of all the combinations of the first event (flipping a coin) and the second event (flipping another coin).

Key point!

The list of outcomes for a single event is also called the sample space. The sample space for rolling a six-sided number cube is: 1, 2, 3, 4, 5, and 6. However, the term sample space is usually used when referring to compound events.

Let's look at the possibilities if the first coin lands on heads. The second coin could be heads, or tails:

Let's look at the possibilities if the first coin lands on tails. The second coin could be heads, or tails:

So, there are four different outcomes:

Heads-Heads	Heads-Tails
Tails-Heads	Tails-Tails

One of these outcomes is the favorable outcome (heads-heads). So, if two coins are flipped, the probability is one out of four that both coins will land on heads.

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

$$P(\text{Head, Head}) = \frac{1}{4}$$

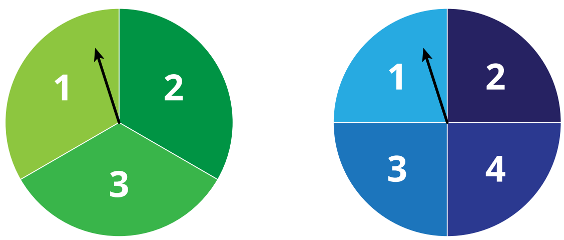
When we make an organized list of all of the outcomes for an event, it is called the **sample space**. The sample space is especially useful for independent events because it helps us see all of the different combinations.

TREE DIAGRAMS

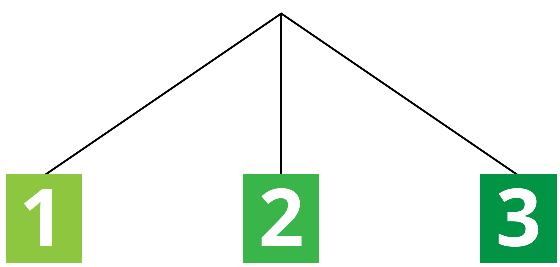
One way to organize the sample space is to make a list, as we did with flipping two coins. Making a list is handy if there are not a lot of outcomes because it can be done quickly.

Another way to organize the sample space is by using a **tree diagram**. A tree diagram is helpful when there are more outcomes to keep track of. It shows each outcome for the first event, and pairs each with all of the outcomes for the second event. This way, we know that each possible outcome is accounted for.

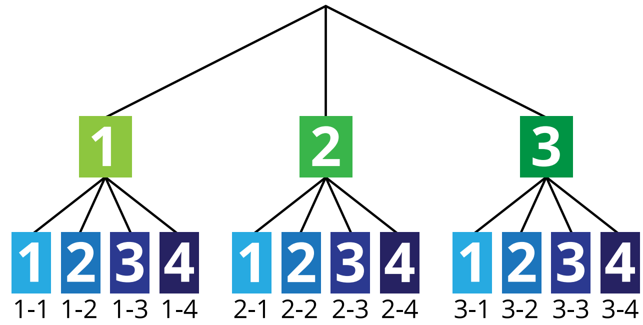
Let's see what the outcomes are if we spin the green spinner, and then the blue spinner:



The first spinner has three outcomes: 1, 2, and 3. These will be the first "branches" of the "tree."



From each outcome of the first event (spinning the green spinner) we will have a "branch" to each outcome of the second event (spinning the blue spinner): 1, 2, 3, and 4



The tree diagram shows all of the possible outcomes. If we spin a 1 on the green spinner, the outcomes for the blue spinner are 1, 2, 3, and 4. These are the outcomes 1-1, 1-2, 1-3, and 1-4. If we spin 2 on the green spinner, the outcomes for the blue spinner are still 1, 2, 3, and 4. These are the outcomes 2-1, 2-2, 2-3, and 2-4. Lastly, the green spinner can land on 3, and the outcomes for the blue spinner are still 1, 2, 3, and 4. These are the last four outcomes: 3-1, 3-2, 3-3, and 3-4. There are a total of 12 outcomes.

Let's try an example using the two spinners from the previous page.

Example:

If you spin the green spinner and then the blue spinner, what is the probability that the numbers will be the same?

Solution:

To find the probability we will find the number of favorable outcomes compared to the total number of outcomes.

