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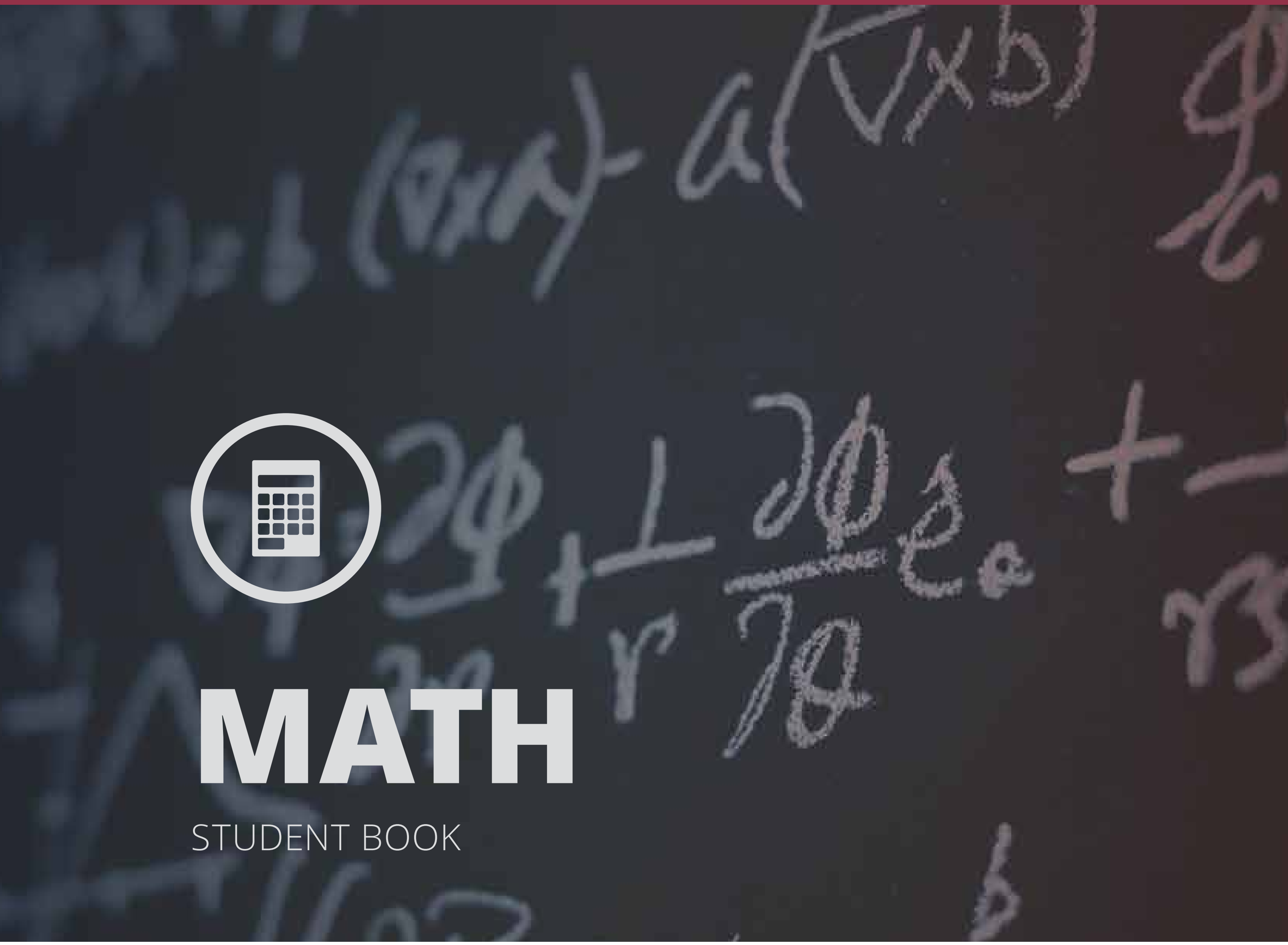

MATH

STUDENT BOOK

7th Grade | Unit 8

Alpha Omega
PUBLICATIONS

Math 708
Geometry

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Geometry

Introduction

In this unit, students will be introduced to geometry. They will learn basic terms and notation for points, lines, line segments, rays, angles, planes, polygons, and circles. Students will learn about the sum of angles for any polygon, as well as find angle measures in regular polygons. Students will also classify triangles by side and angle, learn about types of quadrilaterals, and solve for missing angle measures.

