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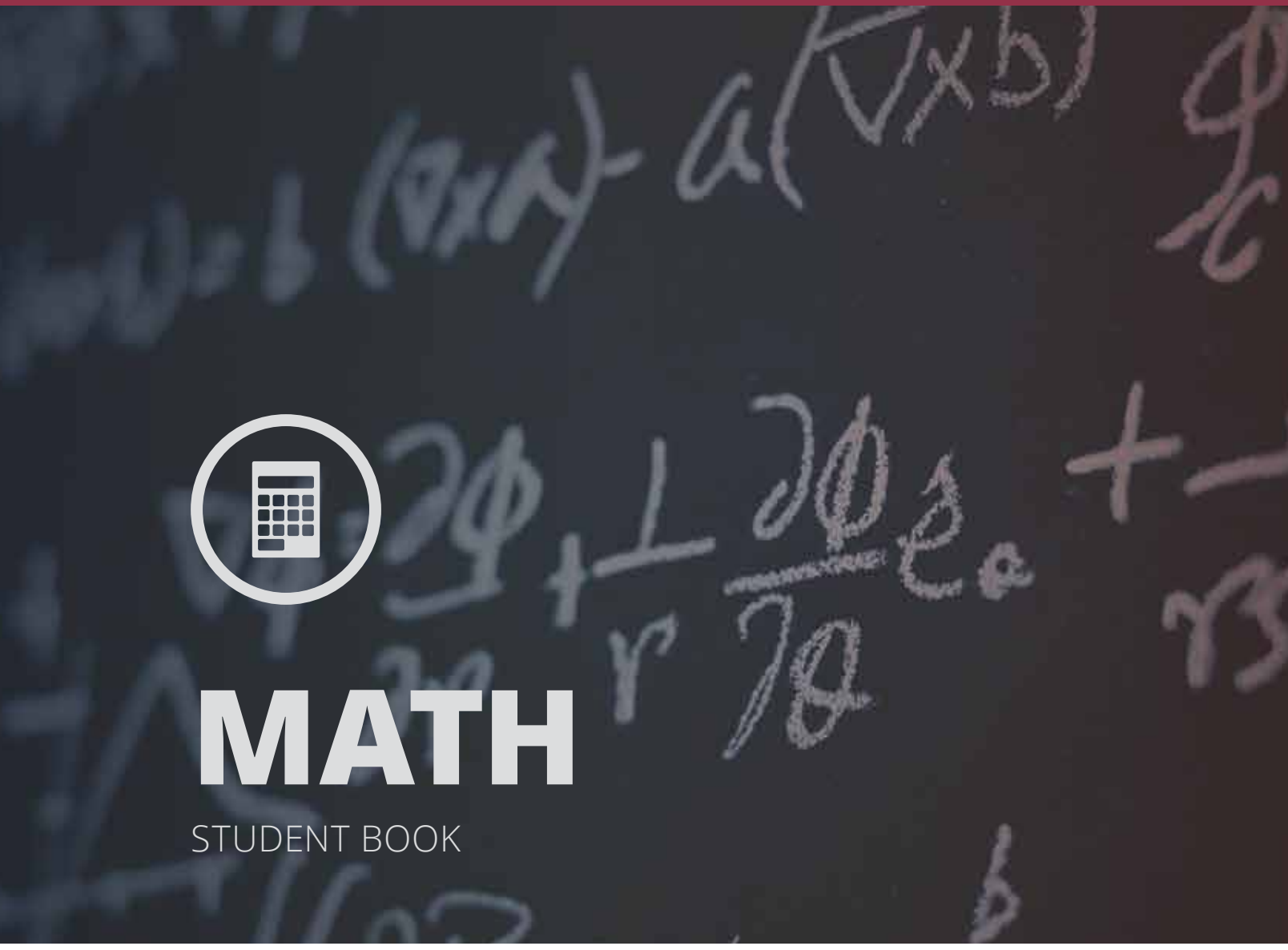

MATH

STUDENT BOOK

7th Grade | Unit 4

Alpha Omega
PUBLICATIONS

Math 704

Patterns and Equations

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Patterns and Equations

Introduction

In this unit, students will start exploring the world of algebra. They will use variables to represent unknown numbers and learn how to set up and solve equations and inequalities in order to find the value of those unknown numbers. They will also learn how to represent solutions on a graph. In addition, they will explore sequences and functions and how equations can be used to represent them.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Use variables to represent unknown numbers.
- Translate between word phrases or sentences and mathematical expressions, equations, or inequalities.
- Evaluate expressions and formulas for specific values.
- Identify arithmetic and geometric sequences and the equations that can be used to describe them.
- Identify functions and function rules.
- Solve equations using the four operations.
- Solve inequalities and graph the solution sets on a number line.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. Variable Expressions

Working with Variables and Expressions

In Ondi and Carlton's health class, they have to keep track of how many calories they eat each day for a week. Ondi forgot to record everything she ate yesterday, so now she has to go back and try to remember. Unfortunately, she's having a hard time!

Objectives

- Use a variable to represent an unknown number.
- Translate a word phrase into a mathematical expression.

