LIFEPAC®

MATH

STUDENT BOOK

7th Grade | Unit 2

Alpha Omega PUBLICATIONS

Math 702
Fractions

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Fractions

Introduction

In this unit, students will learn about the set of numbers that falls between whole numbers. These numbers are known as fractions. Students will learn about the different parts of a fraction, as well as the different types of fractions, such as proper fractions, improper fractions and mixed numbers. Once students are able to identify the different types of fractions, they will learn how to calculate equivalent fractions and simplify, or reduce, fractions. Finally, they will learn how to compute with fractions and mixed numbers.

Objectives

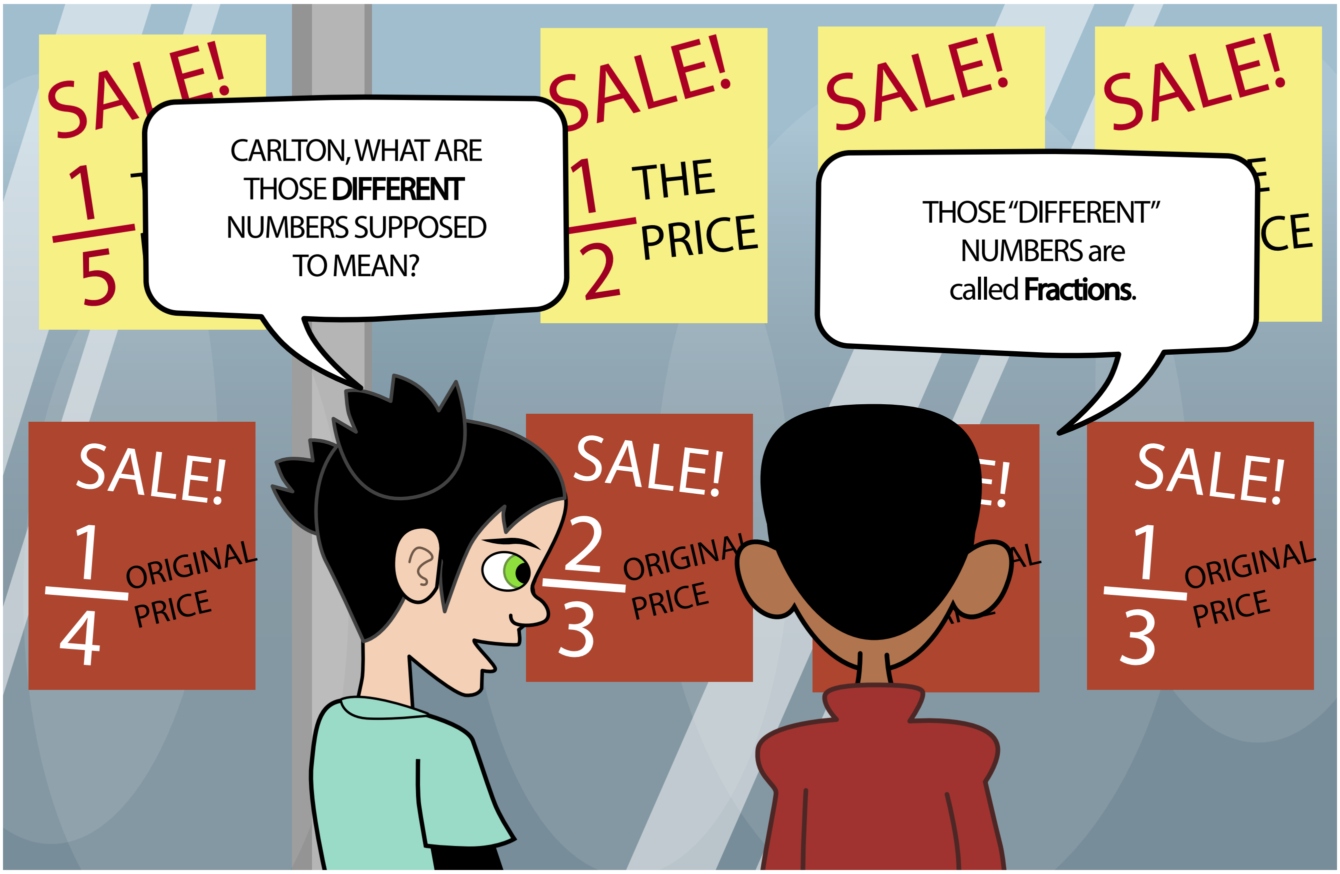
Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Identify parts of fractions and mixed numbers.
- Identify the different types of fractions.
- Perform operations with fractions and mixed numbers.
- Simplify fractions.
- Compare and order fractions.
- Find equivalent fractions.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. Working with Fractions

Fractions and Mixed Numbers



Objectives

- Identify the different parts of fractions and mixed numbers.
- Convert between mixed numbers and improper fractions.
- Round fractions and mixed numbers.

Vocabulary

denominator—the number under the fraction line; tells how many equal parts the whole was broken into

fraction—a number that shows part of a whole

improper fraction—a fraction in which the numerator is larger than the denominator

mixed number—a number with an integer part and a fraction part

numerator—the number above the fraction line; tells how many parts of the whole exist

simplified fraction—a fraction written in lowest terms (i.e., the numerator and denominator do not have any more common factors)

Proper Fractions

Did you know that every time you eat pizza, you are entering the world of *fractions*? Fractions are used to show part of a whole. Pizza is a great example of the use of fractions. Take a look.

The pizza is considered to be one whole, or 1. The one pizza can then be cut into smaller pieces so that it can be divided among several people more easily. What if the pizza is cut into eight equal slices?