Students will then be introduced to transformations in the coordinate plane. They will explore symmetry in polygons, including line and rotational symmetry. They will also investigate reflections, noting the similarities to line symmetry, and work with translations in the coordinate plane. Students will learn how the coordinates are affected in these transformations and apply this knowledge to compound transformations.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Identify basic geometric components and shapes.
- Use angle and circle properties to determine missing angle measures and to find angle sums.
- Identify corresponding parts of similar and congruent figures.
- Use the properties of similar and congruent figures to solve problems.
- Determine if a figure has line symmetry or rotational symmetry.
- Determine the coordinates of an image following a reflection, translation, or compound transformation.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. Basic Geometry

Introduction to Geometry

geometry \jē-'ä-mə-trē\

1: noun — a branch of mathematics concerned with the measurement, properties, and relationships of points, lines, angles, shapes, and figures

2: exclamation — what the acorn said when it grew up: "Gee, I'm a tree!"

In this unit, you will begin your exploration of the branch of mathematics known as geometry. You will begin by learning about the building blocks of geometry: *points*, *lines*, and *planes*.

Objectives

- Identify basic geometric components.
- Use correct geometric terminology and notation.
- Classify angles by their measures.

Vocabulary

acute angle—an angle measuring less than 90°
angle—two rays with a common endpoint
collinear—on the same line
dimensions—the measurements of an object (e.g., length, width, or height)
endpoint—a point that marks the end of a line segment or ray
line—an infinite set of points forming a straight path that continues in two directions
line segment—a part of a line bounded by two endpoints
obtuse angle—an angle measuring greater than 90°
plane—a flat surface that continues in all directions
point—a position in space
ray—a part of a line that has one endpoint and continues in one direction
right angle—an angle measuring 90°
straight angle—an angle measuring 180°
vertex—the point where two line segments, lines, or rays meet to form an angle

Point

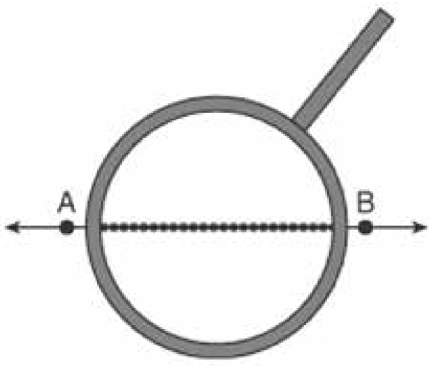
In geometry, a point defines a place in space. A point has no *dimensions* or measurements, but you can name its location with a capital letter and draw a representation of a point with a dot.

The point *P* can be represented as •*P*.

Line

An infinite series of *collinear* points, or points lined up in a row, is called a line. A line can be named by any two points on the line because there can be only one line between any two points. The symbol $\leftrightarrow$ is used to indicate a line.

Key point! You can think of a line as an infinite series of points. However, even if you could magnify the line, you wouldn't see actual points because they have no dimensions.

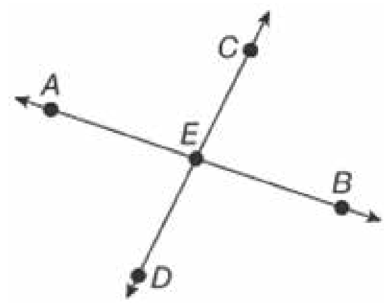


The line *AB* can be represented as $\overleftrightarrow{AB}$. The same line could also be named line *BA* or $\overleftrightarrow{BA}$.

The arrows indicate that the line keeps on going infinitely in both directions.

A line can also be named by a single lowercase letter, such as line *a*.

If two lines intersect, the intersection will be a point.

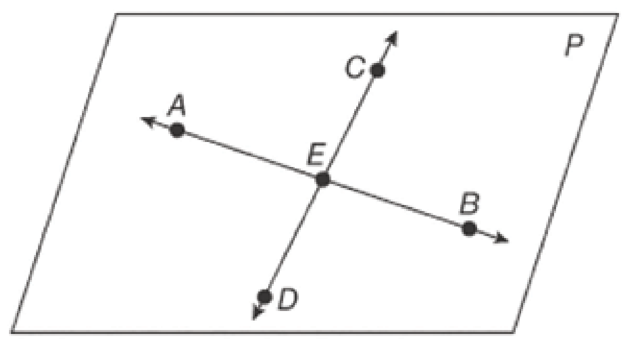


$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point *E*.