Looking at the tree diagram, there are three favorable outcomes: 1-1, 2-2, and 3-3. There are 12 total outcomes in the sample space:

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

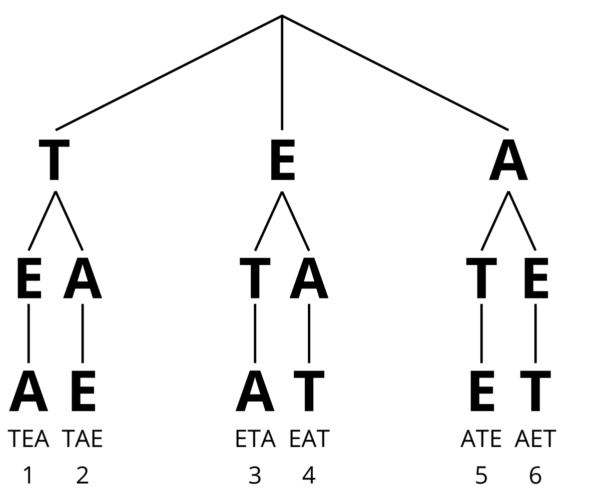
We can rewrite the fraction as a percent.

$\frac{3 \div 4}{12 \div 4} = \frac{1}{4}$ Simplify the ratio by dividing by $\frac{4}{4}$.

$\frac{1 \times 25}{4 \times 25} = \frac{25}{100}$ Multiply by $\frac{25}{25}$ to get a denominator of 100.

So, if we spin both spinners, the probability that both numbers will match is $\frac{1}{4}$, or 25%.

Tree diagrams can also be used to find all of the combinations for groups of things. For instance, how many different ways can the letters T, E, and A be arranged?



Think about it!

The number of events may determine which sample space tool to use. Since outcomes are listed along two sides of the table, tables can only be used to show outcomes for two independent events. However, tree diagrams can show outcomes for two or more events.

MAKE A TABLE

Another way to organize the sample space is to use a table. This method is useful when there are many outcomes and using a tree diagram would be too cumbersome.

A table lists the one event's outcomes along the side of the table, and the other event's outcomes along the top of the table. In the same way that a times table works, each place where the events' outcomes meet is an outcome for both events.

Let's look at the number of outcomes for rolling two 6-sided number cubes.

	1	2	3	4	5	6
1	1-1	2-1	3-1	4-1	5-1	6-1
2	1-2	2-2	3-2	4-2	5-2	6-2
3	1-3	2-3	3-3	4-3	5-3	6-3
4	1-4	2-4	3-4	4-4	5-4	6-4
5	1-5	2-5	3-5	4-5	5-5	6-5
6	1-6	2-6	3-6	4-6	5-6	6-6

Key point!

The order of the numbers in a roll matter! Notice that a roll such as 4-6 is different than a roll of 6-4. A roll of 4-6 is a 4 on the first number cube and a 6 on the second number cube. A roll of 6-4 is a 6 on the first number cube and a 4 on the second number cube.

We can see that there are 36 total outcomes, 6 rows of 6.

Let's look at an example using two 6-sided number cubes.

Example:

If you roll two 6-sided number cubes, what is the probability that the numbers will be the same?

Solution:

To find the probability, we will find the number of favorable outcomes compared to the total number of outcomes.

Looking at the table above, we can see that there are six favorable outcomes: 1-1, 2-2, 3-3, 4-4, 5-5, and 6-6.

There are 36 total outcomes in the sample space.

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{6}{36}$$

We can simplify the ratio by dividing by $\frac{6}{6}$.

$$\frac{6 \div 6}{36 \div 6} = \frac{1}{6}$$

So, if we roll two 6-sided number cubes, the probability that both numbers will match is a $\frac{1}{6}$.

Let's Review!

Before going on to the practice problems, make sure you understand the main points of this lesson.

- ✓ We can find the probability of independent events by making an organized list all of the outcomes, called the sample space.
- ✓ The sample space can be organized using a list, a table, or a tree diagram.

Match each word to its definition.

1.20 ________ an event where two or more events happen one after the other, or at the same time

________ a compound event where the likelihood of one event does not affect the other(s)

________ an organized listing of all possible outcomes for an experiment

________ an organizing tool used to find the sample space for compound events

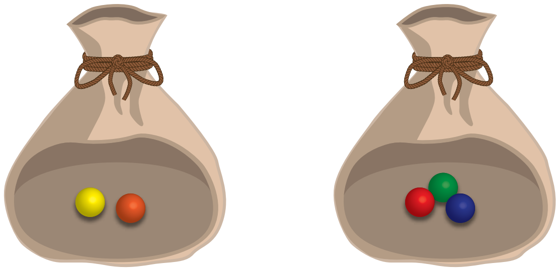
a. sample space
b. independent events
c. tree diagram
d. compound event

Circle the letter for each correct answer.