Fractions consist of two numbers: a *numerator* and a *denominator*. The denominator is the bottom number of the fraction. The denominator tells how many equal parts the whole has been divided into. The pizza was cut into eight equal pieces, so the denominator is 8.

The numerator, or top number, tells you how many pieces of the whole are being talked about. For example, suppose you eat three slices of pizza. You can represent the three slices of pizza you ate by writing the fraction $\frac{3}{8}$. This tells someone who is reading the fraction that you had three of the eight slices of pizza that were available.

Take a look at another example. The picture to the right shows five friends. Can you think of some fractions that you can use to describe the five friends? You could say that $\frac{3}{5}$ of the friends have brown hair. You could also say $\frac{2}{5}$ of them are wearing white shoes.

Fractions are also used in measurements. You might need to use a fraction to describe the length, width, or height of an object. Recipes also commonly involve fractions. Measurements and recipes often use what are called benchmark fractions, or fractions that are very common. The most common denominators are halves (2), thirds (3), fourths (4), and eighths (8). Understanding how to write and interpret fractions will help you outside of math class.

Mixed Numbers

Have you ever seen a whole number followed by a fraction? When you see a whole number followed by a fraction, you are looking at a mixed number. Mixed numbers are used to explain a relationship that is larger than a whole.

You will encounter mixed numbers in many real-world applications. Maybe you have seen mixed numbers used in recipes or baseball statistics. Either way, it is helpful to understand what they mean.

The mixed number $1\frac{3}{8}$ tells you that you have a little more than 1. Take a look at what this looks like in terms of pizza.

Suppose that $1\frac{3}{8}$ pizzas have pepperoni on them.

As you can see from the photo, $1\frac{3}{8}$ is equal to one whole pizza with pepperoni and then $\frac{3}{8}$ of a second pizza with pepperoni.

What do you think the mixed number $1\frac{7}{8}$ looks like in terms of pepperoni pizza? If $1\frac{7}{8}$ of the pizzas had pepperoni on them, you would see one entire pizza with pepperoni and then 7 out of 8 slices of the second pizza with pepperoni. It would look like the following picture.

Mixed numbers can have any number as their whole number, not just 1 as you saw in the previous example. See what it would look like if $2\frac{5}{8}$ pizzas had pepperoni on them.

As you can see from the examples, the number in front of the fraction, or the whole number, represents the number of wholes while the fraction tells you how many parts of the next whole are being talked about.

Improper Fractions

An *improper fraction* is a fraction that has a numerator that is larger than the denominator. Improper fractions are used to describe items that are larger than one whole.

Example:

- $\frac{11}{8}$

The improper fraction of $\frac{11}{8}$ has a numerator of 11 and a denominator of 8. This fraction means that you have 11 parts of something that contains 8 parts in a whole. How can you take 11 of something if it only has 8 parts? There are two ways to look at it.

You can see that $\frac{11}{8}$ is really 1 whole and $\frac{3}{8}$ of another whole.

In fact, you can change the improper fraction into a mixed number first. It is very easy to convert an improper fraction into a mixed number; it's just a simple division problem.

If you divide 8 into 11 you can see, $\frac{11}{8}$ is equivalent to $1\frac{3}{8}$. It doesn't matter if you change an improper fraction to a mixed number or if you leave it in improper form. It means the same thing, but it is important to understand it both ways.

Example:

- Convert $2\frac{5}{8}$ into a mixed number.

Solution:

- The denominator 8 tells you each whole has been divided into 8 parts. The number 2 tells you there are 2 wholes. Since each whole can be divided into 8 parts, you have 2(8) = 16 parts from the 2 wholes. The numerator of the fraction, 5, tells you there are an additional 5 pieces. You now have 16 + 5 = 21 total pieces.

- To express this as an improper fraction, write the total number of pieces (21) as the numerator and the number of pieces per whole (8) as the denominator. $\frac{21}{8}$

Notice that this is the same thing as multiplying the denominator by the whole number and then adding the numerator to get the new numerator. The denominator stays the same.

Sometimes improper fractions convert into whole numbers.

Example:

- Convert $\frac{12}{4}$ into a whole number.

Solution:

- This fraction says that you have twelve things being put into groups of four:

- There are twelve stars in the picture above. Now see how many groups of four you can make:

- You can see that there are three groups of four with nothing left over. This means that you can express $\frac{12}{4}$ as 3.

You can also convert mixed numbers into improper fractions. Multiply the whole number by the denominator and then add the numerator to the product. Finally, put the sum over the existing denominator. Take a look at the following problem to help you better understand this concept.

$$1\frac{3}{5}$$

$$= 1 \cdot 5 + 3$$

$$= 8$$

$$1\frac{3}{5} = \frac{8}{5}$$

Rounding Fractions

Now that you understand proper fractions, improper fractions, and mixed numbers, you can begin to use them to describe things. But first, it will also be helpful for you to understand how to round fractions. By knowing how to round fractions, you can estimate with them. (You will learn more about estimating fractions later.)

The first thing you need to determine when rounding fractions is if the fraction is closest to 0, $\frac{1}{2}$, or 1.

Remember, you can always draw a picture of the fraction to help you decide where best to round the fraction to.

Let’s Review

Before moving on to the practice problems, make sure you can do each of the following:

- Identify proper fractions, improper fractions, and mixed numbers.
- Identify the numerator and the denominator of a fraction.
- Convert between improper fractions and mixed numbers.
- Round fractions and mixed numbers to the nearest number: 0, $\frac{1}{2}$, or 1.

Complete the following activities.