Plane

A plane is a flat surface continuing in all directions. Any two intersecting lines will be contained in a plane. A plane can be named by a single capital letter, such as plane *P*.

Vocabulary! You can think of a plane as a sheet of paper with no thickness (just like a line) that goes on forever in all directions.



Ray

A *ray* of sunshine starts at the sun and moves straight ahead.

Keep in mind! You can't change the order of the letters when naming a ray as you can with a line. The first point is the endpoint, and the ray goes toward the second point. So the letters also indicate which direction the ray is going.

A *ray* in geometry is similar. It is half of a line that has one *endpoint* and continues forever away from the point in one direction. A ray is named by its endpoint and any other point on the ray. The symbol $\rightarrow$ indicates a ray.

Ray *AB* can be represented as $\overrightarrow{AB}$.

Line Segment

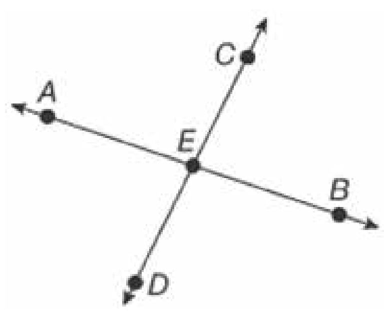
A *line segment* is a part of a line that has two endpoints and includes all the points between the endpoints. A line segment is named by the endpoints and shows a short line over the letters.

Line segment *AB* can be represented as $\overline{AB}$. The same line segment could also be named line segment *BA* or $\overline{BA}$.

Practice using some of these terms.

Example:

- In the following figure, name a point, a line, a ray, and a line segment.



Solution:

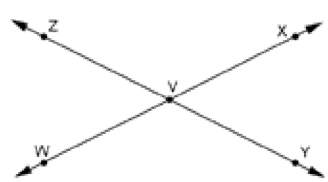
- Look at the figure and think about the definition of each term.
- Point: Although there are an infinite number of points on each line, there are five labeled points: point *A*, point *B*, point *C*, point *D*, and point *E*.

Keep in mind! Although any two points define a line, you can't name a line in a drawing unless it is shown. For example, point C and point A do not define a line in the drawing.

- Line: Any two points on a line can name the line: $\overleftrightarrow{AB}$, $\overleftrightarrow{AE}$, $\overleftrightarrow{EB}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{EA}$, and $\overleftrightarrow{BE}$.
- Ray: Again, you can choose two points on one of the lines, but one must be the endpoint: $\overrightarrow{DE}$, $\overrightarrow{DC}$, $\overrightarrow{CE}$, $\overrightarrow{CD}$, $\overrightarrow{ED}$, and $\overrightarrow{EC}$.
- Line Segment: You can choose any two collinear points, but this time both must be endpoints: $\overline{AB}$, $\overline{DE}$, $\overline{CD}$, and $\overline{BE}$.

Example:

- In the following figure, name as many lines as possible.



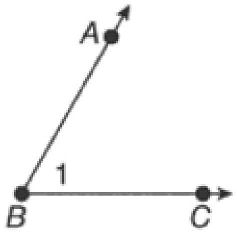
Solution:

$\overleftrightarrow{ZY}, \overleftrightarrow{YZ}, \overleftrightarrow{ZV}, \overleftrightarrow{VZ}, \overleftrightarrow{VY}, \overleftrightarrow{YV}, \overleftrightarrow{WX}, \overleftrightarrow{XW}, \overleftrightarrow{WV}, \overleftrightarrow{VW}, \overleftrightarrow{XV}, \overleftrightarrow{VX}$

Remember! Each line can be written forward and backward.

Angles

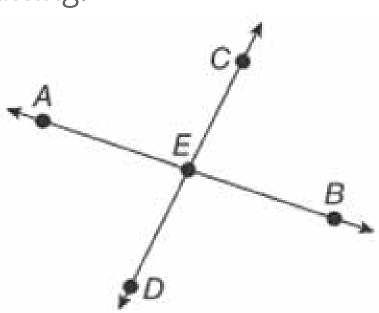
Two rays with a common endpoint form an *angle*. The endpoint is called the *vertex*. There will be angles anywhere lines intersect. The symbol ∠ is used to indicate an angle. Angles can be named three different ways.