Vocabulary

constant—a number; a term containing no variables
expression—a single term; multiple terms connected by an addition or subtraction sign
term—a number, a variable, or the product of a number and variable(s)
variable—a letter used to represent an unknown number

Terms and Expressions

Sometimes in math, there are situations in which you have to work with unknown numbers or numbers whose values can change. For example, in the cartoon, Ondi couldn't remember how many cookies she ate. But her total number of daily calories depended on the number of cookies she ate. So Carlton recommended that Ondi use the letter c to represent the number of cookies. That letter is called a *variable*. A variable is a letter that is used to represent an unknown number.

There are a few other vocabulary words you should look at before you continue working with variables. From the situation above, $60 \cdot c$ is called a *term*. A term can be made up of one number, also called a *constant*, or one variable. It can also be the product of a number and variables, like the term $60 \cdot c$. This could also be expressed as $60c$.

Key point! When a number and a variable, or multiple variables, are written next to each other, it means that they are multiplied together. So 12mn is the same as 12 · m · n.

Example:

- Translate the following phrase to a mathematical expression.
- a number increased by nine

Solution:

- Use the variable *n* to represent "a number." "Increased by nine" translates to addition, or + 9. So the phrase can be written as *n* + 9.
- Here are some more examples of terms.

Examples:

- 4*x*
- -9
- *y*
- 12*mn*

Make note! Notice in the terms that have both a number and a variable that the number comes first. Also, multiple variables are usually written in alphabetical order.

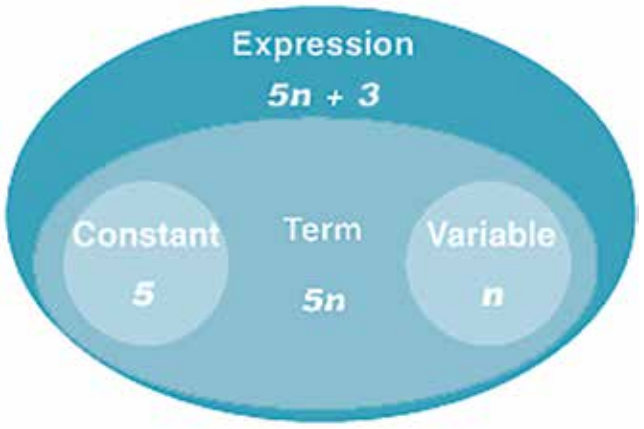
An *expression* is formed when multiple terms are connected by an addition or subtraction sign. One term by itself can also be called an expression. So actually, constants, variables, and terms are all considered expressions! Here are some examples of expressions.

Examples:

- 5
- -*n*
- 3*a*
- 12*b* + 3
- 8*g* - 2*h*

You can use a Venn diagram to show the relationship between constants, variables, terms, and expressions. Remember that in a Venn diagram, a smaller circle inside a larger circle means that every part of the smaller circle is also part of the larger circle. So every constant or variable is also considered a term. And every term is also considered an expression.

Vocabulary! A constant is any term that is just a number. A variable is any term that is just a letter.



Vocabulary! A constant is any term that is just a number. A variable is any term that is just a letter.

Translating Terms and Expressions

Did you know that math has its own language? That might sound strange, but if you think about it, math uses numbers, symbols, and variables to mean specific things, just like other languages. Being able to communicate using mathematical language is important, just like it is important to be able to communicate using verbal language. That means that you often

need to translate between word phrases and mathematical expressions in order to solve problems. Take a look at the three parts of mathematical language and ways to translate them.

The first part of mathematical language is numbers. You've been working with numbers and translating written numbers into numerals for a long time. For example, if you saw the phrase "thirty-seven," you would know that it could also be represented as "37."

The second part of mathematical language is symbols. Symbols are used to represent things like the four basic operations. Often, key words can help you in the translation of symbols. For example, you are already familiar with the terms *sum*, *difference*, *product*, and *quotient*. These are key words that tell you what operation to use. There are many other key words that can help you, too. Take a look at Figure 1, which lists words and phrases that indicate the different operations.

The final part of mathematical language is variables. Earlier in this lesson, you learned that variables represent unknown numbers. When translating, a variable is used to represent any part of the problem or phrase that you don't know the exact value of. For example, the phrase "a number" is often used to represent the unknown part and can be translated as the variable.

This might help! It really doesn't matter which letter you use as the variable. However, it is usually helpful to use a letter that matches the situation. Remember Carlton in the cartoon? He used the letter *c* to represent the unknown number of cookies that Ondi had eaten.

Look at some examples.

Example:

- Translate the following phrase to a mathematical expression.
- five less than twice a number

This might help! The words *from* and *than* tell you that you need to switch the order of the terms in the expression. For example, even though the word *five* comes first in the word phrase, the word *than* means that 5 will come after $2n$ in the expression. Since subtraction is not commutative, this will make a big difference!