1.21 If two coins are flipped, what is the probability that both coins will land on tails?

a. 100% b. 75% c. 50% d. 25%

1.22 What is the sample space for the event of randomly choosing a marble from each of these bags?



a. yellow-green, yellow-red, yellow-blue, orange-green, orange-red, orange-blue

b. yellow-orange, red-green, yellow-red, green-blue, orange-red, yellow-blue

c. yellow-red, yellow-green, yellow-orange, yellow-blue

d. orange, yellow, red, blue, green

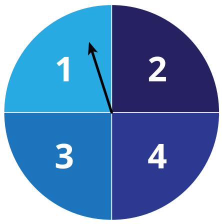
1.23 How many different ways are there to arrange the letters D, D, and A?

a. 1
b. 3
c. 6
d. 9

1.24 How many outcomes are there for the event of rolling a 6-sided number cube and then spinning a 4-part spinner?

a. 10
b. 12
c. 24
d. 64

1.25 If this spinner is spun twice, what is the probability that the numbers that are spun don't match?



a. $\frac{9}{16}$
b. 75%
c. $\frac{3}{8}$
d. 25%

1.26 If two 10-sided number cubes are rolled, which of the following sample space tools should be used to *best* show all of the outcomes?

a. a list
b. a tree diagram
c. a table
d. Venn diagram

1.27 If three independent events happen one after the other, which of the following sample space tools cannot be used?

a. a list
b. a tree diagram
c. a table
d. Venn diagram

1.28 If a coin is flipped and then a six-sided number cube is rolled, what is the probability that the outcome will be Heads and a 2?

a. 33.3%
b. $\frac{1}{12}$
c. 12%
d. $\frac{1}{8}$

PROJECT: THEORETICAL VS. EXPERIMENTAL PROBABILITY

In this section, we have looked at the probability of events. We found that the theoretical probability (P) of an event occurring is expressed as a ratio of the number of favorable outcomes to the total number of outcomes.

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

For instance, the probability of a coin landing on heads is because there is one favorable outcome (heads) out of two total outcomes (heads, tails).

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{1}{2}$$

So, if a coin is tossed 100 times, how many times will it land on heads? How many times *should* it land on heads?

In this project, you will carry out the probability experiment of tossing a coin and comparing the theoretical probability to the actual results.

THEORETICAL PROBABILITY AND EXPERIMENTAL PROBABILITY

We know that if we toss a coin, the probability that it will land on heads is the ratio. This also means that if the experiment is repeated, one out of every two **trials** should be heads. We would expect that half of the coin tosses would be heads.

Since probability is a ratio, we can find an equivalent ratio for any number of trials and predict the results. If we toss a coin 50 times, how many times would we expect to get heads?

The ratio of 1 to 2 should also be true for 50 tosses, so we can set up a proportion.Set up the proportion.

$$\frac{1}{2} = \frac{x}{50}$$

Cross multiply.

$$2x = 50$$

Divide by 2.

$$x = \frac{50}{2} = 25$$

So, we would *expect* to get 25 heads if we toss a coin 50 times. In fact, our predication for the results is often called the expected results.

Of course, in reality we probably would not get heads 25 times. But, that is the fun of probability: How close will the actual results be to the expected results?

To find out, we compare the expected results for a number of trials to the actual results for that number of trials, called the **experimental probability**:

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

One of the big ideas in probability is that the experimental probability gets closer and closer to the theoretical probability with more and more trials. In other words, it should be easier to predict results if there are a large number of trials.

In this project, you will carry out a probability experiment with an increasing number of trials, and see if the statement above is true.

COIN TOSS EXPERIMENT: EXPECTED RESULTS VS. ACTUAL RESULTS

The experiment will be to toss a coin and find the frequency of heads.

NUMBER OF TRIALS	50	100	200	400
Theoretical probability				
Expected results				
Actual results				
Difference				
Experimental probability				
Difference %				

For each column of the table:

1. Find the theoretical probability.
2. Calculate the expected number of heads.
3. Perform the number of trials, tallying the number of heads on scratch paper and recording the actual results in the table.
4. Subtract the expected results from the actual results.
5. Calculate the experimental probability.
6. Find the difference between the theoretical probability and the experimental probability.

Let's take a look at what the first column might look like:

Find the theoretical probability. We found that the theoretical probability is. We'll use that ratio to find the expected results, and we'll use percent for the probability. To find percent, we need a denominator of 100.

$$\frac{1}{2} = \frac{x}{100}$$

Multiply by $\frac{50}{50}$ to get a denominator of 100.

$$\frac{1}{2} \times \frac{50}{50} = \frac{50}{100} = 50\%$$

Calculate the expected number of heads. As we did at the start, we can set up a proportion for the number of trials.

$$\frac{1}{2} = \frac{x}{50}$$

$$x = 25$$

Perform the number of trials. Let's say we get 23 heads.

