LIFEPAC®

MATH

STUDENT BOOK

7th Grade | Unit 6

Alpha Omega PUBLICATIONS

Math 706
Probability and Graphing

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Probability and Graphing

Introduction

In this unit, students will be introduced to basic probability. They will determine theoretical and experimental probability and learn that experimental probability approaches theoretical probability as the number of trials increases. Students will determine the probability for compound events and find sample space using a tree diagram and a table. Students will learn about the counting principle and apply it to finding the probability of compound events. They will also learn the difference between independent and dependent events.

Students will be introduced to the coordinate plane and use it to graph linear functions. They will plot ordered pairs and find the location of points in the coordinate plane. Students will graph linear equations and determine the slope of a line using the slope formula. They will also learn about direct variation functions and their characteristics.

Objectives

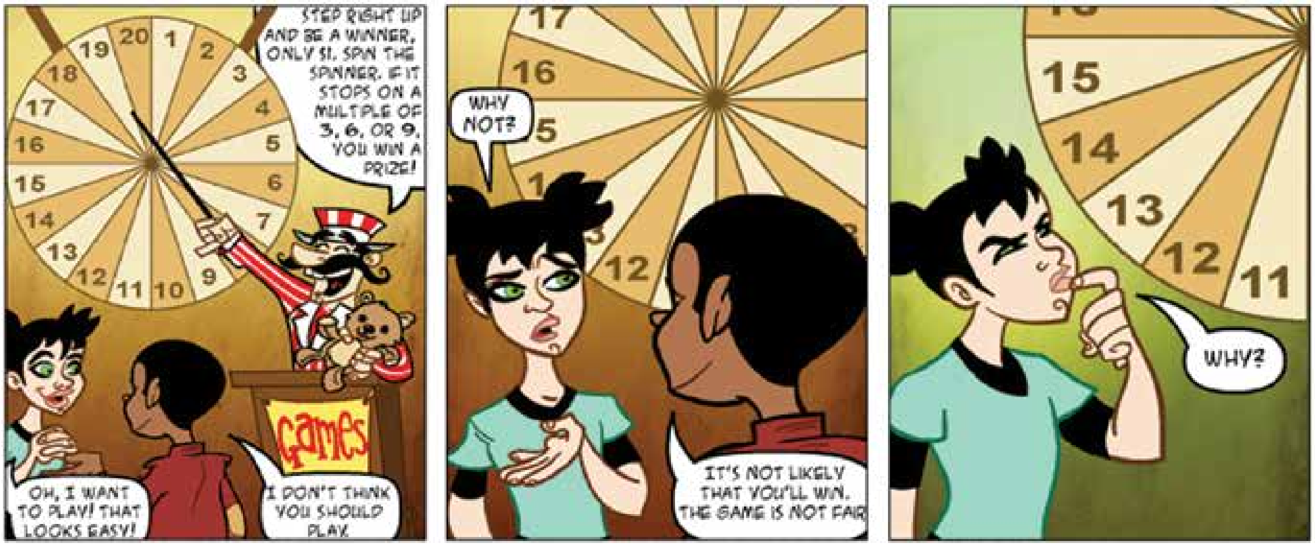
Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Determine the theoretical and experimental probability of an event.
- Determine the sample space for an experiment.
- Determine if events are independent or dependent.
- Determine the probability of independent and dependent events.
- Plot ordered pairs on a rectangular coordinate system.
- Use a table to graph a linear equation.
- Determine the slope of a linear function, including direct variation.
- Determine if a function is a direct variation.
- Graph direct variations.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. Probability

Theoretical Probability



How likely is it that Ondi will win the game? What are her chances, and how can you figure that out? In this lesson, you will learn how to find the likelihood that *events* will occur. The measure of this likelihood is called *probability*.

Objectives

- Determine the theoretical probability of an event.

Vocabulary

complementary events—two disjoint events of which one or the other must occur
disjoint events—events that have no outcomes in common
event—a specific outcome or group of outcomes
experiment—any activity that has two or more outcomes
favorable outcome—outcome for a specific event
outcome—any possible result of an experiment
probability—the measure of the likelihood of an event
theoretical probability—a ratio representing the likelihood of an event

In the area of mathematics known as probability, the carnival game that Ondi wanted to play is called an *experiment*. There are 20 possible *outcomes*, or results of the experiment because the spinner could stop on any number. The event, or specific outcome, that Ondi would need to win is any multiple of 3 from 1 and 20. The outcomes for this event are called *favorable outcomes*.

Probability is a measure of how likely an event is to occur. If all outcomes are equally likely, the probability (*P*) of the event is expressed as a ratio:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

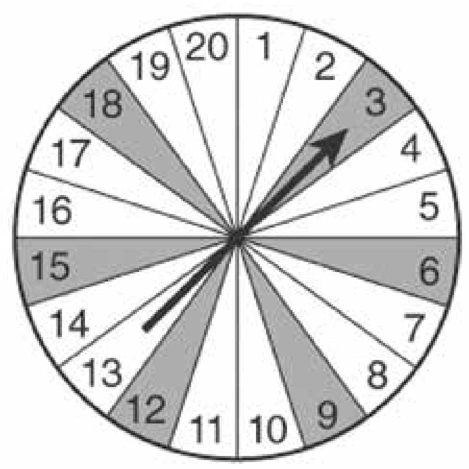
For Ondi, the event is spinning a multiple of 3 on the wheel. Take a look at the probability of Ondi winning the carnival game.

Example:

- What is the probability of spinning a multiple of 3 on a spinner with 20 equally spaced sections numbered from 1 to 20?

Solution:

- You need to find the number of favorable outcomes compared to the total number of outcomes.
- You know there are 20 spaces on the spinner, so the total number of outcomes is 20.
- To find the number of favorable outcomes, look at the multiples of 3 from 1 to 20:
  3, 6, 9, 12, 15, 18
- There are 6 multiples of 3, so there are 6 favorable outcomes.
- Shading in the multiples of 3 on the wheel makes this easier to see.



Reminder! Percents, decimals, and fractions can each express the same ratio:

$$\frac{1}{4} = \frac{25}{100} = 25\% = 0.25$$

Now compare the number of favorable outcomes to the total number of outcomes:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(\text{multiple of 3}) = \frac{6}{20}$$

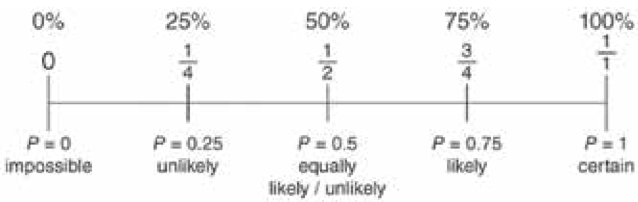
Since probability is a ratio, it can be written as a fraction, decimal, or percent. When you express the probability of an event as a ratio, it is called the *theoretical probability*.

Reminder! To change a fraction to a percent, rewrite it with a denominator of 100. To change a percent to a decimal, move the decimal point two places to the left.

$\frac{6 \div 2}{20 \div 2} = \frac{3}{10}$	Divide by a common factor of 2.
$\frac{3 \cdot 10}{10 \cdot 10} = \frac{30}{100}$	Multiply by 10 to get a denominator of 100.
$\frac{30}{100} = 30\%$	Convert the fraction to a percent.
$30\% = 0.3$	Move the decimal point two places to the left.

So there is a $\frac{3}{10}$, 30%, or 0.3 chance of spinning a multiple of 3 on the wheel.

Probability will always be a number from 0 to 1. The closer the probability is to 1, the more likely it is that the event will occur. You can see this relationship on a number line.

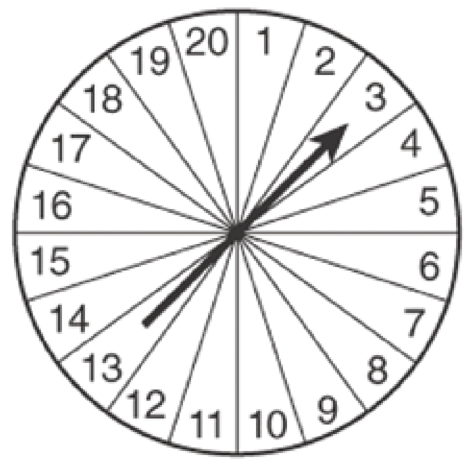


Events that have less than a 50% probability are less likely to occur. Events that have more than a 50% probability are more likely to occur.