1.1 The number $\frac{5}{7}$ can be best described as a(n) _____.

☐ proper fraction ☐ improper fraction ☐ mixed number

1.2 Which fraction has a numerator of 9 and a denominator of 13?

☐ $\frac{13}{9}$ ☐ $\frac{9}{13}$

1.3 Which number is equivalent to the fraction $\frac{15}{7}$?

☐ $\frac{7}{15}$ ☐ $1\frac{2}{7}$ ☐ 2 ☐ $2\frac{1}{7}$

1.4 Which of the following most closely rounds to the number 1?

☐ $\frac{1}{3}$ ☐ $\frac{3}{6}$ ☐ $\frac{8}{9}$ ☐ $\frac{6}{13}$

1.5 A mixed number can have the same value as an improper fraction.

○ True
○ False

1.6 Which group contains numbers that *all* round most closely to $\frac{1}{2}$?

☐ $\frac{1}{5}, \frac{2}{9}, \frac{4}{8}$ ☐ $\frac{3}{5}, \frac{2}{4}, \frac{4}{7}$ ☐ $\frac{3}{4}, \frac{5}{6}, \frac{7}{9}$ ☐ $\frac{1}{2}, \frac{2}{7}, \frac{9}{10}$

1.7 Which number is equivalent to the fraction $\frac{14}{5}$?

☐ $2\frac{4}{5}$ ☐ 2 ☐ 3 ☐ $1\frac{9}{5}$

1.8 Which of the following is an example of an improper fraction?

☐ 3 ☐ $\frac{9}{5}$ ☐ $\frac{5}{9}$ ☐ $1\frac{1}{2}$

Complete the following problems in the space provided.

1.9 Convert $1\frac{2}{5}$ to an improper fraction.

1.10 Convert $4\frac{2}{7}$ to an improper fraction.

1.11 Convert $\frac{12}{5}$ to a mixed number.

1.12 Convert $\frac{9}{2}$ to a mixed number.

1.13 Convert $\frac{21}{8}$ to a mixed number.

Equivalent Fractions

Take a second and think about the words "hi" and "hello." One of the first things that may come to your mind is that they are two different words, but they mean the same thing. You can probably think of a few more examples of different words that mean the same thing. This also happens with fractions. You can have two different fractions that actually mean the same thing.

This lesson will help you understand how the same fraction can be written into other forms without changing its value.

Objectives

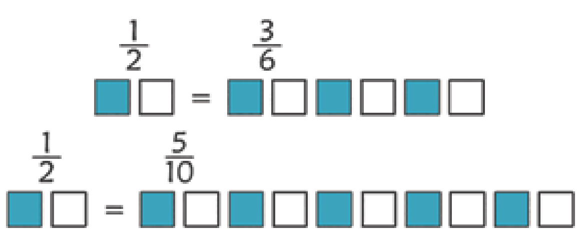
- Identify equivalent fractions.
- Identify fractions written in simplest form.

Vocabulary

equivalent fractions—fractions with the same numerical value; fractions that are equal to each other

simplest form—a fraction in lowest terms

When you have two different fractions that mean the same thing, they're called *equivalent fractions*. Take a look at some fractions that are equivalent to $\frac{1}{2}$.



$\frac{2}{4}$ $\frac{6}{12}$ $\frac{7}{14}$ $\frac{10}{20}$ $\frac{17}{34}$ $\frac{49}{98}$ $\frac{50}{100}$

Even though each of the fractions have different numerators and denominators, they are all equal to one-half of the whole.

An easy way to identify equivalent fractions is to multiply both the numerator and denominator by the same number. Go through this next example to better understand how this works.

Example:

- Find an equivalent fraction for $\frac{2}{3}$.

Solution:

- The first step is to determine a number to multiply both the numerator and the denominator by. Remember that it has to be the same number. Use the number 2:
- $\frac{2}{3} \cdot \frac{2}{2} = \frac{4}{6}$
- So $\frac{4}{6}$ is an equivalent fraction for $\frac{2}{3}$.

You can find multiple equivalent fractions for every fraction. Take a look at the following example of finding three equivalent fractions for the same fraction.

Example:

- Find three equivalent fractions for $\frac{1}{4}$.

Solution:

- $\frac{1}{4} \cdot \frac{2}{2} = \frac{2}{8}$
- $\frac{1}{4} \cdot \frac{3}{3} = \frac{3}{12}$
- $\frac{1}{4} \cdot \frac{4}{4} = \frac{4}{16}$
- Three equivalent fractions for $\frac{1}{4}$ are $\frac{2}{8}$, $\frac{3}{12}$, and $\frac{4}{16}$.

Example:

- Find three equivalent fractions for $\frac{4}{5}$.

Solution:

- $\frac{4}{5} \cdot \frac{2}{2} = \frac{8}{10}$
- $\frac{4}{5} \cdot \frac{3}{3} = \frac{12}{15}$
- $\frac{4}{5} \cdot \frac{4}{4} = \frac{16}{20}$
- Three equivalent fractions for $\frac{4}{5}$ are $\frac{8}{10}$, $\frac{12}{15}$, and $\frac{16}{20}$.

Example:

- What number must go in place of the *?* to make equivalent fractions?.

$$\frac{2}{3} = \frac{?}{12}$$

Solution:

- You know the denominator of the first fraction is 3 and the denominator of the second fraction is 12. As yourself, "What times 3 equals 12?" The answer is 4. This means that 3 was multiplied by 4 to get 12. Multiply the numerator by the same number (4) to get the new numerator. 2(4) = 8

$$\frac{2}{3} = \frac{8}{12}$$

Sometimes an equivalent fraction may have a smaller numerator and denominator than the original fraction. For example, you may remember hearing or reading phrases like "put the fraction in lowest terms" or "be sure to leave the fraction in *simplest form*" or maybe even "reduce all fractions." All of these phrases mean the exact same thing. They're telling you to make sure that the fraction in your answer cannot be expressed as a smaller equivalent fraction. The best way to define *simplest form* is to say that it is a smaller equivalent fraction than the original fraction.