- The angle can be named with three letters. The letters, in order, are a point on one ray, the vertex, and a point on the other ray:
- ∠*ABC* or ∠*CBA*
- The angle can be named with one letter, using just the vertex, as long as it is the only angle in the drawing with that vertex:
- ∠*B*
- The angle can be named with a number. The number is written inside the two rays:
- ∠1

Example:

- Name the angles shown in the drawing.



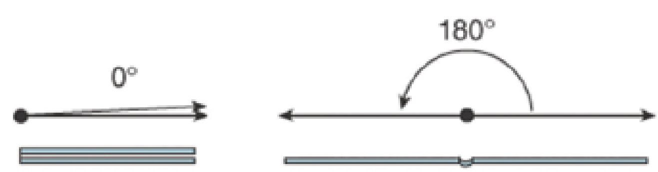
Solution:

- You can't use a numerical name for the angles because none of the angles are labeled numerically. You also can't name the angle by the vertex because point *E* is the vertex for all of the angles.
- So you'll need to use the three-letter designation to name the angles. Use the points on the rays and the vertex *E* to name the angles: ∠*AED*, ∠*AEC*, ∠*CEB*, and ∠*BED*.

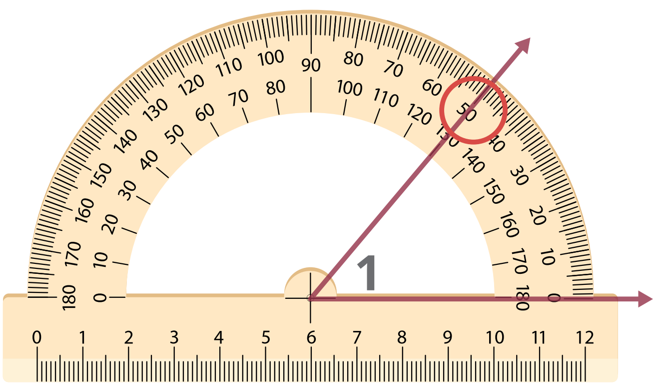
Angle Measurement

Angles are measured in degrees according to how far apart the two rays are. Picture a closed folder on your desk. The edges of the front and back of the folder represent the two rays. When the folder is closed, the angle measure is 0°. As the folder opens, the angle measure increases until the folder is opened flat on the desk and the angle measures 180°. An angle with a measure of 180° is called a *straight angle*.

Key point! The symbol ° above and to the right of the angle measure indicates degrees, just as it does for degrees of temperature.



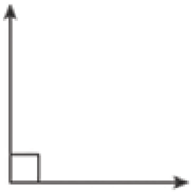
Angles are measured using a tool called a protractor.



There are three types of angles. They are named for how they relate to 90°:

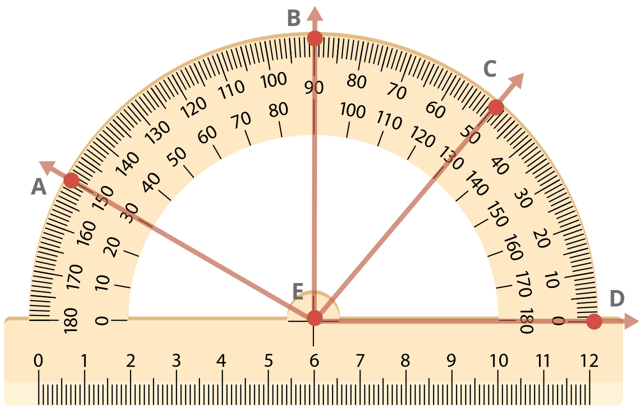
- angle < 90°: *acute angle*
- angle = 90°: *right angle*
- angle > 90°: *obtuse angle*

Did you know! A 90° angle is often shown with a small square at the vertex to indicate that it is a right angle.



Example:

- What are the measures of the angles shown on the following protractor, and what types of angles are they?



Make note! Notice that the protractor is numbered from 0° to 180°, and the measurements go from left to right and from right to left. This is so you can measure angles that open in either direction.