As Carlton said, it is *unlikely* that Ondi would win the game. She has well under a 50% chance of winning.

You can also describe everyday events in terms of likelihood. For example, it is very unlikely that it will snow on a warm summer day. Or suppose you've attended school 48 out of the last 50 school days. Based on your previous attendance, it is very likely that you will be at school on the next school day.

Take a look at a couple of examples using the carnival spinning wheel. For each event, look at the number of favorable outcomes compared to the number of total outcomes.



What is the probability that you would spin the number 30?

There are 20 total outcomes, because there are 20 sections on the wheel. However, you can see from looking at the wheel that there is no space labeled 30, so there are 0 favorable outcomes. You can express this probability using a ratio:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(30) = \frac{0}{20}$$

Make note! In probability, parentheses do not indicate multiplication. The contents of the parentheses are the event. You are not multiplying *P* by 30.

So the probability is 0 out of 20, or 0%. In other words, the outcome is impossible.

What is the probability that you would spin a number less than 21?

Again, there are 20 total outcomes, because there are 20 sections on the wheel. There are 20 favorable outcomes because all of the outcomes are less than 21. You can express this probability using a ratio:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(< 21) = \frac{20}{20}$$

So the probability is 20 out of 20, or 1, or 100%. In other words, the outcome is certain.

Here's another example.

Example:

- There are 2 blue marbles, 1 red marble, and 9 green marbles in a bag. What is the probability of drawing a green marble from the bag?

Solution:

- Find the number of favorable outcomes and the total number of outcomes and compare them to find the probability of the event.
- There are 9 green marbles, so there are 9 favorable outcomes.
- To find the total number of outcomes, you need to find out how many marbles are in the bag:
  - 2 blue + 1 red + 9 green = 12 marbles
- Since there are 12 marbles in the bag, there are 12 total outcomes.
- Now compare the outcomes:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(\text{green}) = \frac{9}{12}$$

Keep in mind! Any proportion can be solved by cross multiplying:

$$\frac{3}{4} = \frac{x}{100}$$
$$300 = 4x$$
$$\frac{300}{4} = x$$
$$75 = x$$

- You can simplify the fraction and change it to a percent or a decimal.

$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$	Divide by a common factor of 3.
$\frac{3 \cdot 25}{4 \cdot 25} = \frac{75}{100}$	Multiply by 25 to get a denominator of 100.
$75\% = 0.75$	Move the decimal point two places to the left.

So there is a $\frac{3}{4}$, 0.75, or 75% chance of drawing a green marble from the bag.

When you have two events of which one or the other *must* occur, the events are called *complementary events*, and their probabilities will always add up to 1:

- *P*(event) + *P*(*not* event) = 1

Try an example involving complementary events.

Example:

- If you roll a regular 6-sided number cube, what is the probability that you will roll a 4? What is the probability that you won't roll a 4?

Solution:

- Compare the number of favorable outcomes to the total number of outcomes for each experiment. Write the probability as a fraction this time.

- There is only one favorable outcome for each of the 6 numbers on the cube, so there is one favorable outcome for rolling a 4. There are 6 sides to the number cube, so there are a total of 6 outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(4) = \frac{1}{6}$$

- The favorable outcomes for *not* rolling a 4 are 1, 2, 3, 5, and 6. So there are 5 favorable outcomes out of 6 total outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(\textit{not } 4) = \frac{5}{6}$$

- Notice that the probability of rolling a 4 and the probability of not rolling a 4 add up to 1:

$$P(4) + P(\textit{not } 4)$$

$$= \frac{1}{6} + \frac{5}{6}$$

$$= 1$$

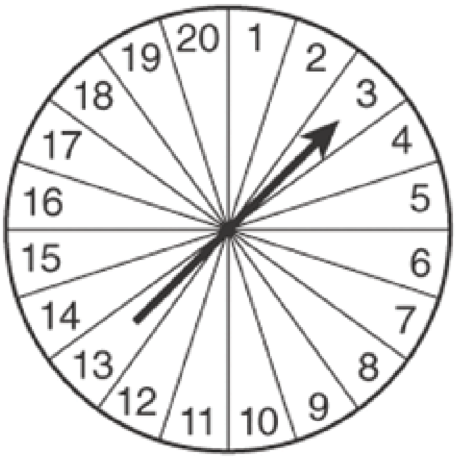
- They add up to 1 because both events together account for all of the outcomes. You could say that all rolls of the number cube are either 4 or not 4.
- Events that have no outcomes in common are called *disjoint events*, and the probability that either event will occur is the sum of the probabilities of the events:

$$P(\text{event 1 } \textit{or} \text{ event 2}) = P(\text{event 1}) + P(\text{event 2})$$

- Try an example involving disjoint events.

Example:

- If you spin the carnival spinning wheel, what is the probability that you will spin a multiple of 5 or a multiple of 7?



Solution:

- Compare the number of favorable outcomes to the total outcomes for each event. This will give you the probability for each event. If there are no outcomes in common, you can add the probabilities of the events.
- The favorable outcomes for a multiple of 5 are 5, 10, 15, and 20. So there are 4 favorable outcomes.
- You know there are 20 total outcomes because there are 20 spaces on the wheel.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

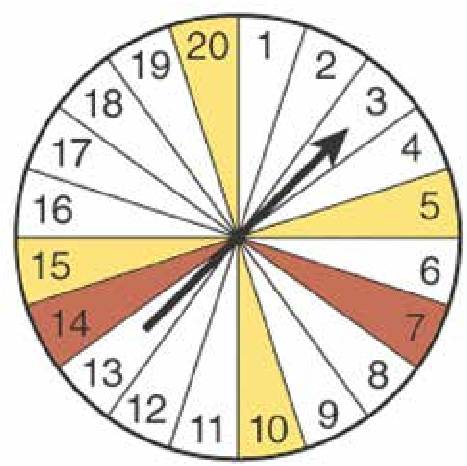
$$P(\text{multiple of } 5) = \frac{4}{20}$$

- The favorable outcomes for a multiple of 7 are 7 and 14. So there are 2 favorable outcomes out of 20 total outcomes.

$P(\text{event}) = \dfrac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

$P(\text{multiple of 7}) = \dfrac{2}{20}$

- If the wheel is colored, you can see the favorable outcomes more easily.
- yellow: multiples of 5
- orange: multiples of 7



- There are no outcomes in common, so you can add the probabilities.
- *P*(event 1 or event 2) = *P*(event 1) + *P*(event 2)
- *P*(multiple of 5 *or* multiple of 7) = *P*(multiple of 5) + *P*(multiple of 7)
- *P*(multiple of 5 *or* multiple of 7) = $\dfrac{4}{20} + \dfrac{2}{20}$
- *P*(multiple of 5 *or* multiple of 7) = $\dfrac{6}{20}$
- This was the same probability as the game Ondi wanted to play, and you found that 6 out of 20 was 30%. So there is a 30% chance of spinning a multiple of 5 or 7.

Example:

- A side game at the fair requires the game operator to guess the month of the guest's birth within 2 months. If the game operator is off by more than two months, the guest wins a prize. What is the probability that the game operator will randomly guess a person's birth month within two months of the correct month?

Solution:

- There are 12 possible months to choose from, so there are 12 possible outcomes. Of these possible outcomes, the game operator must either guess the correct month, or one of the two months on either side of the correct month. For example, if the guest was born in July, the game operator could guess July (the correct month), August, September (the 2 months after July), June, or May (the 2 months before July). This allows 5 possible favorable outcomes.
- Compare the number of favorable outcomes with the number of possible outcomes and form a ratio.

$$\frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{5}{12}$$

$$P(\text{birth month within 2 months}) = \frac{5}{12}$$

or $41.\overline{6}\%$ or $0.41\overline{6}$

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Theoretical probability is expressed as a ratio and can be written as a fraction, decimal, or percent.
- To find the theoretical probability of an event, compare the number of favorable outcomes to the total number of outcomes:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

Complete the following activities.

1.1 Select all that apply. If there are 8 chocolate chip cookies out of 20 total cookies in a jar, what is the probability that you will randomly choose a chocolate chip cookie?

☐ $\frac{2}{5}$ ☐ 40% ☐ 0.8 ☐ 0.4

1.2 Select all that apply. In a carnival game, there is a 45% probability of winning a prize. Which of the following is true?

☐ The probability that you won't win is $\frac{11}{20}$.

☐ The probability that you will win is 4.5.

☐ The probability that you won't win is 0.55.