The key to finding the simplest form of a fraction is that instead of multiplying the numerator and denominator by the same number, you now divide both the numerator and the denominator by the same number. Remember that to keep the fractions equivalent, you have to do the same thing to the numerator that you do to the denominator. Take a look at some examples.

Example:

- Put the fraction $\frac{12}{16}$ in simplest form.

Solution:

- Begin by identifying a number that both the numerator and the denominator can be divided by. In the case of $\frac{12}{16}$, both the numerator and the denominator can be divided by 2.

$$\frac{12}{16} \div \frac{2}{2} = \frac{6}{8}$$

- You now have the fraction $\frac{6}{8}$. Again, you can see that both the numerator and the denominator can be divided by 2.

$$\frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$$

- This fraction cannot be reduced any further, so the fraction $\frac{12}{16}$ in simplest form is $\frac{3}{4}$.

You may have noticed in the very beginning that both the numerator of 12 and the denominator of 16 were divisible by another number, 4. Now see what happens if you divide by 4:

$$\frac{12}{16} \div \frac{4}{4} = \frac{3}{4}$$

The answer is still $\frac{3}{4}$, but you only had to divide once this time.

It doesn't matter if you divide the numerator and the denominator one time or multiple times. The important thing is to make sure that the fraction is in simplest form before moving on to the next problem.

Let's Review

Before moving on to the practice problems, be sure you can do each of the following:

- Identify equivalent fractions.
- Put a fraction in simplest form.

Complete the following activities.

1.14 Which fraction is not equivalent to $\frac{5}{6}$?

☐ $\frac{15}{18}$ ☐ $\frac{20}{30}$ ☐ $\frac{10}{12}$ ☐ $\frac{30}{36}$

1.15 Equivalent fractions are always larger than the original fraction.

○ True
○ False

1.16 The fraction $\frac{24}{28}$ in simplest form is _____.

☐ $\frac{6}{9}$ ☐ $\frac{48}{56}$ ☐ $\frac{12}{14}$ ☐ $\frac{6}{7}$

1.17 Which of the following fractions is an equivalent fraction for $\frac{2}{7}$?

☐ $\frac{6}{21}$ ☐ $\frac{4}{7}$ ☐ $\frac{10}{28}$ ☐ $\frac{8}{11}$

1.18 Which fraction is *not* in simplest form?

☐ $\frac{10}{21}$ ☐ $\frac{6}{7}$ ☐ $\frac{3}{5}$ ☐ $\frac{4}{6}$

1.19 Which list has three equivalent fractions for $\frac{2}{5}$?

☐ $\frac{4}{10}, \frac{6}{20}, \frac{10}{25}$

☐ $\frac{6}{15}, \frac{8}{16}, \frac{10}{25}$

☐ $\frac{4}{10}, \frac{6}{15}, \frac{8}{20}$

☐ $\frac{6}{15}, \frac{8}{20}, \frac{12}{25}$

1.20 Both the numerator and the denominator of $\frac{18}{21}$ can be divided by _____.

☐ 2 ☐ 3 ☐ 4 ☐ 6

1.21 You can only divide a numerator and denominator one time when finding the simplest form.

○ True

○ False

Write an equivalent fraction in simplest form for each fraction below.

1.22 $\frac{9}{18}$

1.23 $\frac{4}{12}$

1.24 $\frac{5}{15}$

1.25 $\frac{15}{20}$

1.26 $\frac{12}{18}$

Divisibility Rules and Prime Factorization

Have you ever looked at a large number and wished it was smaller so it wasn't so intimidating?

This lesson will show you how many, but not all, numbers can be broken down into smaller numbers, making them look less intimidating!

1,008,459,053,472

58,264,875,346,547

109,438,324,421,476

289,524,673,041

Objectives

- Factor numbers.
- Identify a number as prime or composite.
- Use a factor tree to find the prime factorization of a number.
- Identify the basic divisibility of a number.

Vocabulary

composite number—a number that has more factors than just 1 and itself
factor—a number that divides evenly into another number
factor tree—an organized way of finding the prime factorization of a number
prime factorization—the product of prime factors of a number
prime number—a number that has only two factors: 1 and itself

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Divisibility Rules

Have you ever had to share a bag of candy with your brother or sister? When you divided the candy up, did it come out *evenly*?

What does it mean to come out evenly? It means that each person gets the same amount and there is none left over. In the case of the candy, without even knowing it, you determined whether the amount of candy in the bag was *divisible* by 2. This lesson will give you some tests you can use to determine if a number is divisible (having no remainder) by another number. You will then use the divisibility test to guide you as you begin factoring numbers.

Divisibility tests are useful in factoring, adding and subtracting fractions with unlike denominators, and reducing fractions to lowest terms. They help you determine when a whole number can be divided by another whole number with no remainder. The divisibility tests that follow have been proven to work. You will have an opportunity to test them yourself and write your own.

RULE: Any even number is divisible by 2.

If a number is even, it can be divided into *two* groups with nothing left over. If the number in the ones place is 0, 2, 4, 6, or 8, the number is even, which means it can be divided by 2 with no remainder. For example, 468 is an even number. If you divide 468 by 2, the result is 234 with no remainder. Therefore, 438 is divisible by 2.

RULE: When the sum of all of the digits in a number is divisible by 3, then the number is divisible by 3.

Choose a number in which the sum of the digits is a number that is a multiple of 3. For example, test 231. First, add up the digits:

- 2 + 3 + 1 = 6
- The number 6 is divisible by 3, so 231 is divisible by 3. If you divide 231 by 3, the result is 77 with no remainder.