Solution:

- Compare the angles to 90° to decide which measure to use and how to classify them.
- *∠DEC* is less than 90°. It measures 50° and is an acute angle.
- *∠DEB* is 90°, so it is a right angle.
- *∠DEA* is greater than 90°. It measures 150° and is an obtuse angle.

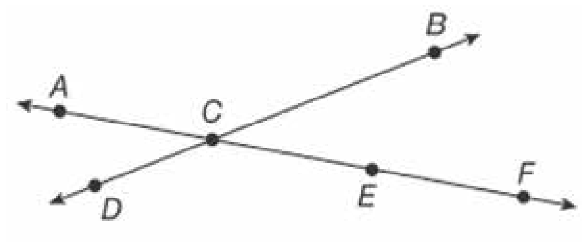
Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Geometry is a branch of mathematics that deals with the properties of points, lines, angles, and planes.
- Angles are measured in degrees from 0° for a closed angle to 180° for a straight angle.
- Angles are named as they relate to 90°.
    - Angles greater than 90° are obtuse angles.
    - Angles equal to 90° are right angles.
    - Angles less than 90° are acute angles.

Complete the following activities.

1.1 Select all that apply. Which of the following name a line in the drawing?



- ☐ $\overleftrightarrow{EC}$
- ☐ $\overleftrightarrow{EB}$
- ☐ $\overleftrightarrow{DB}$
- ☐ $\overleftrightarrow{FC}$

1.2 Select all that apply. Which of the following name a ray in the drawing above?

- ☐ $\overrightarrow{FC}$
- ☐ $\overrightarrow{CD}$
- ☐ $\overrightarrow{AF}$
- ☐ $\overrightarrow{FD}$

1.3 Select all that apply. Which of the following name a line segment in the drawing above?

- ☐ $\overline{AF}$
- ☐ $\overline{CE}$
- ☐ $\overline{AB}$
- ☐ $\overline{EF}$

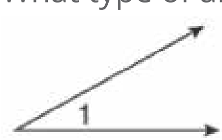
1.4 Select all that apply. Which of the following name an angle in the drawing above?

- ☐ ∠ACB
- ☐ ∠CDE
- ☐ ∠ECB
- ☐ ∠BDA

1.5 What is the intersection of $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ in the drawing above?

- ☐ point *A*
- ☐ point *D*
- ☐ point *C*
- ☐ point *E*

1.6 What type of angle is ∠1?



- ☐ obtuse
- ☐ straight
- ☐ acute
- ☐ right

1.7 Which measurement is the measure of an obtuse angle?

- ☐ 75°
- ☐ 87°
- ☐ 137°
- ☐ 90°

1.8 Use a protractor to find the measure of the angle below.

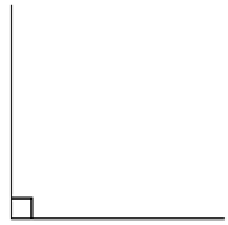
- ☐ 170°
- ☐ 10°
- ☐ 15°
- ☐ 165°

1.9 What does the notation $\overrightarrow{PQ}$ mean?

1.10 What does the notation •R mean?

Identify each angle below as acute, right, or obtuse.

1.11



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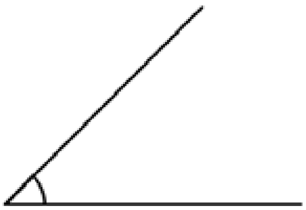