☐ The probability that you will win is $\frac{9}{20}$.

1.3 In a raffle, Scott buys 10 tickets and his friend Tom buys 6 tickets. If there are 80 tickets sold, what is the probability that Scott *or Tom will win?*

☐ $\frac{1}{8}$ ☐ $\frac{1}{5}$ ☐ $\frac{3}{40}$ ☐ $\frac{4}{5}$

1.4 A jar contains 100 coins. If it is very *unlikely* that you will randomly choose a quarter out of the jar, how many quarters could be in the jar?

☐ 50 ☐ 35 ☐ 85 ☐ 10

1.5 At a school, there are 526 students and 263 are girls. About how likely is it that a randomly chosen student will be a boy?

☐ unlikely

☐ somewhat likely

☐ very likely

☐ equally as likely as unlikely

1.6 What is the probability of rolling a 2 or not rolling a 2 using a regular 6-sided number cube?

☐ $\frac{1}{6}$ ☐ 33.3% ☐ $\frac{5}{6}$ ☐ 100%

1.7 Chris has 2 pairs of black socks, 4 pairs of red socks, and 18 pairs of white socks in a dresser drawer. If he reaches in his drawer without looking, what is the probability that he will choose a pair of white socks?

☐ 75% ☐ 33.3% ☐ 25% ☐ 50%

1.8 There are 6 red marbles, 4 blue marbles, and 15 green marbles in a jar. If you reach in and randomly draw one, what is the probability that you will choose a red marble?

☐ 66.7% ☐ 40% ☐ 30% ☐ 24%

1.9 There are 12 boys and 13 girls in a class. If the teacher randomly chooses a student's name out of a hat, what is the probability it will be a girl?

☐ 48% ☐ 50% ☐ 52% ☐ 92%

1.10 You are one of 50 people with an entry into a random drawing for a new bicycle. What is the probability that you will win the drawing?

1.11 You must roll an even number on a standard 6-sided game die to win the game. What is the probability that you will win the game on your next roll?

1.12 What is the probability of a football player correctly guessing whether the coin toss will be heads or tails?

1.13 What is the probability of rolling either an even number or a 5 on a standard 6-side game die?

1.14 A deck of cards has 52 cards (13 cards in each of 4 suits: clubs, diamonds, spades, and hearts). What is the probability of drawing a card with a diamond on it?

Experimental Probability

Was Ondi lucky to win the game? It was not likely that she would win with only a 30% chance, but she did. In this lesson, you will explore the difference between what you *think* will happen in an experiment and what actually *does* happen. The actual result of an experiment is called the *experimental probability*.

Objectives

- Determine the experimental probability of an event.

Vocabulary

experimental probability—a ratio representing the actual results of an experiment
prediction—a guess for the results of an experiment based on the theoretical probability
success—a trial in which a desired outcome occurs
trial—each test of an experiment

Theoretical probability is based on mathematical reasoning. You compare the number of favorable outcomes to the total number of outcomes for an experiment:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

Reminder! Theoretical probability is based on each outcome among the total number of outcomes being equally likely. For instance, when a number cube is rolled, each number from 1 to 6 (the outcomes) is equally likely to come up. The probability for each event is the same.

Experimental probability, on the other hand, is determined by what actually happens when you perform several *trials* of the experiment. Each time the desired outcome occurs, it is called a *success*.

$$\text{experimental probability of an event} = \frac{\text{number of successes}}{\text{number of trials}}$$

For many experiments, you can't determine the theoretical probability; you can only determine the experimental probability. For instance, suppose you want to find out what soda people prefer out of three choices. There are a total of three outcomes, but you don't know if each outcome is equally likely (If you did, you wouldn't need to do the experiment!), so you can't say the probability of choosing a soda is 1 out of 3.

You can't find the *theoretical probability*, but you can determine the *experimental* probability.

Example:

- People were given three different sodas to taste and asked which tasted best. Given the results shown, what is the experimental probability for soda B?
- soda A: 45
- soda B: 88
- soda C: 67

Solution:

- You need to compare the number of successes to the total number of trials:
- There were 88 successes because 88 people chose soda B.
- You can find the number of trials by adding the results for each soda:
    - 45 + 88 + 67 = 200

Connections! When planning an experiment, it is helpful to choose a number of trials that can easily be converted to a percent. Choose a multiple or factor of 100, such as the following: 10, 25, 50, 100, 200, 500

- There were 200 trials.
- Now find the experimental probability:

  $$\text{experimental probability of an event} = \frac{\text{number of successes}}{\text{number of trials}}$$

  $$\text{experimental probability} = \frac{88}{200}$$

- You can rewrite the fraction as a percent by changing the denominator to 100:

  $$\frac{88 \div 2}{200 \div 2} = \frac{44}{100} = 44\%$$

- So the experimental probability for soda B was 44%.
- You can use this information to make a *prediction* about how many people will choose soda B if you perform more trials.

Example:

- If 44% of people like soda B, given the choice of sodas A, B, and C, predict how many people will choose soda B out of 1,000 people.

Connections! People who predict trends use this same strategy. Suppose you read that 6 million people watched a certain TV show last night. That number is actually based on a small, carefully chosen survey, and then proportions were used to predict the results for all TV watchers.

Solution:

- To solve the problem, you can use a proportion. Your first ratio is 44 out of 100 because 44% of people like soda B. This ratio needs to be the same as the ratio of the number of people out of 1,000 who you predict will like soda B. If the two ratios are equal, you have a proportion.

$\frac{44}{100} = \frac{x}{1{,}000}$ Set up the proportion.

$44{,}000 = 100x$ Cross multiply.

$440 = x$ Divide both sides by 100.

- So you can predict that 440 people would like soda B if 1,000 people were asked to choose between sodas A, B, and C.

In the same way, you can use the theoretical probability to make predictions about the results of an experiment. You can use a proportion to compare the theoretical probability to the ratio of predicted outcomes for a given number of total outcomes. Then you can compare the prediction to the experimental results and experimental probability.

Example:

- A recent survey indicated that 48% of Americans agree with the President's economic policy. If you were to survey 500 people, how many people would you expect to agree with the President's economic policy based on this survey?

Solution:

- Begin by setting up a proportion.

$$\frac{\text{percent that agree}}{\text{total (100\%)}} = \frac{\text{number that agree}}{\text{total number surveyed}}$$

$$\frac{48}{100} = \frac{x}{500}$$

- Cross multiply and solve for *x*.

$$\frac{48}{100} = \frac{x}{500}$$
$$100x = 24{,}000$$
$$x = 240$$

- Based on the results of the survey, you would expect to find 240 out of 500 people you survey agree with the President's economic policy. The actual number you get after surveying 500 people will most likely be different.

Example:

- If a regular 6-sided number cube is rolled 240 times, predict how many times each number on the cube would be rolled. Then compare the predictions to the actual results.

Solution:

- First you need to determine the theoretical probability. There are 6 total outcomes on the number cube, and each number has one favorable outcome.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{6}$$

- So there is a 1 in 6 chance for each number on the cube to be rolled. To predict the results of the experiment, you can compare this ratio to the number of successes out of 240 trials.

$$\frac{1}{6} = \frac{x}{240}$$ Set up the proportion.

$240 = 6x$ Cross multiply.

$40 = x$ Divide both sides by 6.

- So you can predict that there would be 40 successes for each number on the number cube. You can summarize this in a table.

	1	2	3	4	5	6
Predicted Results	40	40	40	40	40	40

- In an experiment a number cube was rolled 240 times and the results were recorded.
- We can make a new table to compare the predicted results to the actual results:

	1	2	3	4	5	6
Predicted Results	40	40	40	40	40	40
Actual Results	37	45	58	32	36	32
Difference	-3	+5	+18	-8	-4	-8

- The experiment did not get the predicted results for any of the numbers. That's okay because the theoretical probability is based on what you *expect* will happen while the experimental probability is based on what *did* happen.
- In fact, theoretical probability and experimental probability rarely match. Part of an experiment is the randomness of the event. A coin could land heads 10 times in a row; each flip is a random event. However, in the long run, with more and more trials, the experimental and theoretical probabilities get closer to each other.
- Remember that Ondi's actual results didn't match the predicted result because the theoretical probability only gave her a 30% chance to win. But she still won!

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- To find the experimental probability of an event, compare the number of successes to the number of trials.
- Theoretical probability and proportions can be used to make predictions about experimental results.
- Experimental probability can be used to predict future experimental results.
- The theoretical probability and experimental probability of an event are rarely the same. As more trials are performed, though, the two probabilities do get closer together.