Addition (+)	Subtraction (-)	Multiplication (·)	Division (÷)
sum	difference	product	quotient
increased by	decreased by	times	divided by
more than	less than	of	per
more	less	multiplied by	
added to	subtracted from	by	
plus	minus		
greater than	fewer than		

Figure 1 | Words and Phrases that indicate operations

Solution:

- Use the variable *n* to represent "a number." Twice something means to multiply by two, so twice a number is $2 \cdot n$, or $2n$. "Five less than" translates to subtraction, or - 5. So the phrase can be rewritten as $2n - 5$.

Example:

- Translate the following phrase to a mathematical expression.
- six more than half a number

This might help! The word *half* tells you that you need divide the tem by 2. Set up a fraction with the term as the numerator and the number 2 as the denominator. If the question asked for a third, you would put a 3 in the denominator. A fourth would be a 4 in the denominator, etc.

Solution:

- Use the variable *n* to represent "a number." Half of something means to divide by two, so half a number is $n \div 2$ or $\frac{n}{2}$. "Six more than" translates to addition, or +6. So the phrase can be rewritten as $\frac{n}{2} + 6$.
- Sometimes, you'll need to translate within an actual problem situation, like Carlton did. Here are a few hints that can help:
  - "Total" means to add all the parts together.
  - Splitting something up means to divide it into equal parts.
  - Any part that has an unknown value should be represented by a variable.

Example:

- Three friends share an apartment. They also share all the monthly costs of the apartment, including the rent and utilities. Write an expression that represents what each friend pays each month.

Solution:

- Start by choosing a variable to represent the value that is unknown, the total monthly cost. You can use any letter you want, but for this example, use *c* to represent the cost.
- The cost, *c*, is divided three ways each month. So every month, each of the three friends pays $c \div 3$.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Variables, constants, and terms are all types of expressions.
- Numbers and variables written right next to other variables are multiplied together.
- Mathematical language has three parts: numbers, symbols, and variables.
- Word phrases can be translated into mathematical phrases.

Complete the following activities.

1.1 Every term is _____ .

☐ constant ☐ variable ☐ expression ☐ term

1.2 The expression -7*y* is a _____ .

☐ constant ☐ variable ☐ expression ☐ term

1.3 All of the following represent the same expression *except* _____.

☐ -8*ab* ☐ -8(*a*)(*b*) ☐ 8 - *ab* ☐ -8 · *a* · *b*

1.4 Which of the following expressions represents the phrase "the quotient of a number and -5"?

☐ *n* - 5 ☐ *n* ÷ -5 ☐ -5*n* ☐ *n* + (-5)

1.5 If the cost of a raffle ticket is $2, which of the following expressions could be used to represent the cost of *m* tickets?

☐ 2*m* ☐ 2 + *m* ☐ *m* - 2 ☐ *m* ÷ 2

1.6 The cost of a raffle ticket is $2. There is also a one-time $5 fee to be part of the raffle. Which of the following expressions could be used to represent the total cost of being in the raffle and buying *m* tickets?

☐ 10*m* ☐ 5*m* + 2 ☐ 7 + *m* ☐ 2*m* + 5

1.7 Which of the following expressions represents the phrase "19 less than a number"?

☐ *n* - 19 ☐ 19 - *n* ☐ *n* ÷ 19 ☐ *n* + 19

1.8 Which of the following expressions represents the phrase "six added to five times a number"?

☐ 5 + 6*x* ☐ 6 + 5*x* ☐ 11*x* ☐ 6 + 5 + *x*

1.9 Which of the following phrases could represent the expression *k* - 4?

☐ a number subtracted from four ☐ a number decreased by four

☐ four less a number ☐ a number less than four

1.10 Dylan is six years older than his sister. If his sister's age is represented by the variable *s*, which of the following expressions represents Dylan's age?

☐ 6*s* ☐ *s* - 6 ☐ *s* + 6 ☐ 6 - *s*

Write an expression for each situation below.

1.11 If a giant candy bar costs $2, what is the cost of buying *c* candy bars?

1.12 A movie theater charges $4 for a drink and $6 for pail of popcorn. What is the total charge for *d* drinks and p popcorns?

1.13 The county fair charges $10 admission and $2 per ride. What is the cost for one admission and *r* rides?

1.14 The county fair charges $10 admission and $2 per ride. What is the cost for *a* admissions and *r* rides?

1.15 A concession stand starts the day with *h* hamburgers. How many hamburgers do they have left at the end of the day if they sold 58 hamburgers?

Translating Word Sentences

In this lesson, you'll be translating word sentences into the language of math. In order to do that, you'll have to use what you know about translating phrases.

Objectives

- Translate between word sentences and mathematical equations.
- Write an equation to represent a word problem.

Vocabulary

equation—a mathematical statement that shows two expressions are equal using an equal sign

Translating sentences is almost exactly the same as translating phrases. But there is one more symbol that you have to use: the equal sign! A word phrase translates to an expression. A word sentence translates to an *equation*. An equation is a complete thought. It tells you that two expressions are equal to each other. So an equation is two expressions joined by an equal sign.

Vocabulary! Remember that a variable, or letter, represents the value that is unknown.