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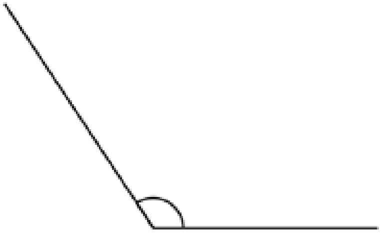
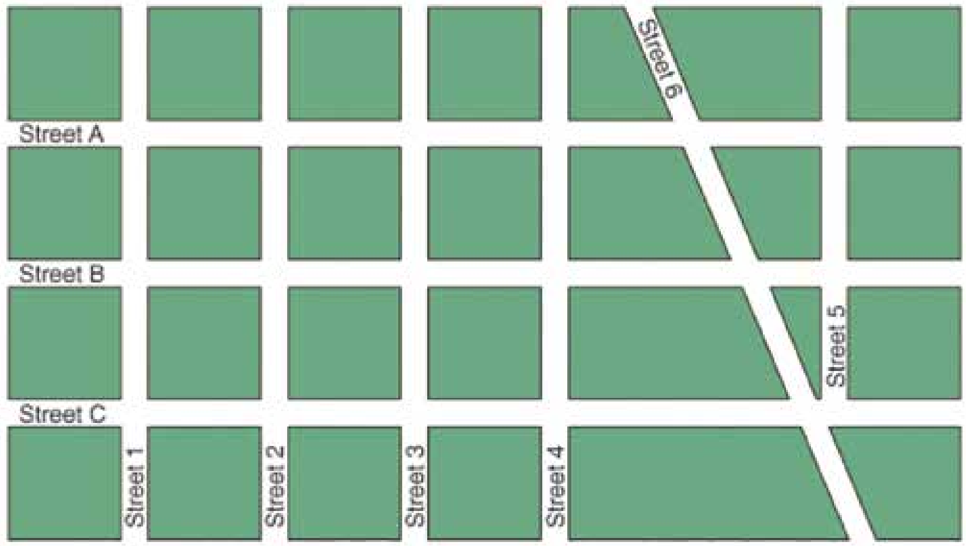


Special Pairs of Angles



If you've ever looked at a city map, you've probably noticed that some streets intersect, but others never do. Some streets intersect at right angles, but others intersect diagonally.

In this lesson, you will look at lines that have some of the same properties as streets. You will also look at the special angles that result when lines cross.

Objectives

- Identify special pairs of angles.
- Use angle properties to determine missing angle measures.

Vocabulary

adjacent angles—two angles that have a common vertex and side but are not overlapping

complementary angles—two angles whose sum is 90°

congruent angles—angles that have the same measure

corresponding angles—two angles in the same position on different lines

parallel lines—lines that never cross one another and are the same distance apart at all times

perpendicular lines—lines that intersect and create right angles

supplementary angles—two angles whose sum is 180°

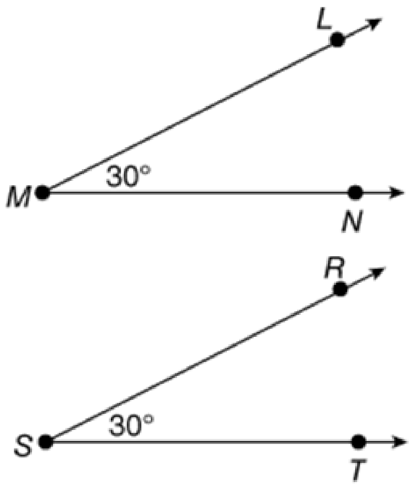
transversals—lines that intersect two or more lines to create angles

vertical angles—congruent angles that are opposite from one another at the intersection of two lines

Special Pairs of Angles

Several of the pairs of angles you'll be looking at are *congruent angles*. Congruent angles have the same measure. The symbol ≅ is used to indicate congruency.

To indicate that ∠*LMN* is congruent to ∠*RST*, you would write ∠*LMN* ≅ ∠*RST*.



The measure of an angle is represented by a lowercase m:

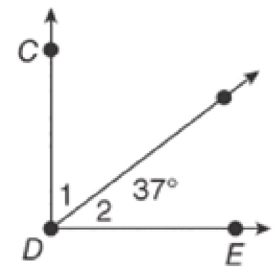
To indicate that the measure of ∠*LMN* is 30°, you would write m ∠*LMN* = 30°.

The first pair of special angles you will look at in Figure 1 are *complementary angles*. Complementary angles are any two angles whose angle measures add up to 90°, or make up a right angle.

Complementary angles can be *adjacent angles*, like the pair on the left of Figure 1. This means they share the vertex and a side.

Example:

- If m ∠*CDE* = 90°, and m∠2 = 37°, what is the measure of ∠1?



Solution:

- The angles are complementary because their measures add up to 90°. So you can solve for ∠1.

m∠1 + m∠2 = 90°	definition of complementary angles
x + 37° = 90°	Substitute in the known value and a variable you can solve for.
x = 53°	Subtract 37° from both sides.

- So the measure of ∠1 is 53°.