Reminder: A multiple of a number is the product of that number and another whole number. For example, 24 is a multiple of 1, 2, 3, 8, 12, and 24, but 1, 2, 3, 8, and 12 are not multiples of 24. The multiples of 24 are 24, 48, 72, 96, and so on:

- $24 \cdot 1 = 24$
- $24 \cdot 2 = 48$
- $24 \cdot 3 = 72$
- $24 \cdot 4 = 96$
- The number of multiples a number has is unlimited.

RULE: If the last two digits of a number are divisible by 4, then the entire number is divisible by 4.

Take a look at the number 596. To see if the entire number is divisible by 4, you just need to focus on the last two digits, 96:

- 96 ÷ 4 = 24

There is no remainder, so 596 is divisible by 4. This checks because 596 divided by 4 is 149 with no remainder.

RULE: Any whole number that has a 0 or 5 in the ones place is divisible by 5.

Any number that is a multiple of 5 will be divisible by 5. If you look at the multiples of 5, what do you notice?

- 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, ...

All the multiples of 5 have a 0 or a 5 in the ones place. For example, 34,670 has a 0 in the ones place, so it is divisible by 5. If you divide 34,670 by 5, the result is 6,934 with no remainder.

This might help! For multiples, think multiply. You're multiplying any number you choose by a whole number to get a multiple.

RULE: If the number is divisible by *both* 2 and 3, then it is divisible by 6.

This is an easy test because as long as the number is divisible by both 2 and 3, then it is divisible by 6. For example, 426 ends in an even number, so it is divisible by 2. To see if it is divisible by 3, all you have to do is add up the digits to see if their sum is divisible by 3:

- 4 + 2 + 6 = 12

Dividing 12 by 3 results in 4 with no remainder, so 426 is divisible by 3. Because 426 is divisible by both 2 and 3, it is automatically divisible by 6. If you divide 426 by 6, the result is 71 with no remainder.

RULE: If the sum of all the digits in a number is divisible by 9, then the number is divisible by 9.

This test is similar to the divisibility test for 3. Choose a number in which the sum of the digits is a number that is a multiple of 9. For example, test 8,334:

- 8 + 3 + 3 + 4 = 18

Since 18 is divisible by 9, 8,334 is also divisible by 9. If you divide 8,334 by 9, the result is 926 with no remainder.

RULE: Any whole number that has a 0 in the ones place is divisible by 10.

This test is similar to the divisibility test for 5. Any number that is a multiple of 10 will be divisible by 10. If you look at the multiples of 10, what do you notice?

- 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, ...

All the multiples of 10 have a 0 in the ones place. For example, 123,980 has a 0 in the ones place, so it is divisible by 10. If you divide 123,980 by 10, the result is 12,398 with no remainder.

Divisibility can be very helpful when trying to find the factors of a number.

Factors

All numbers are composed of smaller numbers called *factors*. Factors are numbers that can divide evenly into another number. Even the number 1 has a factor—it's 1. But you're going to look at numbers larger than 1 and see what smaller numbers make them up.

There are a couple of ways to determine the factors of a number. One way is to make a list. Take a look at some numbers and find the factors that make them up by creating lists.

Example:

- What are the factors of 24?

Solution:

- Begin by listing all pairs of numbers that can be multiplied together to get 24:
- 1 and 24
- 2 and 12
- 3 and 8
- 4 and 6
- This means that the number 24 has the factors 1, 2, 3, 4, 6, 8, 12, and 24.

Example:

- What are the factors of 27?

Solution:

- 1 and 27
- 3 and 9
- The factors of 27 are 1, 3, 9, and 27.

Example:

- What are the factors of 47?

Solution:

- 1 and 47
- The factors of 47 are 1 and 47.

Now that you know how to find factors, you can look at ways of classifying numbers as either prime or composite.

Prime and Composite Numbers

All positive numbers larger than 1 can be identified as either prime or *composite numbers*. A *prime number* is a number whose only factors are 1 and itself. A composite number is a number that has more factors than just 1 and itself. This will make more sense when you look at some examples.

Make note! A prime number has two factors: 1 and itself. A composite number has three or more factors: 1, itself, and at least one other whole number. Because the number 1 only has one factor, it is neither prime nor composite.

Begin with the number 2. The only factors that divide evenly into 2 are 1 and 2. Because the only factors the number 2 has are 1 and itself, the number 2 is considered to be a prime number.

Now look at the number 15. This number has other factors besides 1 and 15. Both 3 and 5 are also factors of 15. So 15 has the factors 1, 3, 5, and 15. Having these factors makes 15 a composite number.

Take a look at the numbers from 1 to 100 and see which ones are prime and which are composite.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

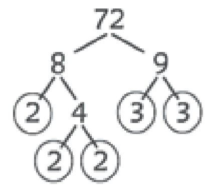
Factor Trees

Another way to find the factors of numbers is to use a *factor tree*. A factor tree is an organized diagram that breaks larger numbers into smaller and smaller numbers until each number is a prime number. When you show all the prime numbers that make up the larger number, you are showing the *prime factorization* of that number. Take a look at how factor trees are used to find the prime factorization of a number.

Example:

- What is the prime factorization of 72?

Solution:



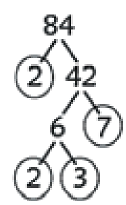
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

- The prime factorization from above can also be written as follows:
- $72 = 2^3 \times 3^2$

Example:

- What is the prime factorization of 84?

Solution:



$$84 = 2 \times 2 \times 3 \times 7$$

- The prime factorization from above can also be written as follows:
- $84 = 2^2 \times 3 \times 7$

It is helpful to know how to find factors when beginning with a factor tree. It doesn't matter if you start with different numbers than a classmate does. At the end, you will have the same prime factorization. Be sure to double check that each of your ending factors is a prime number before moving to the next branch.