Subtract the expected results from the actual results.

$$23 - 25 = -2$$

So, we got heads 2 less times than we expected**.**

Calculate the experimental probability. We'll record this as a percent also**.**

$$\text{Exp. probability} = \frac{\text{Event occurence}}{\text{Total num. of trials}} = \frac{23}{50}$$

Multiply by $\frac{2}{2}$ to get a denominator of 100.

$$\frac{23}{50} \times \frac{2}{2} = \frac{46}{100} = 46\%$$

Find the difference between the experimental probability and the theoretical probability.

$$46\% - 50\% = -4\%$$

So, we got 4% less than we expected. The first column would look like this:

Number of trials	50
Theoretical probability	50%
Expected results	25
Actual results	23
Difference	-2
Experimental probability	46%
Difference %	-4%

Match each word to its definition.

1.29 ________ probability based on results of trials a. experimental probability

________ each time an experiment is performed b. trial

Answer the following questions.

1.30 How did the experimental probability change as the number of trials increased?

____

____

____

1.31 Were there any unexpected results? ______________________________

____

____

____

1.32 Why do you think you got the results you did? ______________________

____

____

____

____

TEACHER CHECK ________ initials ________ date

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

SELF TEST 1: PROBABILITY

Circle the letter for each correct answer (each answer, 7 points).

1.01 What is the probability of drawing a green marble from the bag #1?

a. $\frac{1}{4}$ b. $\frac{1}{2}$

c. $\frac{1}{3}$ d. $\frac{1}{6}$

1.02 What is the probability of *not* drawing a blue marble from bag #2?

a. $\frac{1}{4}$ b. $\frac{2}{3}$

c. $\frac{1}{3}$ d. $\frac{3}{4}$

1.03 What is the sum of the probability of drawing a green marble and the probability of not drawing a green marble in bag #2?

a. 100% b. 50%

c. 25% d. 0%

1.04 If a six-sided number cube is rolled, what is the probability that a three will be rolled?

a. $\frac{1}{6}$ b. $\frac{1}{4}$ c. $\frac{1}{3}$ d. $\frac{1}{2}$

1.05 If the probability of an event is $\frac{1}{4}$, and there are 28 total outcomes, how many favorable outcomes are there?

a. 1 b. 4 c. 7 d. 28

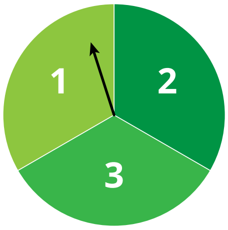
1.06 If the probability of an event is 26%, what is the probability of its complement occurring?

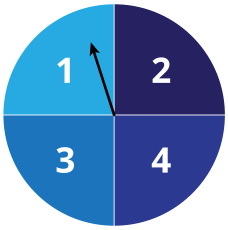
a. 100% b. 74% c. 26% d. 24%

1.07 Which percent would represent an event that is very unlikely?

a. 52% b. 82%

c. 12% d. 41%





1.08 If the green spinner is spun and then the blue spinner is spun, creating a two-digit number, what is the probability that the resulting number will be 14 or less?

a. $\frac{1}{6}$ b. $\frac{1}{4}$

c. $\frac{1}{3}$ d. $\frac{1}{2}$

1.09 If two 6-sided number cubes are rolled, what is the probability that a 2 is rolled on the first cube and a 5 is rolled on the second cube?

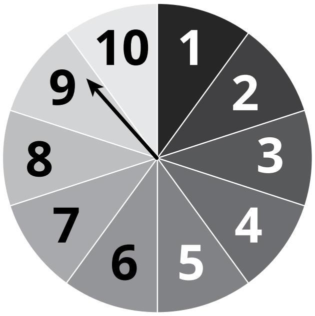
a. $\frac{1}{6}$ b. $\frac{1}{12}$ c. $\frac{1}{18}$ d. $\frac{1}{36}$

1.010 If two coins are flipped, what is the probability that both coins will not land on heads?
a. 0% b. 25% c. 50% d. 75%

1.011 What is the sample space for the event of flipping a coin and then spinning a four-part spinner, numbered 1, 2, 3, 4?
a. H-1 H-2 H-3 H-4
b. H-1 H-2 H-3 H-4 T-1 T-2 T-3 T-4
c. H-1 T-2 H-3 T-4
d. 1-H 2-H 3-H 1-T 2-T 3-T

1.012 How many different ways are there to arrange the letters A, B, A, and B?
a. 4 b. 6 c. 8 d. 16

1.013 If this spinner is spun, what is the probability that the spinner will land on a multiple of 4?
a. 40% b. $\frac{1}{4}$
c. 20% d. $\frac{1}{10}$



Place a check mark next to each correct answer (you may select more than one answer, this question 7 points).

1.014 Which of the following events is impossible with the above spinner?
☐ Spinning an odd number.
☐ Spinning a number that is neither odd nor even.
☐ Spinning a number greater than 1.
☐ Spinning a number less than 1.

Answer true or false (this question, 2 points).

1.015 ___________ The probability of an event must be expressed as a percent.

80/100 **SCORE** ___________ **TEACHER** ___________ ___________
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