Complete the following activities.

1.15 Theoretical probability and experimental probability are the same.

○ True
○ False

1.16 What is the experimental probability of drawing a red marble, given the following results?

Marble Color	Blue	Green	Red
Times Drawn	8	5	7

☐ $\frac{7}{13}$ ☐ $\frac{7}{20}$
☐ $\frac{7}{8}$ ☐ $\frac{13}{20}$

1.17 A coin is flipped 25 times and lands on heads 16 times. Based on the experimental probability, how many heads would you predict for 200 flips of the coin?

☐ 100 ☐ 72 ☐ 128 ☐ 64

1.18 A spinner is divided into 4 equal sections numbered 1 to 4. The theoretical probability of the spinner stopping on 3 is 25%. Which of the following is most likely the number of 3's spun in 10,000 spins?

☐ 3,108 ☐ 2,538 ☐ 2,400 ☐ 1,367

1.19 Predict the number of times you will win a prize in 500 spins, based on the theoretical probability.

☐ 300
☐ 150
☐ 157
☐ 287

1.20 Select all that apply. A card is randomly drawn from a deck of cards. (There are 52 cards in a deck, with 13 cards of each suit.) You record the following results for the experiment.

	Hearts	Clubs	Spades	Diamonds
Times Drawn	28	32	18	22

Which of the following are true?

- ☐ The experimental probability of drawing a club is 32%.
- ☐ The theoretical probability of drawing a diamond is 25%.
- ☐ Based on the experimental probability, there would be 90 spades drawn in 500 trials.
- ☐ Based on the theoretical probability, there would be 120 hearts drawn in 500 trials.

1.21 Select all that apply. Suppose you throw wadded sheets of paper at a wastebasket 10 feet away, and you make 14 out of 25 shots. Which of the following is true?

- ☐ The theoretical probability of making a basket is 50%.
- ☐ The experimental probability of making a basket is 56%.
- ☐ You would predict that you would make 70 baskets in 125 trials.
- ☐ You will make 28 baskets in 50 throws.

1.22 A coin is flipped 20 times. The results are 12 heads and 8 tails. The theoretical probability of getting heads is 60%.

- ○ True
- ○ False

1.23 A fast food restaurant states that 1 out of every 4 game pieces is an instant winner on their peel-off game. How many instant winning game pieces would you theoretically expect to have if you have a total of 20 game pieces?

1.24 The theoretical probability of a basketball player scoring a point on his free throw is 80%. How many points would you expect this player to score from free throws if he shoots 10 free throws in a game?

Use this information to answer the following questions.

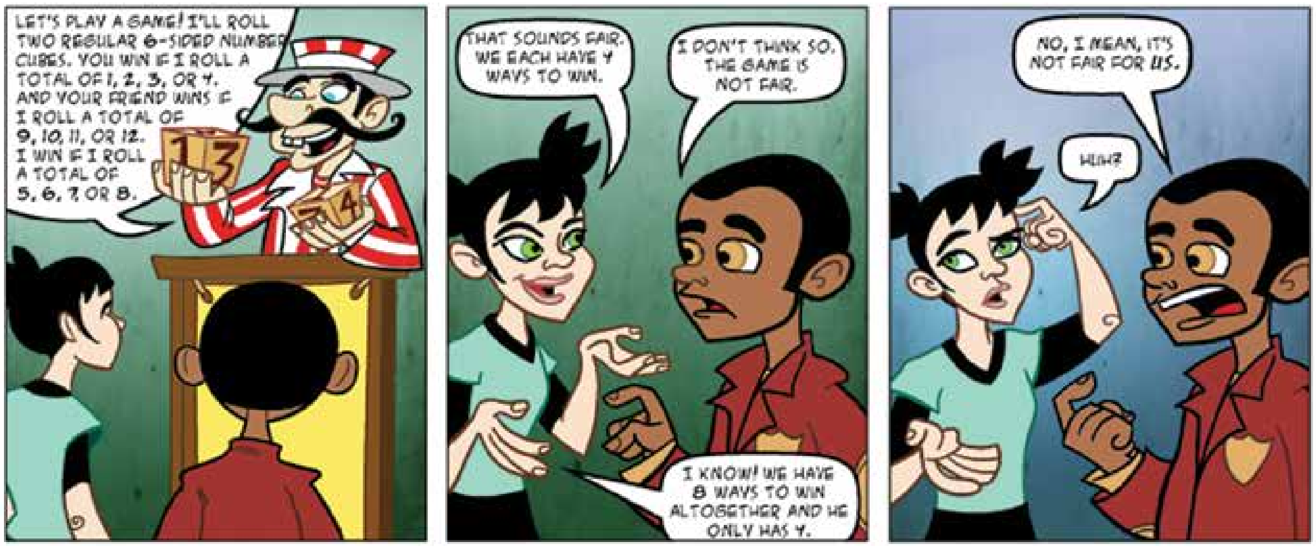
A bag of colored candies has 38 brown pieces, 6 red pieces, 19 yellow pieces, 8 blue pieces, 17 orange pieces, and 12 green pieces.

1.25 What is the probability of randomly picking a brown piece?

1.26 What is the probability of randomly picking a red or yellow piece?

1.27 How many green pieces would you expect to find in a bag of 250 candies?

Sample Space



Is the game fair? It's hard to tell because there are more outcomes than it might look like at first. In this lesson, you will learn how to find all of the outcomes for experiments like this game and decide if they are fair.

Objectives

- Determine the sample space for an experiment.

Vocabulary

counting principle—principle that states the number of outcomes for a compound event is found by multiplying the total number of outcomes for each event together

compound event—an event consisting of two or more events that can happen at the same time or one after the other

fair game—a game in which each participant has the same probability of winning

sample space—an organized listing of all possible outcomes for an experiment

tree diagram—an organizing tool used to find the sample space for compound events

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You know that the probability of an event is the number of favorable outcomes compared to the total number of outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

To find the total number of outcomes for one event, you can just list them. A list of all outcomes is called the *sample space*. If you toss a coin, the sample space is heads or tails. If you roll a number cube, the sample space is 1, 2, 3, 4, 5, and 6.

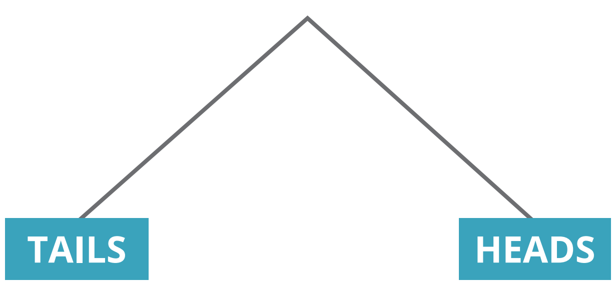
Usually, when you refer to sample space, you are thinking about a *compound event*, or an experiment in which *two or more* events happen at the same time or one after the other. The probability is still found by comparing the number of favorable outcomes to the total number of outcomes. However, finding the sample space is a little different.

For example, what if you toss a coin and roll a number cube at the same time? What is the sample space? To find out, you need to list all of the combinations of the first event with the second event. You could get heads and a 2, or tails and a 3, or heads and a 6.... This could get confusing!

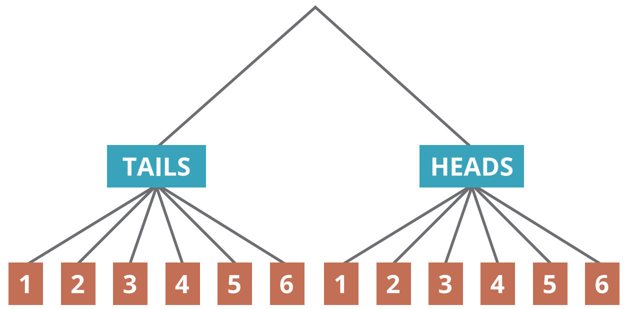
You need an organized way to find the sample space. You can use a *tree diagram* to help you.

A tree diagram shows each outcome for the first event and pairs each of these outcomes with all of the outcomes for the second event. In this way, you know that each possible outcome is accounted for.

So if you toss a coin and roll a number cube, what would the tree diagram look like? Tossing a coin has two outcomes: heads and tails. These will be the first branches of your tree diagram.



From each outcome of tossing a coin, you will have a branch to each outcome of rolling a number cube: 1, 2, 3, 4, 5, and 6.