You can also find prime factors of a number by repeat division. Look at the following examples using the same two numbers as the previous example.

Example:

- What is the prime factorization of 72?

Solution:

- Divide 72 by the smallest number that will divide evenly into it. In this case, the number is 2. Continue this process until you get a prime number as your answer. The list of prime numbers you have generated is the prime factorization.
- $$\begin{array}{r|l} 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$
- The prime factorization from the list of prime numbers can also be written as follows:
- $72 = 2^3 \times 3^2$

Example:

- What is the prime factorization of 84?

Solution:

- $$\begin{array}{r|l} 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array}$$
- The prime factorization can also be written as follows:
- $84 = 2^2 \times 3 \times 7$

Let's Review

Before moving on to the practice problems, be sure you can do each of the following:

- Find the factors of a number.
- Identify a number as prime or composite.
- Find the prime factorization of a number.

Complete the following activities.

1.27 Which list shows all the factors of 28?

- ☐ 1, 4, 7, 28
- ☐ 1, 2, 4, 6, 14, 28
- ☐ 1, 2, 4, 7, 14, 28
- ☐ 1, 4, 6, 28

1.28 Which list contains only composite numbers?

- ☐ 6, 24, 51, 72, 105
- ☐ 31, 44, 57, 64, 92
- ☐ 12, 37, 54, 80, 117
- ☐ 26, 53, 66, 84, 96

1.29 The number 61 is a prime number.

- ○ True
- ○ False

1.30 Which of the following isn't a composite number?

- ☐ 69
- ☐ 51
- ☐ 41
- ☐ 33

1.31 Which number is missing from the list of factors for 48? 1, 2, 4, 6, 8, 12, 16, 24, 48

- ☐ 3
- ☐ 7
- ☐ 20
- ☐ 14

1.32 Select all that apply. Choose the numbers that are factors of 50.

- ☐ 2
- ☐ 4
- ☐ 10
- ☐ 25
- ☐ 8
- ☐ 15

1.33 Select all that apply. Choose the numbers that are factors of 112.
(Hint: Use the divisibility rules to help you!)

- ☐ 2
- ☐ 4
- ☐ 7
- ☐ 3
- ☐ 6
- ☐ 8

1.34 How many factors does 144 have?

- ☐ 6
- ☐ 8
- ☐ 12
- ☐ 15

1.35 The number 12,570 is divisible by 2, 3, and 6.

- ○ True
- ○ False

1.36 What is the prime factorization of 96?

- ☐ $8 \cdot 12$
- ☐ $2^5 \cdot 3$
- ☐ $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
- ☐ $2^5 \cdot 3^2$

Complete the following activities.

1.37 What is the prime factorization of 18?

1.38 What is the prime factorization of 24?

1.39 What is the prime factorization of 36?

1.40 What is the prime factorization of 45?

1.41 What is the prime factorization of 64?

Greatest Common Factor and Least Common Multiple

In this lesson, you will compare the factors of two or more numbers. To do this, you need to be able to find the factors of a number. You must also know the difference between a factor and a multiple. Using prime factorization will help you identify the *greatest common factor* and *least common multiple* of a set of numbers.

Objectives

- Find the GCF of a set of numbers.
- Find the LCM of a set of numbers.
- Define the difference between the GCF and the LCM of a set of numbers.

Vocabulary

factor—a number that divides evenly into another number

greatest common factor—the largest factor that any given numbers have in common

least common multiple—the smallest multiple that any given numbers have in common

multiple—the product of a number and another whole number

Do you remember what the words *factor* and *multiple* mean? Take a moment to review these terms.

What is a factor? If a number divides into another number with no remainder, it is said to be a factor of that number. For example, the factors of 24 are all the numbers that divide evenly into 24: 1, 2, 3, 4, 6, 8, 12, and 24:

- $1 \cdot 24 = 24$
- $2 \cdot 12 = 24$
- $3 \cdot 8 = 24$
- $4 \cdot 6 = 24$

What is a multiple? A multiple of a number is the product of that number and another whole number. For example, the multiples of 24 are 24, 48, 72, 96, and so on:

- $24 \cdot 1 = 24$
- $24 \cdot 2 = 48$
- $24 \cdot 3 = 72$
- $24 \cdot 4 = 96$

Now that you've reviewed those terms, you can apply them to some new terms—the greatest common factor, or GCF, and the least common multiple, or LCM.

Greatest Common Factor

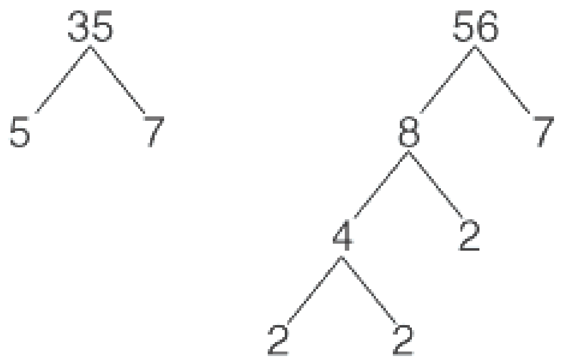
The factors of a number are all the numbers that go evenly into a number. Factoring is simply identifying or determining those numbers. The greatest common factor, or GCF, of a set of numbers is the largest factor that any given numbers have in common. There are different ways to determine the GCF. Take a look at a few methods.

Example:

- Find the GCF of 35 and 56.