Do you remember the three parts of mathematical language? They are numbers, symbols, and variables. The equal sign is a symbol. To determine which symbols to use in an expression or equation, look for key words. Key words can also help you determine when to use the equal sign. Figure 2 will remind you of the key words that indicate the different operations. Notice that the last column shows words that indicate the equal sign.

Vocabulary! Remember that a variable, or letter, represents the value that is unknown.

Now look at the difference between phrases and sentences. Remember that a phrase translates to an expression while a sentence is a complete thought and translates to an equation.

Addition (+)	Subtraction (-)	Multiplication (·)	Division (÷)	Equality (=)
sum	difference	product	quotient	equals
increased by	decreased by	times	divided by	is
more than	less than	of	per	the result is
more	less	multiplied by		yields
added to	subtracted from	by		is equal to
plus	minus			the solution is
greater than	fewer than			

Figure 2 | Words and Phrases that indicate operations

Phrase: a number decreased by eleven

Translation: n - 11

Sentence: A number decreased by eleven is equal to fifteen.

Translation: n - 11 = 15

Phrase: half of a number

Translation: $\frac{1}{2}n$

Sentence: Half of a number equals negative twelve.

Translation: $\frac{1}{2}n = -12$

Can you see the difference between a phrase and a sentence, or an expression and an equation? A phrase, or expression, doesn't tell you much. It's only half of a thought. But when you complete the thought, you have so much more information. In fact, given a sentence, or equation, you can actually find the unknown number! That's something you'll work on later. First, you need to be able to translate sentences and equations. Here is another example.

Key point! An equation gives you two expressions that are equal to each other. That's why equations are so important. With one known expression, you have enough information to find the value of an unknown part in the other expression!

Example:

- Translate the following sentence to an equation.
- Nine fewer than a number is equal to negative five.

Solution:

- Remember that "fewer *than*" means you need to switch "nine" and "a number" in the first part of the equation. So nine fewer than a number is written as n - 9.
- So this sentence translates to n - 9 = -5.

You can also translate an equation to a word sentence. For any given equation, there may be many possible translations. For example, the equation -4 + n = 7 could be translated as "the sum of negative four and a number is equal to seven." It could also be translated as "if negative four is added to a number, the result is seven." Can you think of any other variations for this sentence?

Example:

- Translate the following sentence to a mathematical equation.
- six more than half a number is thirteen

Solution:

- Use the variable n to represent "a number." Half of something means to divide by two, so half a number is $n \div 2$ or $\frac{n}{2}$. "Six more than" translates to addition, or +6. The word *is* means *equals*. So the sentence can be rewritten as $\frac{n}{2} + 6 = 13$.

Example:

- Express $3n + 6 = -9$ as a word sentence.

Solution:

- Remember that there are many possible variations when translating from an equation to a word sentence. Here is one possibility:
- Three times a number increased by six equals negative nine.
- Being able to translate between sentences and equations becomes extremely helpful when solving word problems. In this next example, you can practice writing an equation that represents a given word problem.

Example:

- Kyle and Marcus went to a concert last night. The cost for parking was $5 plus an additional $1.50 for each hour. The total cost they paid for parking was $9.50. Write an equation that could be used to find how many hours, h, they were at the concert.

Solution:

- Kyle and Marcus had to pay $5 *plus* $1.50 for each hour, h. You can represent this part with the expression $5 + 1.5h$. That expression represents the total they paid for parking. You also know that the total was equal to $9.50, so $5 + 1.5h = 9.5$.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- A word sentence translates to an equation.
- An equation shows that two expressions are equal.
- When translating from an equation to a sentence, different variations are possible.

Complete the following activities.

1.16 A number increased by negative eight is equal to fourteen. Which equation could be used to find the number?

- ☐ $n + 8 = 14$
- ☐ $n - (-8) = 14$
- ☐ $n + (-8) = 14$
- ☐ $8 - n = 14$

1.17 Twice a number is equal to negative four. Which equation could be used to find the number?

- ☐ $2n = 4$
- ☐ $2n = -4n$
- ☐ $2n - 4$
- ☐ $2n = -4$

1.18 One-third of a number yields twelve. Which equation could be used to find the number?

- ☐ $3n = 12$
- ☐ $\frac{1}{3}n = 12$
- ☐ $\frac{1}{3}n = -12$
- ☐ $3n = -12$

1.19 Translate the following sentence to an equation. If a number is divided by negative two, the solution is negative sixteen.

- ☐ $\frac{n}{-2} = -16$
- ☐ $\frac{-2}{n} = -16$
- ☐ $-2n = -16$
- ☐ $n \div 2 = -16$

1.20 Which of the following sentences could *not* be used to represent the equation $x - 5 = -10$?

- ☐ A number subtracted from five is equal to negative ten.
- ☐ Five less than a number is negative ten.
- ☐ A number less five equals negative ten.
- ☐ If a number is decreased by five, the result is negative ten.

1.21 Which of the following sentences could *not* be used to represent the equation $\frac{y}{2} = 14$?

- ☐ Half of a number is equal to fourteen.
- ☐ Two divided by a number yields fourteen.
- ☐ A number divided by two equals fourteen.
- ☐ The quotient of a number and two has a result of fourteen.