There are a total of 12 outcomes. Use this tree diagram to help you work through the next example.

Example:

- If you flip a coin and roll a number cube, what is the probability that you would get tails and an even number?

Solution:

- Find the number of favorable outcomes compared to the total number of outcomes.
- Looking at the tree diagram, there are 3 favorable outcomes: tails-2, tails-4, and tails-6. You also know that there are 12 total outcomes in the sample space.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(\text{event}) = \frac{3}{12}$$

- You can rewrite the fraction as a percent.

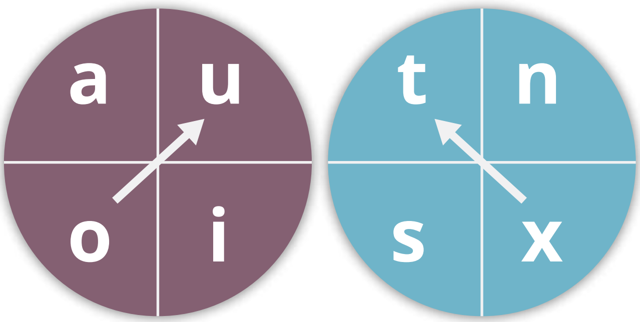
$$\frac{3}{12} = \frac{1}{4} = 25\%$$

- So if you toss a coin and roll a number cube, there is a 25% probability of getting tails and an even number.

Notice that the tree diagram has two benefits: It shows the total number of outcomes, and it also shows each individual outcome so you can easily see which are favorable outcomes.

Example:

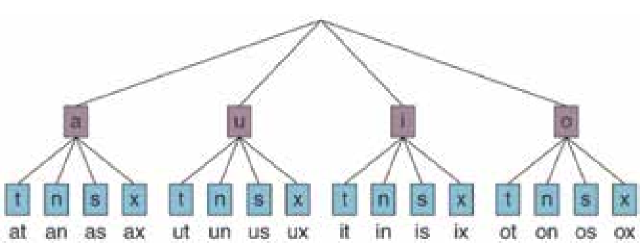
- If you spin the spinner on the left and then the spinner on the right, what is the probability that you will spin two letters that spell a word?



Solution:

- This is a good situation for a tree diagram. You need to know the total number of outcomes to find the probability, but you also need to see all of the combinations to know which ones will make words.
- There are 4 outcomes for the first spinner.

- Each of these will branch to the 4 outcomes of the second spinner.



- There are 16 total outcomes in the sample space.
- Looking at the tree diagram, you can see there are 10 words:
  - at, an, as, ax, us, it, in, is, on, ox
- So the probability is $\frac{10}{16}$ that you would spin a word. Rewrite that as a percent:

$\frac{10}{16} = \frac{x}{100}$	Set up the proportion.
$1{,}000 = 16x$	Cross multiply.
$62.5 = x$	Divide both sides by 16.

There is a 62.5% probability of making a word if you spin the first spinner and then the second spinner.

Now take a look at the game Ondi and Carlton were thinking about playing. Carlton did not think the game was fair.

A game is fair if each person has the same probability of winning. See if the carnival game is a *fair game* by determining whether each person has the same probability of winning.

Example:

- Two regular number cubes are rolled. The numbers rolled on each number cube are then added together. If Ondi, the carnival worker, and Carlton are assigned to the following roll totals, do they have an equal chance of winning?
  - Ondi: 1, 2, 3, or 4
  - carnival worker: 5, 6, 7, or 8
  - Carlton: 9, 10, 11, 12

Solution:

- First, you will determine the sample space. Then you can find the favorable outcomes for each group of numbers.

Think about it! You could use a tree diagram, but it would be fairly large. Can you see why?

- There are two events happening at once (the roll of each number cube), so this is a compound event. This time, use a table to list all of the outcomes.
- The table, as shown in Figure 1, needs to list the outcomes for the first event (the roll of the first number cube: 1, 2, 3, 4, 5, 6) along the top. Put the outcomes for the second event (the roll of the second number cube: 1, 2, 3, 4, 5, 6) along the side. Each cell where the two number cube outcomes meet is an outcome for rolling both number cubes.
- You can see that there are 36 total outcomes (6 rows of 6). Now look at the favorable outcomes for each group of numbers. Notice that there are no outcomes with a total of 1, Figure 2.

This might help! An outcome of 4-6 means a 4 on the first roll and a 6 on the second roll. An outcome of 6-4 means a 6 on the first roll and a 4 on the second roll.

- Coloring in the table will help you see the combinations for each person's group of numbers, see Figure 3.

	1	2	3	4	5	6
1	1 + 1 = 2	2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6	6 + 1 = 7
2	1 + 2 = 3	2 + 2 = 4	3 + 2 = 5	4 + 2 = 6	5 + 2 = 7	6 + 2 = 8
3	1 + 3 = 4	2 + 3 = 5	3 + 3 = 6	4 + 3 = 7	5 + 3 = 8	6 + 3 = 9
4	1 + 4 = 5	2 + 4 = 6	3 + 4 = 7	4 + 4 = 8	5 + 4 = 9	6 + 4 = 10
5	1 + 5 = 6	2 + 5 = 7	3 + 5 = 8	4 + 5 = 9	5 + 5 = 10	6 + 5 = 11
6	1 + 6 = 7	2 + 6 = 8	3 + 6 = 9	4 + 6 = 10	5 + 6 = 11	6 + 6 = 12

Figure 1 | Outcome for Rolled Cubes

Ondi	carnival worker	Carlton
1: no outcomes	5: 1-4, 2-3, 3-2, 4-1	9: 3-6, 4-5, 5-4, 6-3
2: 1-1	6: 1-5, 2-4, 3-3, 4-2, 5-1	10: 4-6, 5-5, 6-4
3: 1-2, 2-1	7: 1-6, 2-5, 3-4, 4-3, 5-2, 6-1	11: 5-6, 6-5
4: 1-3, 2-2, 3-1	8: 2-6, 3-5, 4-4, 5-3, 6-2	12: 6-6
6 total outcomes	20 total outcomes	10 total outcomes

Figure 2 | Possible Outcomes

Key	Ondi		carnival worker		Carlton	
	1	2	3	4	5	6
1	1 + 1 = 2	2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6	6 + 1 = 7
2	1 + 2 = 3	2 + 2 = 4	3 + 2 = 5	4 + 2 = 6	5 + 2 = 7	6 + 2 = 8
3	1 + 3 = 4	2 + 3 = 5	3 + 3 = 6	4 + 3 = 7	5 + 3 = 8	6 + 3 = 9
4	1 + 4 = 5	2 + 4 = 6	3 + 4 = 7	4 + 4 = 8	5 + 4 = 9	6 + 4 = 10
5	1 + 5 = 6	2 + 5 = 7	3 + 5 = 8	4 + 5 = 9	5 + 5 = 10	6 + 5 = 11
6	1 + 6 = 7	2 + 6 = 8	3 + 6 = 9	4 + 6 = 10	5 + 6 = 11	6 + 6 = 12

Figure 3 | Combinations by Each Person's Group

Keep in mind! The event of Ondi winning and the event of Carlton winning are disjoint events because they have no outcomes in common.

- Carlton was right. The probability of Ondi or Carlton winning is still less than the probability of the carnival worker winning.
- Probability of Ondi or Carlton winning:
  - *P*(Ondi wins *or* Carlton wins) = *P*(Ondi) + *P*(Carlton)
  - *P*(Ondi wins *or* Carlton wins) = $\frac{6}{36} + \frac{10}{36}$
  - *P*(Ondi wins *or* Carlton wins) = $\frac{16}{36}$
  - *P*(Ondi wins *or* Carlton wins) = $\frac{4}{9}$
- Probability of the carnival worker winning:
  - $\frac{20}{36} = \frac{5}{9}$

In this next example, you will figure out how this carnival game could be a fair game.

Example:

- If three people roll two number cubes, how can the game be made fair?

Solution:

- In order for the game to be fair, each person needs to have the same probability of winning. Since there are a total of 36 outcomes, each person needs 12 ways to win (36 ÷ 3 = 12). It really doesn't matter which outcomes are assigned, as long as each person has 12 favorable outcomes.
- Notice each row of the table in Figure 4 has 6 outcomes, so you could give each person 2 rows (6 · 2 = 12) of outcomes from the sample space.
- You could set up these rules for the game:
  - The first person would win by rolling any number on the first number cube and rolling a 1 or 2 on the second number cube.
  - The second person would win by rolling any number on the first number cube and rolling a 3 or 4 on the second number cube.
  - The third person would win by rolling any number on the first number cube and rolling a 5 or 6 on the second number cube.