Another pair of special angle similar to complementary angles are *supplementary angles*, as shown in Figure 2. Supplementary

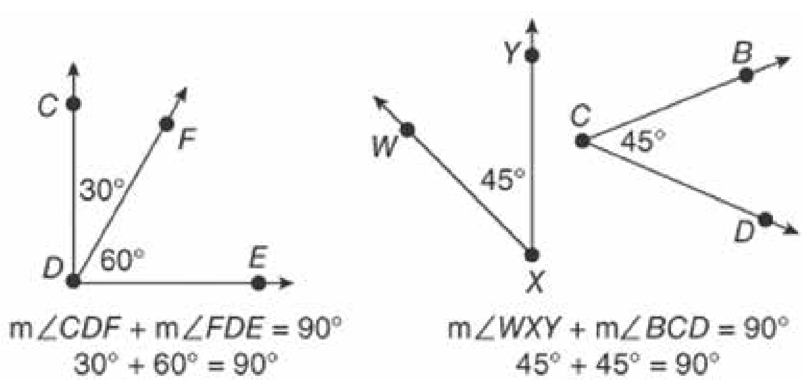
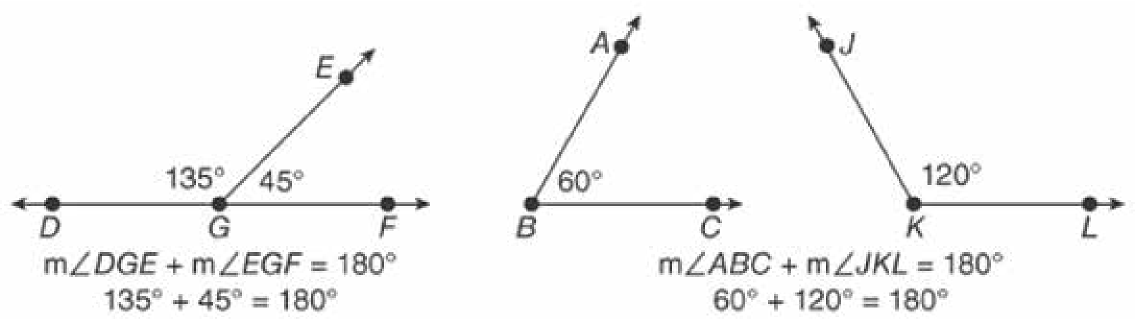


Figure 1 | Complementary Angles

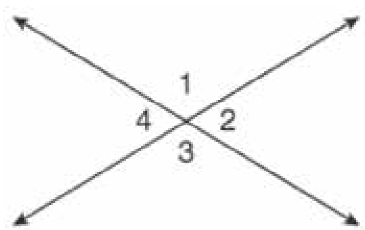
Figure 2 | Supplementary Angles



angles are any two angles whose angle measures add up to 180°.

Supplementary angles can be adjacent, in which case they form a straight angle or line, like the pair on the left in Figure 2. Or they can be any two angles whose measures add up to 180°, like the angles on the right in Figure 2.

When two lines intersect, four angles are created, some of which are supplementary. Do you see any supplementary angles in the following image?



Notice that ∠1 and ∠2 are adjacent and supplementary. This is because, together, they form a straight line, or straight angle, which is 180°.

Notice that ∠1 and ∠4 are also supplementary for the same reason. This means that ∠2 and ∠4 are congruent:

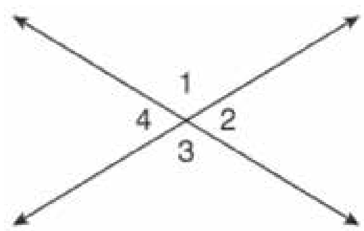
$m\angle 1 + m\angle 2 = 180°$ — definition of supplementary angles

$m\angle 1 + m\angle 4 = 180°$ — definition of supplementary angles

$m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4$ — Set the pairs of supplementary angles equal to each other.

$m\angle 2 = m\angle 4$ — Subtract m∠1 from both sides.

∠2 and ∠4 are called *vertical angles*. Vertical angles are the angles opposite each other when two lines intersect. They are always congruent.

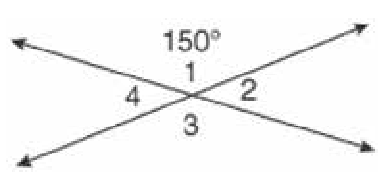


Think about it! Vertical angles are congruent because both angles are supplementary to the same angle: ∠1 and ∠2 add up to 180°, and ∠1 and ∠4 add up to 180°.

When two lines intersect, you can find all of the angle measures if you know just one angle measure.