1.22 The cost to mail a letter is a base charge of $0.42 plus $0.17 for each ounce. Write an equation that could be used to find how many ounces a letter that costs $0.93 weighs. (Let x represent the number of ounces.)

- ☐ $0.59x = 0.93$
- ☐ $0.59 = 0.93x$
- ☐ $0.42x + 0.17 = 0.93$
- ☐ $0.42 + 0.17x = 0.93$

1.23 Chad washes windows after school to make some extra money. He charges $5.50 to wash each window. If the customer provides the supplies, Chad deducts $3.25 from the total cost. One customer paid a total of $27.50 and *did not* provide supplies. Which equation could be used to find the number of windows, *w*, that Chad washed for this customer?

- ☐ $5.5w + 3.25 = 27.5$
- ☐ $5.5w = 27.5$
- ☐ $5.5w - 3.25 = 27.5$
- ☐ $5.5 - 3.25w = 27.5$

1.24 Chad washes windows after school to make some extra money. He charges $5.50 to wash each window. If the customer provides the supplies, Chad deducts $3.25 from the total cost. One customer paid a total of $35.25 and *did* provide supplies. Which equation could be used to find the number of windows, *w*, that Chad washed for this customer?

- ☐ $5.5w + 3.25 = 35.25$
- ☐ $5.5w = 35.25$
- ☐ $5.5w - 3.25 = 35.25$
- ☐ $5.5 - 3.25w = 35.25$

1.25 Write a sentence to represent the equation $4m = -8$.

1.26 A giant candy bar costs $2. Write an equation to show how many candy bars, *c*, were purchased if the total cost was $18.

1.27 A movie theater charges $4 for a drink and $6 for pail of popcorn. Write an equation to show the number of drinks, *d*, and popcorns, *p*, that were purchased for a total cost of $14.

1.28 The county fair charges $10 admission and $2 per ride. Write an equation to show the number of rides, *r*, purchased if the total cost was $36.

1.29 The county fair charges $10 admission and $2 per ride. Write an equation to show the cost of *a* admissions and *r* rides if the total cost is $76.

1.30 A concession stand starts the day with *h* hamburgers. Write an equation to show the concession stand sold 58 hamburgers and had 7 hamburgers left at the end of the day.

Evaluating Expressions

In this lesson, you'll be using the order of operations to simplify expressions that contain variables.

Objectives

- Substitute values in for variables in an expression.
- Evaluate expressions for specific variables.

Vocabulary

evaluate—to find the value of a mathematical expression using the order of operations

To simplify or *evaluate* an expression means to find its value using the order of operations. You've used the order of operations a lot in the past. In fact, you've already simplified lots of expressions! But you probably haven't simplified expressions that contain variables. Don't worry, though! There really is only one new step that you'll have to worry about. Let's take a look.

Consider the following problem.

Charlie mows lawns during the summer. He charges $5 for gas plus $8 for each hour it takes to mow the lawn.

You can use the expression $5 + 8h$ to represent the total cost to mow one lawn. (Let h represent the number of hours.)

There is nothing more you can do with the expression $5 + 8h$ because you don't know the number of hours. You used a variable, h, because the number will change depending on the job. But if you did know the number of hours for a specific job, you could substitute that number in for h and find the total cost of that job. That's what you're going to do today!

Find the customer's cost for a lawn that takes Charlie three hours to mow. If it takes Charlie three hours, then h is equal to 3. So substitute 3 in for h in the original expression. When you substitute a number in for a variable, always put the number inside parentheses. Then use the order of operations to simplify the expression:

Key point! When substituting a number in for a variable, always put parentheses around the number that you are substituting in. This will help you make sure that all calculations are done correctly and in the correct order.

$$\begin{aligned} & 5 + 8h \\ &= 5 + 8(3) \\ &= 5 + 24 \\ &= 29 \end{aligned}$$

Charlie will bill the customer $29 for a lawn that takes three hours to mow. How about a lawn that takes four and a half hours to mow? Again, substitute the number of hours in for h:

$$\begin{aligned} & 5 + 8h \\ &= 5 + 8(4.5) \\ &= 5 + 36 \\ &= 41 \end{aligned}$$

Charlie will charge the customer $41 for a lawn that takes four and a half hours to mow. Notice that the value of the expression depends on the value you substitute in for *h*. When you substitute different numbers in for the variable, you get different answers for the value of the expression.

Step by Step! To evaluate an expression, follow these two steps:

1. Substitute the given values in for each variable. Use parentheses!
2. Simplify the expression using the order of operations.

Sometimes there will be more than one variable in an expression. Follow the same steps as listed. Substitute the given value for each variable into the expression. Then use the order of operations to evaluate the expression. Take a look at a few more examples.

Example:

- If $x = 4$ and $y = -3$, what is the value of $x^2 + y^2$?

Solution:

- Substitute the given values of *x* and *y* into the expression:
  - $(4)^2 + (-3)^2$
- Simplify the expression using the order of operations:
  - $(4)^2 + (-3)^2$
  - $= 16 + (-3)^2$
  - $= 16 + 9$
  - $= 25$

Example:

- Evaluate the expression $m - 2n$ for $m = -1$ and $n = \frac{1}{2}$.