Key	First Person		Second Person		Third Person	
	1	2	3	4	5	6
1	1 + 1 = 2	2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6	6 + 1 = 7
2	1 + 2 = 3	2 + 2 = 4	3 + 2 = 5	4 + 2 = 6	5 + 2 = 7	6 + 2 = 8
3	1 + 3 = 4	2 + 3 = 5	3 + 3 = 6	4 + 3 = 7	5 + 3 = 8	6 + 3 = 9
4	1 + 4 = 5	2 + 4 = 6	3 + 4 = 7	4 + 4 = 8	5 + 4 = 9	6 + 4 = 10
5	1 + 5 = 6	2 + 5 = 7	3 + 5 = 8	4 + 5 = 9	5 + 5 = 10	6 + 5 = 11
6	1 + 6 = 7	2 + 6 = 8	3 + 6 = 9	4 + 6 = 10	5 + 6 = 11	6 + 6 = 12

Figure 4 | Outcomes by Row for a Fair Game

Example:

- In the table of sums from Figure 4, you will see that there are an equal number of odd sums as there are even sums from the roll of two dice. A game based on odd or even sums would be a fair game because both players have an equal chance of winning. If the game were based on odd or even products (multiplication), would the game still be fair?

Solution:

- Begin by making a table of products similar to the table of sums, as shown in Figure 5.
- Products that are even are shaded in orange in Figure 5. Notice that there are 9 spaces that remain white, and 27 spaces that are orange. This would not be a fair game because whoever had evens would have an unfair advantage.

Have you noticed a pattern in each of the compound events you've looked at? Take a look.

In the first experiment, you tossed a coin and rolled a number cube. There were 2 outcomes for the first event, 6 outcomes for the second event, and 12 total outcomes.

- 2 outcomes · 6 outcomes = 12 outcomes

In the second experiment, you spun two spinners with 4 letters on each spinner. There were 4 outcomes for the first event, 4 outcomes for the second event, and 16 total outcomes.

- 4 outcomes · 4 outcomes = 16 outcomes

In the third experiment, you rolled two number cubes. There were 6 outcomes for the first event, 6 outcomes for the second event, and 36 total outcomes.

- 6 outcomes · 6 outcomes = 36 outcomes

Key point! The counting principle also works for more than two events. For example, if you have three events, just multiply the outcomes for all three events together to find the total number of outcomes.

In each case, if you multiply the number of outcomes for each event together, you get the total number of outcomes for the compound event. This is known as the *counting principle*:

- (outcomes for event 1)(outcomes for event 2) = total number of outcomes

The counting principle is especially handy when there are a large number of outcomes. Try this next example.

x	1	2	3	4	5	6
1	$1\times1=1$	$1\times2=2$	$1\times3=3$	$1\times4=4$	$1\times5=5$	$1\times6=6$
2	$2\times1=2$	$2\times2=4$	$2\times3=6$	$2\times4=8$	$2\times5=10$	$2\times6=12$
3	$3\times1=3$	$3\times2=6$	$3\times3=9$	$3\times4=12$	$3\times5=15$	$3\times6=18$
4	$4\times1=4$	$4\times2=8$	$4\times3=12$	$4\times4=16$	$4\times5=20$	$4\times6=24$
5	$5\times1=5$	$5\times2=10$	$5\times3=15$	$5\times4=20$	$5\times5=25$	$5\times6=30$
6	$6\times1=6$	$6\times2=12$	$6\times3=18$	$6\times4=24$	$6\times5=30$	$6\times6=36$

Figure 5 | Outcomes of Products

Example:

- A 20-sided icosahedron and an 8-sided octahedron are rolled. How many possible outcomes are there?

Solution:

- You can use the counting principle to find the total number of outcomes. There are 20 outcomes for the first event and 8 outcomes for the second event.
  - 20 outcomes · 8 outcomes = 160 outcomes
- There are 160 ways to roll a 20-sided icosahedron and an 8-sided octahedron. Imagine if you had to make a tree diagram or a table!

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Sample space is an organized list of all possible outcomes.
- Tree diagrams and tables can be used to determine sample space.
- A fair game is one in which each person has the same probability of winning.
- The counting principle states that to find the total number of outcomes for a compound event, you must multiply the number of outcomes for each event by each other.

Complete the following activities.

1.28 What is the probability of tossing two coins and having them both land on heads?
☐ 33.3% ☐ 25% ☐ 50% ☐ 75%

1.29 How many ways are there to choose a card from a deck of cards (52 cards in a deck) and roll a regular 6-sided number cube?
☐ 58 ☐ 52 ☐ 156 ☐ 312

1.30 The table below shows the sample space for spinning a 4-part spinner (labeled A, B, C, D) and then a 5-part spinner (labeled V, W, X, Y, Z). What is the probability of spinning AZ?

	A	B	C	D
V	AV	BV	CV	DV
W	AW	BW	CW	DW
X	AX	BX	CX	DX
Y	AY	BY	CY	DY
Z	AZ	BZ	CZ	DZ

☐ 25%
☐ 20%
☐ 10%
☐ 5%

1.31 Suppose you are going to make a sandwich. You have mayonnaise and mustard to spread on the bread. You have white, wheat, and rye bread. You have ham, turkey, and pastrami. How many different sandwiches can you make using one spread, one type of bread, and one type of meat?
☐ 18 ☐ 15 ☐ 9 ☐ 8

1.32 Which expression would you use to find the number of outcomes for flipping 4 coins?
☐ $2 + 2 + 2 + 2$ ☐ $2 \cdot 2 \cdot 2 \cdot 2$ ☐ $2(4)$ ☐ $2 + 4$

1.33 What is the probability of rolling a sum of 5 if two regular 6-sided number cubes are rolled?
☐ $\frac{1}{5}$ ☐ $\frac{1}{9}$ ☐ $\frac{5}{36}$ ☐ $\frac{1}{6}$

1.34 Select all that apply. Which of the following are true?
☐ A table can be used to show sample space.
☐ A tree diagram can be used to show sample space.
☐ Sample space is the probability of two events happening.
☐ The counting principle can be used to find the number of outcomes in the sample space.

1.35 Two people play a game in which they randomly draw numbers from 1 to 13. If the number is even, player 1 wins. Otherwise, player 2 wins. This is a fair game.

- ○ True
- ○ False

1.36 A restaurant gives kids a choice from 4 different entrées, 5 different sides, and 3 different drinks. How many different combinations are possible for a meal that includes one entrée, one side, and one drink?

1.37 What is the probability of rolling a sum of 7 on two standard 6-sided dice?

1.38 A school dress code offers students a choice between pants or shorts in either tan or navy, and collared shirts in either red, white, or blue. How many possible different combinations can a student make and still be within the dress code?

1.39 A combination lock requires a 3-digit number. How many different combinations are possible for the lock using the digits 0-9 in each of the three positions?

1.40 How many unique 2-letter combinations are possible using the 26 letters of the English alphabet?

Independent and Dependent Events

Suppose it's dark when you wake up, and you're too tired to turn the light on. You have 6 pairs of socks in your drawer, and you know only one pair is white. You reach in and pull out a pair of socks that are too dark to be white. You put the socks back and try again. This time you get another pair of dark socks! Were they the same pair?

Suppose you didn't put the first pair of socks back. Would your probability of drawing a white pair of socks the second time be the same as if you'd put them back in the drawer?

In this lesson, you will learn the difference between *independent events* (the first example) and *dependent events* (the second example). You will also find the probability of these kinds of events by using the counting principle.

Objectives

- Determine if events are independent or dependent.
- Determine the probability of independent and dependent events.

Vocabulary

dependent events—compound events in which one event affects the likelihood of the other event

independent events—compound events in which one event does not affect the likelihood of the other event

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Independent Events vs. Dependent Events

In the sock example, there is a 1 in 6 probability of getting a pair of white socks because there is 1 pair of white socks out of 6 total pairs:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{6}$$

Suppose you pull a pair of socks out of the drawer, and they are not white. If you put them back and then reach into the drawer to pull out another pair, the probability is the same as the first event because there are 6 total outcomes (6 pairs of socks) both times. These two events are independent events because the first event (choosing from 6 pairs) does *not* affect the outcome of the second event (choosing from 6 pairs of socks again). The second event is *independent* of what happens in the first event.