Solution:

- Begin by identifying the prime factors for each of the numbers:



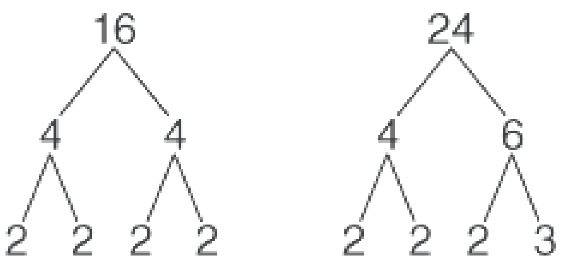
- The prime factors of 35 are 5 and 7.
- The prime factors of 56 are 2, 2, 2, and 7.
- The GCF is 7 because 7 is the only prime factor that both 35 and 56 have in common.

Example:

- Find the GCF of 16 and 24 using prime factorization.

Solution:

- First, find the prime factorization for both numbers:



- The prime factorization of 16 is $2 \cdot 2 \cdot 2 \cdot 2$.
- The prime factorization of 24 is $2 \cdot 2 \cdot 2 \cdot 3$.
- Second, identify the prime factors that both numbers have in common and multiply them. With 16 and 24, the common prime factors are $2 \cdot 2 \cdot 2$, which equals 8. So the GCF of 16 and 24 is 8.

Example:

- Find the GCF for 12, 18, and 36.

Solution:

- First, find the prime factorization for all the numbers:
- $12 = 2 \cdot 2 \cdot 3$
- $18 = 2 \cdot 3 \cdot 3$
- $36 = 2 \cdot 2 \cdot 3 \cdot 3$
- Second, identify the prime factors that all the numbers have in common and multiply them. With 12, 18, and 36, the common prime factors are $(2 \cdot 3)$, which equals 6. The GCF of 12, 18, and 36 is 6.
- Another way to find the GCF is to list all the factors of the given numbers. The largest number they have in common is the GCF.

Example:

- Find the GCF of 12 and 24.

Solution:

- List the factors of 12:
- 1, 2, 3, 4, 6, 12
- List the factors of 24:
- 1, 2, 3, 4, 6, 8, 12, 24
- The largest number in both lists is 12. So 12 is the GCF of 12 and 24.

Example:

- Find the GCF for 15 and 16.

Solution:

- Factors of 15: 1, 3, 5, 15
- Factors of 16: 1, 2, 4, 8, 16
- It's easy to see there are no common factors other than 1. This means that the greatest common factor of 15 and 16 is 1.

Make note! The number 4 multiplied by itself equals 16, but the 4 only needs to be listed once.

Now that you better understand GCF, take a look at finding the least common multiple of a set of numbers.

Least Common Multiple

The least common multiple, or LCM, of a set of numbers is the smallest number into which the numbers can be divided evenly. Another way to state this is that the LCM is the smallest multiple that any given numbers have in common.

There are different ways to determine the LCM. One method is to list all the multiples. Another method is to use prime factorization. So far, this probably sounds exactly like finding the GCF. But be careful because multiples are different than factors!

Example:

- Find the LCM of 16 and 12.

Solution:

- List the first five multiples of both numbers:
- Multiples of 16: 16, 32, 48, 64, 80, ...
- Multiples of 12: 12, 24, 36, 48, 60, ...
- The smallest number 16 and 12 have in common is 48. So the LCM is 48.

In this example it was pretty easy to find the LCM. However, with the listing method, you have to be really quick with multiplying larger numbers, or it may take a while to find the LCM. The good thing about having more than one method to choose from is that you can choose whichever one is easiest for you.

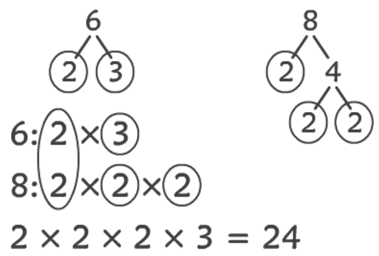
To find the LCM using prime factorization, follow these steps:

- Find the prime factorization of each number.
- Find the common prime factor(s).
- Multiply the common prime factor(s) and the extra factors.

Example:

- Find the LCM of 6 and 8 using prime factorization.

Solution:



Take a look at some more examples of finding the LCM using both methods.

Example:

- Find the LCM of 3 and 4 using a list of multiples.

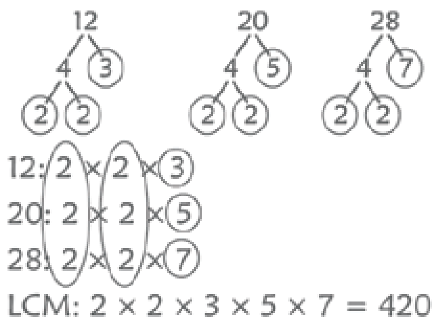
Solution:

- Multiples of 3: 3, 6, 9, 12, 15, ...
- Multiples of 4: 4, 8, 12, 16, ...
- The LCM of 3 and 4 is 12.

Example:

- Find the LCM of 12, 20, and 28 using prime factorization.

Solution:

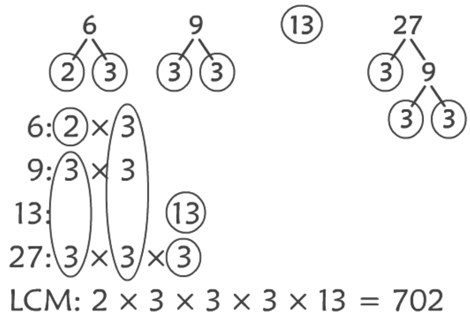


Notice how using prime factorization to identify the LCM is easier than using a list of common multiples. In the previous example, to discover that 420 is the LCM of the numbers would have required a very long list of multiples.

Example:

- Find the LCM for 6, 9, 13, and 27.