Solution:

- Substitute the given values of *m* and *n* into the expression:
  - $-1 - 2(\frac{1}{2})$

Example:

- Evaluate the expression $2(a+3) - b^2$ when $a = 6$ and $b = 3$.

Solution:

- Substitute the given values of *a* and *b* into the expression:

$$2(a+3) - b^2$$
$$2(6+3) - 3^2$$
$$= 2(9) - 9$$
$$= 18 - 9$$
$$= 9$$

This might help! Remember that subtracting a number is the same as adding its opposite. So instead of subtracting 1, you can add -1.

- Simplify the expression using the order of operations:
  - $-1 - 2(\frac{1}{2})$
  - $= -1 - (1)$
  - $= -1 + (-1)$
  - $= -2$

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- You can evaluate an expression that has variables by substituting a specific value in for each variable.
- After substituting, use the order of operations to simplify the expression.
- The value of the expression depends on the value that was substituted in. If a different value is substituted, there will usually be a different result.

Complete the following activities.

1.31 Evaluate each expression for $g = -7$ and $h = 3$ and match it to its value.

_____ $g + h$	-10
_____ $g - h$	46
_____ $h - g$	-4
_____ gh	10
_____ $g + h^2$	-21
_____ $g^2 - h$	2

1.32 If $w = -2$ and $v = -8$, which of the following expressions shows the values correctly substituted in for the variables in the expression $w^2 - v + 1$?

- ☐ $-2^2 - (-8) + 1$
- ☐ $(-2)^2 - (8) + 1$
- ☐ $-2^2 - 8 + 1$
- ☐ $(-2)^2 - (-8) + 1$

1.33 Evaluate the expression $w^2 - v + 1$ for $w = -2$ and $v = -8$.

- ☐ 5
- ☐ 13
- ☐ -11
- ☐ -3

1.34 What is the value of $a^3 - b^2$ for $a = 3$ and $b = \frac{1}{2}$?

- ☐ 8
- ☐ $8\frac{3}{4}$
- ☐ $26\frac{3}{4}$
- ☐ 26

1.35 The cost to rent a paddle boat at the park is \$8 for each hour. The expression $8h$, where h represents the number of hours, can be used to represent the total cost for renting the boat. Which of the following expressions could be used to find the cost for renting the boat for 2 hours?

- ☐ 8(2)
- ☐ 8 + (2)
- ☐ $\frac{8}{(2)}$
- ☐ 82

1.36 The cost to mail a letter is a base charge of $0.42 plus $0.17 for each ounce. Write an expression to represent the cost for mailing a letter that is *w* ounces. Then find the cost for mailing a letter that is four ounces.

- ☐ 0.17 + 0.42*w*; $1.85
- ☐ 0.17 + 0.42*w*; $2.36
- ☐ 0.42 + 0.17*w*; $1.10
- ☐ 0.42 + 0.17*w*; $2.36

1.37 The total cost of dinner is divided among five friends. Write an expression to represent the cost for each person for a dinner that costs *d* dollars. Then find the cost for each person for a dinner that had a total cost of $56.25.

- ☐ $\frac{d}{5}$; $11.25
- ☐ 5*d*; $281.25
- ☐ $\frac{d}{5}$; $281.25
- ☐ 5*d*; $11.25

1.38 Two-fifths of a number is decreased by three. Find the value of the expression if the number is twenty-five.

- ☐ 8.8
- ☐ 13
- ☐ 10
- ☐ 7

1.39 Evaluate the expression $xy - 7$ for x = -5 and y = -3.

1.40 Evaluate the expression $4m + 3p$ for m = -2 and p = 5.

1.41 Evaluate the expression $3(6 - r) + t$ for r = 3 and t = -3.

1.42 Evaluate the expression $5d - 2e + 6$ for d = 6 and e = -2.

1.43 Evaluate the expression $b(c - 1)^2 + 3$ for b = 7 and c = 4.

Using Formulas to Solve Problems

Malcolm's parents just put $1,000 into an account that earns simple interest. They told Malcolm that he can withdraw the money in ten years and use it for whatever he wants. Ten years sounds like a really long time to Malcolm, but his parents said that the longer the money stays in the account, the more interest it will make. The account has a rate of 5%, or 0.05. Can you help Malcolm figure out how much interest his account will earn in ten years?

In this lesson, you'll be solving problems using a formula, or rule.

Objectives

- Use a formula to solve a problem.

Vocabulary

formula—an expression that uses variables to state a rule

To help Malcolm find out how much money his account will earn in ten years, you'll use a *formula*. A formula is a commonly known equation that uses variables to state a rule. For example, the formula for finding simple interest is $i = prt$. Each variable represents part of the rule:

- i represents the amount of the interest.
- p represents the principal, or the amount of money invested.
- r represents the rate (as a decimal).
- t represents the time of the investment (in years).

So the rule for finding simple interest is that the interest is equal to the principal

multiplied by the rate multiplied by the time.