Now, suppose after you pull out the first pair of socks, you don't put them back. This time, when you reach for another pair, the probability has changed. Why? Because now there are only 5 pairs of socks in the drawer, and there is a 1 in 5 probability of choosing the white pair of socks.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{5}$$

These two events are dependent events because the first event (choosing from 6 pairs, leaving 5 pairs in the drawer) affects the outcome of the second event (choosing from 5 pairs of socks). The second event *depends* on what happens in the first event.

Example:

- Identify whether the events in the following scenario are independent or dependent.
- If you spin this spinner twice, what is the probability that you will spin green and then blue?

Solution:

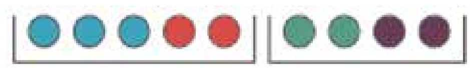
- The events are independent because the outcome of the first spin does not affect the results of the second spin. There are 4 colors to choose from each time.

Finding the Probability of Independent Events

Independent events are a type of compound event. In fact, all of the compound events you have looked at previously have been independent events. Using a table will let you see all of the possible outcomes for an experiment, which can help you understand how to find the probability.

Example:

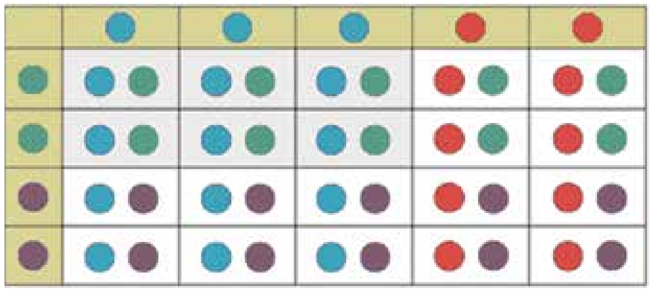
- A box contains 3 blue marbles and 2 red marbles. A second box contains 2 green marbles and 2 purple marbles. You randomly choose a marble from each box.



- What is the probability that you will choose a blue marble and a green marble?

Solution:

- First, the events are independent because choosing a marble from the first box does not affect the probability of choosing from the second box.
- Make a table for the sample space so you can see the number of favorable outcomes and the total number of outcomes:



- There are 6 favorable outcomes for a blue marble and a green marble (shaded in gray in the table) out of a total of 20 outcomes:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$= \frac{6}{20} = \frac{3}{10}$$

- So there is a $\frac{3}{10}$ probability that you will draw a blue marble and then a green marble.

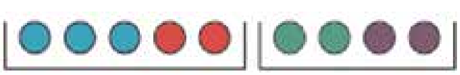
- Now look at the same event using the counting principle. The counting principle tells you that the total number of outcomes for a compound event is the number of outcomes for each event multiplied together.
- Take a look at the original ratio for the probability of an event:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

- You can use the counting principle to rewrite this ratio for the probability of a compound event:

$$P(\text{event 1 } \textit{and} \text{ event 2}) = \frac{\text{number of favorable outcomes}}{(\text{\# of outcomes for event 1})(\text{\# of outcomes for event 2})}$$

- You can also use the counting principle to find the number of favorable outcomes for compound events in the same way.



- Notice in the example that there are 3 favorable outcomes (the blue marbles) in the first box and 2 favorable outcomes in the second box (the green marbles). Multiplying (3 · 2 = 6) gives you 6 favorable outcomes.
- You can use this to rewrite your probability ratio again:

$$P(\text{event 1and event 2}) = \frac{(\text{\# of favorable outcomes for event 1})(\text{\# of favorable outcomes for event 2})}{(\text{\# of outcome for event 1})(\text{\# of outcomes for event 2})}$$

- Notice that each part of the ratio is the probability for each event. You can split the ratio into these two parts, just as you do when working with multiplying fractions:

$$P(\text{event 1 } \textit{and} \text{ event 2}) = \frac{(\text{\# of favorable outcomes for event 1})}{(\text{\# of outcomes for event 1})} \cdot \frac{(\text{\# of favorable outcomes for event 2})}{(\text{\# of outcomes for event 2})}$$

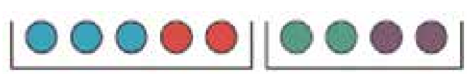
- Now you can simplify the two fractions:
- P(event 1 *and* event 2) = P(event 1) · P(event 2)
- You can call the first event A and the second event B.
- So to find the probability of two independent events, multiply the probability for each event together:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

- Now go back to the marble example using the new equation.

Example:

- A box contains 3 blue marbles and 2 red marbles. A second box contains 2 green marbles and 2 purple marbles. If you randomly choose a marble from each box, what is the probability that you will choose a blue marble and a green marble?



Solution:

- Find the probability for each event by comparing the number of favorable outcomes to the total number of outcomes for each event. Then multiply the probabilities together.

- For the first event, you need to draw a blue marble. There are 3 favorable outcomes out of a total of 5 outcomes: $\frac{3}{5}$.
- For the second event, you need to draw a green marble. There are 2 favorable outcomes out of a total of 4 outcomes: $\frac{2}{4}$.
- Now multiply the probabilities together:
  - $P(A \text{ and } B) = P(A) \cdot P(B)$
  - $P(A \text{ and } B) = \frac{3}{5} \cdot \frac{2}{4}$
  - $P(A \text{ and } B) = \frac{6}{20}$
  - $P(A \text{ and } B) = \frac{3}{10}$
- This is the same result that you got using the table.

Finding the Probability of Dependent Events

You can find the probability for dependent events the same way you did with independent events. Multiply the probability of each event together.

The difference is that the first event will affect the probability of the second event, so you need to take that into account when you find the probability of the second event.

Look at an example.

Example:

- There are six cards, numbered 1 to 6, lying face down. If you turn over one card as the tens place and another card as the ones place, what is the probability that the number will be 42?

Solution:

- The probability of the first event is 1 out of 6, or $\frac{1}{6}$, because only one out of the six cards has 4 on it.
- If you draw the 4, there are only five cards left in the pile. Now the probability of the second event is 1 out of 5, or $\frac{1}{5}$, because only one out of the five remaining cards has a 2 on it.
- Multiply the probability for each event together to find the probability for both events:
  - $\frac{1}{6} \cdot \frac{1}{5} = \frac{1}{30}$
- Remember, with dependent events, the second event *depends* on the first event. The compound event can only happen if the first event happens, so you have to adjust the second event's probability based on that outcome.
- Rewrite the equation to account for this:
  - $P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$

Try another example.

Example:

- A store is giving away 20 gift cards as part of a promotion. Each card has a different value, ranging from \$5 to \$100. If you and a friend each get a gift card, what is the probability that one of you will get the \$100 card and the other will get the \$5 card?

Solution:

- Event *A* is the probability of getting the \$100 card. There is 1 card out of the 20 cards with a favorable outcome.

$$P(A) = \frac{1}{20}$$

- Event B is the probability of getting the \$5 card. If the \$100 card is already removed from the mix, there are 19 remaining cards. There is 1 of the remaining 19 cards with a favorable outcome.

$$P(B) = \frac{1}{19}$$

- The probability of both events occurring is

$$P(A \text{ and } B) = \frac{1}{20} \cdot \frac{1}{19} = \frac{1}{380}$$

Example:

- There are 10 white marbles and 15 black marbles in a bag. If you choose a marble from the bag, *don't replace it*, and then choose another marble, what is the probability that you will get two white marbles?

Solution:

- Find the probability for each event and multiply them together.
- There are 10 favorable outcomes out of 25 total outcomes for the first choice: $\frac{10}{25}$.

Think about it! There are 9 favorable outcomes because there are only 9 white marbles left. There are 24 total outcomes because there are 24 marbles left.

- If you choose a white marble, there are now 9 favorable outcomes (10 – 1 = 9) and 24 total outcomes (25 – 1 = 24): $\frac{9}{24}$.
- Now multiply the probabilities:
  - *P*(*A* and *B*) = *P*(*A*) · *P*(*B* given *A*)
  - *P*(*A* and *B*) = $\frac{10}{25} \cdot \frac{9}{24}$
  - *P*(*A* and *B*) = $\frac{90}{600}$
  - *P*(*A* and *B*) = $\frac{15}{100}$
  - *P*(*A* and *B*) = 15%
- So there is a 15% probability that you will get two white marbles if the first marble isn't replaced.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Independent events are compound events in which one event does *not* affect the likelihood of the other event.
- Dependent events are compound events in which one event *does* affect the likelihood of the other event.
- You can find the probability of independent events by multiplying the probability for each event together:
  - *P*(*A* and *B*) = *P*(*A*) · *P*(*B*)
- You can find the probability of dependent events in the same way as independent events. But you must account for the change in the second event's probability due to the first event happening:
  - *P*(*A* and *B*) = *P*(*A*) · *P*(*B* given *A*)

Complete the following activities.