Solution:



Remember that factors are less than or equal to the given number, and multiples are greater than or equal to the given number. With this in mind, the GCF will always be less than or equal to the smallest of the original numbers, and the LCM will always be greater than or equal to the largest of the original numbers.

Example:

- Find the LCM and GCF for 12 and 18.

Solution:

- The prime factors of 12 are 2, 2, 3.
- The prime factors of 18 are 2, 3, 3.
- To find the GCF, find all prime factors that are common to both. In this case, it is 2, 3. Multiply 2(3) = 6 to find the GCF.
- To find the LCM, find the most number of times each prime factor appears in any one of the original numbers. 2 appears twice in 12 and once in 18. Because 2 is the most number of times it is found, use it twice. 3 appears once in 12 and twice in 18. Because 2 is the most number of times it is found, use it twice. The LCM is 2(2)(3)(3) = 36.

Notice that the GCF (6) is smaller than either of the original numbers and the LCM (36) is greater than either of the original numbers.

Let's Review

Before moving on to the practice problems, be sure you can do each of the following:

- Find the GCF of a set of numbers.
- Find the LCM of a set of numbers.
- Define the difference between the GCF and the LCM.

Complete the following activities.

1.42 The GCF of 28 and 42 is _____.
- ☐ 2
- ☐ 4
- ☐ 7
- ☐ 14

1.43 The GCF of two prime numbers is _____.
- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4

1.44 Which of the following pairs of numbers has a GCF of 1?
- ☐ 30 and 70
- ☐ 24 and 44
- ☐ 12 and 30
- ☐ 16 and 21

1.45 What is the LCM of 5, 13, and 10?
- ☐ 50
- ☐ 65
- ☐ 130
- ☐ 650

1.46 Find the LCM of 8 and 12.
- ☐ 8
- ☐ 12
- ☐ 24
- ☐ 96

1.47 Find the LCM of 9 and 15.
- ☐ 1
- ☐ 3
- ☐ 45
- ☐ 135

1.48 The LCM of two numbers can be equal to the smallest number in the set.
- ○ True
- ○ False

1.49 The GCF and the LCM of 16 and 24 is the same value.
- ○ True
- ○ False

Complete the following activities.

1.50 Find the GCF of 15 and 20.

1.51 Find the GCF of 8 and 24.

1.52 Find the GCF of 10 and 25.

1.53 Find the LCM of 8 and 24.

1.54 Find the LCM of 10 and 25.

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

Self Test 1: Working with Fractions

Check the box of the correct answer for each question (5 points, each numbered activity)

1.01 The number $\frac{5}{3}$ can be best described as a(n) _____.
- ☐ proper fraction
- ☐ mixed number
- ☐ improper fraction

1.02 Which number is equivalent to the fraction $\frac{18}{5}$?
- ☐ $\frac{5}{18}$
- ☐ 3
- ☐ $3\frac{3}{5}$
- ☐ $2\frac{8}{5}$

1.03 The LCM of three numbers is 48. What are the numbers?
- ☐ 2, 3, 4
- ☐ 6, 16, 24
- ☐ 3, 8, 12
- ☐ 6, 8, 12

1.04 Which of the following rounds most closely to 1?
- ☐ $\frac{2}{3}$
- ☐ $\frac{4}{8}$
- ☐ $\frac{1}{5}$
- ☐ $\frac{4}{5}$

1.05 A proper fraction never has the same value as a mixed number.
- ☐ True
- ☐ False

1.06 Which group contains numbers that *all* round most closely to 0?
- ☐ $\frac{1}{5}, \frac{2}{9}, \frac{0}{4}$
- ☐ $\frac{3}{5}, \frac{2}{4}, \frac{4}{7}$
- ☐ $\frac{3}{4}, \frac{5}{6}, \frac{7}{9}$
- ☐ $\frac{1}{2}, \frac{2}{7}, \frac{9}{10}$

1.07 Which of the following is an example of a proper fraction?
- ☐ 3
- ☐ $\frac{9}{5}$
- ☐ $\frac{5}{9}$
- ☐ $1\frac{1}{2}$

1.08 The GCF of the numerator and denominator in $\frac{15}{24}$ is _____.
- ☐ 2
- ☐ 3
- ☐ 5
- ☐ 8

1.09 Both the numerator and the denominator of $\frac{21}{24}$ can be divided by _____.
- ☐ 2
- ☐ 3
- ☐ 4
- ☐ 6

1.010 The following is the prime factorization of which composite number? $2^2 \cdot 3^2 \cdot 5$

☐ 20 ☐ 60 ☐ 120 ☐ 180

1.011 Match these items to the choices below.

________	a number to be multiplied	GCF
________	5	factor
________	6	LCM
________	greatest common factor	composite number
________	least common multiple	prime number

1.012 The number 420 is divisible by all of the following *except* _____.

☐ 3 ☐ 9 ☐ 12 ☐ 30

1.013 A composite number can be written as the product of prime numbers.

☐ True ☐ False

1.014 The number with a prime factorization of 3^4 is _____.

☐ 6 ☐ 12 ☐ 81 ☐ 243

1.015 The LCM of 15 and 30 is _____.

☐ 1 ☐ 15 ☐ 30 ☐ 45

1.016 The GCF of 64 and a number is 16. Which of the following could be the number?

☐ 16 ☐ 24 ☐ 28 ☐ 36

Complete each Activity (5 points, each numbered activity)

1.017 Write an equivalent fraction for $\frac{12}{16}$ in simplest form.

1.018 Convert $3\frac{4}{7}$ to an improper fraction.

1.019 Convert $\frac{32}{5}$ to a mixed number.

1.020 Find the GCF of 24 and 27.

1.021 Find the LCM of 9 and 15.

84/105	**SCORE** ________	**TEACHER** ________ initials ________ date

NOTES

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