Using the information from the beginning of the lesson, you know that Malcolm's parents invested $1,000 at a rate of 0.05, and it will be left alone for ten years. So p = 1,000, r = 0.05, and t = 10. To find i, or the interest, substitute each known value into the formula for its corresponding variable. Remember that when you substitute a specific value in for a variable, you should use parentheses. Then use the order of operations to simplify:

- $i = prt$
  $i = (1{,}000)(0.05)(10)$
  $i = (50)(10)$
  $i = 500$

So in ten years, Malcolm's account will earn $500 in interest. That's half of what is already in there! Just by leaving it alone, the amount will go from $1,000 to $1,500. Wow!

There are a few more common formulas that you'll be using. Take a look at the table below.

Distance	$d = rt$	d = distance, r = rate, t = time
Simple Interest	$i = prt$	i = interest, p = principal, r = rate, t = time
Temperature in Fahrenheit	$F = \frac{9}{5}C + 32$	F = Fahrenheit, C = Celsius
Temperature in Celsius	$C = \frac{5}{9}(F - 32)$	F = Fahrenheit, C = Celsius

Be Careful! Both the formulas for distance and simple interest include rate, but rate refers to the rate of speed in the distance formula and the rate of interest in the simple interest formula. Make sure you choose the appropriate formula for your situation.

There are also variations for the previous formulas. For example, what if you wanted to find the rate or the time, instead of the distance? To find the rate, you could use the formula $r = \frac{d}{t}$. To find the time, you could use the formula $t = \frac{d}{r}$. Notice that the part you want to find is by itself on the left side of the formula. Take a look at a couple more examples.

Key point! To determine which variation of a formula to use, choose the one that has the part you want to find by itself on one side of the formula.

Example:

- Dallas needs to drive 348 miles in six hours. What speed (or rate) will he need to average in order to get to his destination on time?

Solution:

- You know the distance and the time. You need to find the rate. So use the formula $r = \frac{d}{t}$, where d = 348 and t = 6. Substitute the values you know into the formula:

$$r = \frac{d}{t}$$

$$r = \frac{348}{6}$$

$$r = 58$$

This might help! The rate for this problem is measured in miles per hour because the distance in miles was divided by the time in hours. Remember that the division sign can be read as "per."

Dallas must average a speed of 58 miles per hour.

Example:

- What is 5°C expressed in Fahrenheit?

Solution:

- Since you want to find the temperature in Fahrenheit, you'll choose the formula that has F by itself on the left side of the formula:

$$F = \frac{9}{5}C + 32$$

- Substitute the value of 5 in for C and simplify:

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(5) + 32$$

$$F = \frac{45}{5}(5) + 32$$

$$F = 9 + 32$$

$$F = 41$$

- The temperature is 41°F

Example:

- What is $59°$ F expressed in Celsius?

Solution:

- Since you want to find the temperature in Celsius, you'll choose the formula that has C by itself on the left side of the formula:

$$C = \frac{5}{9}(F - 32)$$

- Substitute 59 for F and solve.

$$C = \frac{5}{9}(59 - 32)$$

$$C = \frac{5}{\cancel{9}}\left(\cancel{27}^{3}\right)$$

$$C = 15$$

- The temperature is $15°$ C.

Step by Step! When using a formula to solve a word problem, follow these steps:

1. Choose the appropriate formula.
2. Substitute all known values into the formula.
3. Simplify the expression using the order of operations.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- A formula uses variables to state a rule.
- When solving a word problem, choose the formula that uses the information you are given and has the variable you are trying to find by itself on one side of the formula.
- Once you've chosen the appropriate formula, substitute the values you know into the formula and use the order of operations to simplify.

Complete the following activities.

1.44 Match each formula to its correct description.

_____	$i = prt$	rate of speed
_____	$r = \frac{d}{t}$	distance
_____	$r = \frac{i}{pt}$	rate of interest
_____	$t = \frac{i}{pr}$	simple interest
_____	$t = \frac{d}{r}$	time money is invested
_____	$d = rt$	time spent driving

1.45 Jackie drove 7 hours at an average rate of 60 miles an hour. How far did she travel?

_____ miles

1.46 Hayden invested $750 for three years at an interest rate of 0.035. How much interest did he earn in that time?

$_____

1.47 Convert 45°C to Fahrenheit.

_____ °F

1.48 Convert 50°F to Celsius.

_____ °C

1.49 Terrance has $100 that he can invest in a savings account. His bank is willing to pay interest to his account at a rate of 0.04. How long will Terrance need to leave the money in the account if he wants to earn $100 in interest, doubling his money? Use the formula $t = \frac{i}{pr}$.

- ☐ 25 years
- ☐ 40 years
- ☐ 10 years
- ☐ 5 years

1.50 Tyrone biked 45 miles at an average speed of 18 miles per hour, without stopping. About how long did the ride take him? Use the formula $t = \frac{d}{r}$.

☐ 3 hours ☐ 2.75 hours ☐ 2.5 hours ☐ 2.25 hours

1.51 Maria has a savings account that is earning simple interest. If she started with $200 in the account and has earned a total of $8 in interest in two years, what is the rate of the account? Use the formula $r = \frac{i}{pt}$.