1.41 Select all that apply. Which of the following are independent events?

- ☐ You flip a coin and roll a number cube.
- ☐ You draw a marble from a bag, replace it, and draw again.
- ☐ You randomly choose 2 socks from a drawer, hoping they will match.
- ☐ You draw two numbers from 1 to 10 from a hat without replacing the first.

1.42 You roll a number cube twice. What is the probability of rolling a 2 first and then rolling an odd number?

☐ $\frac{1}{36}$ ☐ $\frac{1}{12}$ ☐ $\frac{1}{9}$ ☐ $\frac{1}{4}$

1.43 There are 4 chocolate chip cookies and 12 oatmeal cookies in a jar. If you reach in and randomly choose 2 cookies *without replacing the first*, what is the probability that both will be chocolate chip?

☐ $\frac{1}{9}$ ☐ $\frac{3}{64}$ ☐ $\frac{1}{20}$ ☐ $\frac{1}{11}$

1.44 There are 5 red marbles, 15 blue marbles, and 5 green marbles in a bag. If you reach in and randomly draw two marbles *without replacing the first*, what is the probability that both will be blue?

☐ $\frac{8}{15}$ ☐ $\frac{3}{5}$ ☐ $\frac{19}{49}$ ☐ $\frac{7}{20}$

1.45 There are 5 red marbles, 15 blue marbles, and 5 green marbles in a bag. You reach in and randomly draw a marble, replace it, and draw another marble. What is the probability that one will be red and one will be blue?

☐ 80% ☐ 20% ☐ 12% ☐ 8%

1.46 You spin a spinner and then flip a coin. The events are independent.

- ○ True
- ○ False

1.47 There are 4 white marbles and 1 black marble in a bag. You draw two marbles, one after the other. Is the probability for both of the marbles to be white *higher* if you replace the first marble drawn or *higher* if you do not replace it?

- ☐ higher if you replace the first marble
- ☐ higher if you don't replace the first marble
- ☐ the same in either case

Complete the following activities.

A box of 8 crayons has one each of red, orange, yellow, green, blue, purple, black, and brown.

1.48 What is the probability of randomly selecting the blue crayon and then randomly selecting the orange crayon if you *do not* replace the first crayon before selecting the second crayon?

1.49 What is the probability of randomly selecting the black crayon and then randomly selecting the purple crayon if you *do* replace the first crayon before selecting the second crayon?

1.50 You need a red crayon, a yellow crayon, and a green crayon to color a picture of a traffic light. What is the probability that you will randomly select the colors in that order if you replace each crayon before selecting the next one?

1.51 What is the probability of selecting the red, yellow, and green crayons in that order if you do not replace the crayons after each selection?

1.52 What is the probability of selecting the red, yellow, and green crayons in any order if you do not replace the crayons after each selection?

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

Self Test 1: Probability

Complete the following activities (5 points, each numbered activity).

1.01 Suppose you are asked to pick 3 numbers from 1 to 20 to win a prize. What is the probability that one of the numbers you will pick is the winning number?

- ☐ 5%
- ☐ 10%
- ☐ 15%
- ☐ 20%

1.02 At a carnival game, there is a 38% probability of winning a prize. What is the probability of not winning a prize?

- ☐ 38%
- ☐ 50%
- ☐ 62%
- ☐ 76%

1.03 Alice and Finn roll two number cubes. Which of the following rules will make the game fair?

- ☐ Alice wins if a total of 5 is rolled. Finn wins if a total of 9 is rolled.
- ☐ Alice wins if a total of 3 is rolled. Finn wins if a total of 10 is rolled.
- ☐ Alice wins if a total of 7 is rolled. Finn wins if a total of 8 is rolled.
- ☐ Alice wins if a total of 4 is rolled. Finn wins if a total of 11 is rolled.

1.04 You have 2 spreads, 5 meats, and 2 kinds of bread. How many different sandwiches can you make using one of each type of ingredient?

- ☐ 9
- ☐ 12
- ☐ 20
- ☐ 40

1.05 There are 6 red marbles, 8 blue marbles, and 11 green marbles in a bag. What is the probability that you will randomly draw either a red or a blue marble?

- ☐ 24%
- ☐ 56%
- ☐ 32%
- ☐ 10%

1.06 What is the experimental probability of drawing a red marble, given the following results?

Marble Color	Blue	Green	Red
Times Drawn	6	6	8

- ☐ $\frac{2}{5}$
- ☐ $\frac{4}{5}$
- ☐ $\frac{2}{3}$
- ☐ $\frac{7}{10}$

1.07 A coin is flipped 40 times, and it lands on heads 16 times. Based on the experimental probability, how many heads would you predict for 200 flips of the coin?

- ☐ 90
- ☐ 80
- ☐ 56
- ☐ 112

1.08 Select all that apply. A spinner is divided into 4 equal sections. Which of the following are true?

- ☐ The spinner will land on each section of the spinner an equal number of times.
- ☐ The theoretical probability is 25% for each section.
- ☐ If the spinner is spun 64 times, you would predict it to land on each section 16 times.
- ☐ The experimental probability is $\frac{1}{4}$ for each section.

1.09 Select all that apply. There are 3 red marbles, 5 green marbles, and 2 blue marbles in a bag. Which of the following are true?

- ☐ The probability of randomly drawing either a red marble or a green marble is 80%.
- ☐ The probability of randomly drawing a red marble and then a green marble is $\frac{1}{6}$.
- ☐ The probability of not drawing a blue marble is 20%.
- ☐ The probability of drawing a green marble is the same as the probability of drawing either a red or a blue marble.

1.010 Select all that apply. A coin is flipped and a number cube is rolled. Which of the following are true?

- ☐ The sample space has 12 different outcomes.
- ☐ The sample space has 8 different outcomes.
- ☐ Each result is equally likely.
- ☐ Heads and an even number are very likely.

1.011 Three coins are flipped. What is the probability that there will be at least two tails?

☐ $\frac{3}{8}$ ☐ $\frac{1}{2}$ ☐ $\frac{1}{8}$ ☐ $\frac{1}{9}$

1.012 Suppose there are 21 students in your class. If the teacher draws 2 names at random, what is the probability that you and your best friend will be chosen?

☐ $\frac{2}{21}$ ☐ $\frac{1}{20}$ ☐ $\frac{1}{105}$ ☐ $\frac{1}{210}$

1.013 Which of the following are dependent events?

- ☐ rolling a number cube and then flipping a coin
- ☐ spinning a spinner and then rolling a number cube
- ☐ drawing a marble from a bag, not replacing it, and then drawing a second marble
- ☐ choosing a number from a hat, replacing it, and then choosing another number

1.014 What is the probability of rolling an odd number and spinning "B," given the sample space below?

	1	2	3	4	5	6
A	A1	A2	A3	A4	A5	A6
B	B1	B2	B3	B4	B5	B6
C	C1	C2	C3	C4	C5	C6
D	D1	D2	D3	D4	D5	D6

☐ $\frac{1}{2}$ ☐ $\frac{1}{8}$

☐ $\frac{1}{6}$ ☐ $\frac{1}{12}$

1.015 A number cube is rolled and a coin is flipped. Predict how many times you would get heads and a number less than 3 in 240 trials.

☐ 60 ☐ 40 ☐ 30 ☐ 20

1.016 What is the probability of rolling doubles on a pair of standard 6-sided dice?

1.017 What is the probability of rolling doubles three times in a row on a pair of standard 6-sided dice?

1.018 You have 8 tickets out of the 150 tickets in a drawing. What is the probability that one of your tickets will be drawn first?

1.019 A baseball player's batting average is 0.333, which means he gets a hit 33% of the time, or $\frac{1}{3}$ of the time. Based on this information, how many hits would you expect the player to have in a series if he has 12 at-bats in the series?

1.020 How many meal combinations are possible that contain one appetizer, one entrée, and one dessert from a menu with 4 appetizers, 5 entrées, and 3 desserts?

80/100 **SCORE** ________ **TEACHER** ________ ________

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