☐ 12.5 ☐ 0.08 ☐ 0.00125 ☐ 0.02

1.52 If Dustin averages 60 miles per hour, how long will it take him to drive 330 miles?

1.53 A passenger train travels 70 miles per hour. At that rate, how far will the train travel in 3.5 hours?

1.54 How much interest will $1500 invested at a rate of 0.03 earn after 7 years?

1.55 What is $95°$ F expressed in Celsius?

1.56 What is $37°$ C expressed in Fahrenheit?

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

Self Test 1: Variable Expressions

Complete the following activities (5 points, each numbered activity).

1.01 _____ states a rule using variables.

- ☐ A variable
- ☐ An expression
- ☐ A formula
- ☐ A term

1.02 _____ terms are also expressions.

- ☐ No
- ☐ Some
- ☐ All

1.03 Which of the following expressions could be used to represent "three less than half a number"?

- ☐ $3 - \frac{1}{2}n$
- ☐ $\frac{1}{2}n - 3$
- ☐ $3n - \frac{1}{2}$
- ☐ $\frac{1}{2} - 3n$

1.04 Which of the following phrases translates to the expression $y + 7$?

- ☐ a number decreased by seven
- ☐ seven more than a number
- ☐ the product of a number and seven
- ☐ a number divided by seven

1.05 Four friends are going to split the cost of a pizza. If p stands for the cost of the pizza, which of the following expressions could represent what each friend will pay?

- ☐ $4p$
- ☐ $p - 4$
- ☐ p
- ☐ $\frac{p}{4}$

1.06 The product of a number and negative three is equal to twenty-seven. Which of the following could represent this statement?

- ☐ $-3n + 27$
- ☐ $-3(27) = n$
- ☐ $-3n = 27$
- ☐ $-3 = 27n$

1.07 Skye ran r laps today at soccer practice. Nadine ran five fewer laps than Skye. If Nadine ran eight laps at practice, which of the following equations could be used to find how many laps Skye ran?

- ☐ $r - 5 = 8$
- ☐ $r + 5 = 8$
- ☐ $r = 8 - 5$
- ☐ $-5r = 8$

1.08 Which of the following statements would *not* translate to the equation $8 \div x = -4$?

- ☐ If eight is divided by a number, the result is negative four.
- ☐ Eight divided by a number is equal to negative four.
- ☐ The quotient of eight and a number equals negative four.
- ☐ The solution of a number divided by eight is negative four.

1.09 If $a = -9$ and $b = -4$, what is the value of $a + b$?

☐ -5 ☐ 5 ☐ -13 ☐ 13

1.010 Simplify $2x^2 - y$ for $x = 3$ and $y = -2$.

☐ 20 ☐ 14 ☐ 16 ☐ 10

1.011 The cost for children under twelve at a certain buffet restaurant is a base charge of a dollar plus twenty-five cents for each year of the child's age. Represent this cost as an expression, using y to stand for the child's age in years. Then use the expression to find the cost for a child who is nine.

☐ $1 + 0.25y$; \$2.25 ☐ $0.25 + y$; \$9.25

☐ $1 + 0.25y$; \$3.25 ☐ $0.25 + y$; \$8.75

1.012 Tommy and his family drove home from his grandparent's house in three and a half hours. If they drove at an average rate of 48 miles per hour, which of the following formulas could be used to find how far away Tommy lives from his grandparents?

☐ $d = rt$ ☐ $t = \frac{d}{r}$ ☐ $r = \frac{d}{t}$

1.013 Tommy and his family drove home from his grandparent's house in three and a half hours. If they drove at an average rate of 48 miles per hour, about how far away does Tommy live from his grandparents?

☐ 144 miles ☐ 192 miles ☐ 168 miles ☐ 180 miles

1.014 Hillary opened a savings account with \$1,500. How much simple interest will she earn if she leaves the money in the account for two years at an interest rate of 0.025? Use the formula $i = prt$.

☐ \$750.00 ☐ \$37.50 ☐ \$375.00 ☐ \$75.00

1.015 Convert 15°C to Fahrenheit. Use the formula $F = \frac{9}{5}C + 32$.

☐ 59°F ☐ 27°F ☐ 85°F ☐ 33°F

1.016 A student brought 2 dozen cookies to share with the students in his class. Write an expression to show how many cookies each student should get if there are *s* students.

1.017 Solve the expression $6w - 5z$ for $w = 4$ and $z = -2$.

1.018 A runner completes a 5-mile race in 40 minutes. What was the runner's average speed in miles per hour? ($40 \text{ min.} = \frac{2}{3}\text{hr.}$)

1.019 A credit card offers no interest financing if the entire balance is paid in full within 12 months. If the balance is not paid in full at the end of 12 months, the customer is charged 18% interest on the original balance. What will be the amount of interest charged on a $500 purchase that is not paid in full at the end of the 12-month period? (18% = 0.18) Use the formula $i = prt$

1.020 Convert $-40°$ F to Celsius. Use the formula $C = \frac{5}{9}(F - 32)$.

80/100 **SCORE** _____ **TEACHER** _____ _____
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