Example:

- If m∠1 = 150°, find the measures of ∠2, ∠3, and ∠4.



Solution:

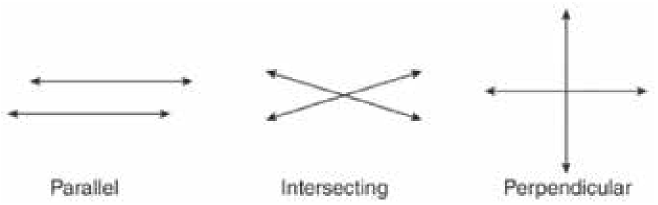
- Remember that vertical angles are congruent and adjacent angles are supplementary when two lines intersect.
- You know that m∠1 = 150° and that ∠1 and ∠2 are supplementary.

m∠1 + m∠2 = 180°	definition of supplementary angles
150° + x = 180°	Substitute in the known value and a variable you can solve for.
x = 30°	Subtract 150° from both sides.

- So m∠2 = ∠30°.
- Since ∠2 and ∠4 are vertical angles, m∠4 = 30°.
- Since ∠1 and ∠3 are vertical angles, m∠3 = 150°.

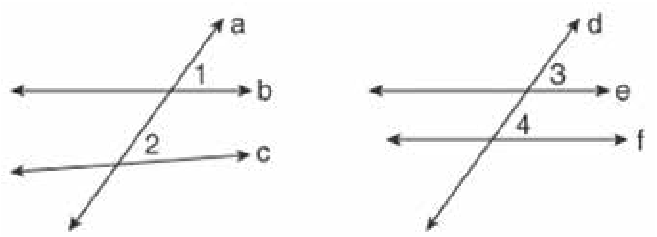
Corresponding Angles

If there are two lines in a plane, three things can happen:



Parallel lines never meet and stay the same distance apart. Lines can also intersect at any angle, just as streets do. If the lines intersect at right angles, they are called *perpendicular lines*.

Sometimes another line cuts, or crosses, a pair of lines, creating angles. The line that crosses the pair of lines is called a *transversal*.



Lines *a* and *d* are transversals. Angles that are in corresponding positions on each line are called *corresponding angles*. Notice that ∠1 and ∠2 are corresponding angles, as are ∠3 and ∠4.

If the two lines intersected by the transversal are parallel, the corresponding angles are congruent. Lines *b* and *c* are not parallel, so ∠1 and ∠2 are not congruent. However, lines *e* and *f* are parallel, so ∠3 and ∠4 are congruent.

You may have noticed that a pair of lines cut by a transversal creates several vertical angles and adjacent supplementary angles. If the two lines are parallel, there are lots of congruent angles, see Figure 3.

Congruent Angles	
Corresponding Angles	**Vertical Angles**
m∠1 ≅ m∠5	m∠1 ≅ m∠4
m∠2 ≅ m∠6	m∠2 ≅ m∠3
m∠3 ≅ m∠7	m∠5 ≅ m∠8
m∠4 ≅ m∠8	m∠6 ≅ m∠7

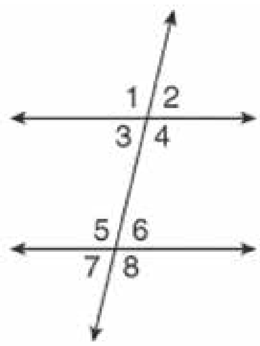


Figure 3 | Congruent Angles

Using the relationships of corresponding and vertical angles, you can show that many other pairs of angles are congruent.

For example, you can show that $\angle 2 \cong \angle 7$.

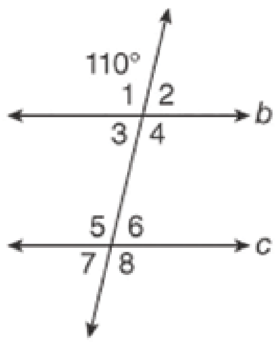
Figure 3, $\angle 2 \cong \angle 6$, and $\angle 6 \cong \angle 7$, so $\angle 2 \cong \angle 7$ by the transitive property.

Connections! Transitive property: If $a = b$ and $b = c$, then $a = c$.

Since you know that there are also pairs of supplementary angles when a transversal crosses two parallel lines, you can solve for all the angle measures if you are given one of them.

Example:

- If lines *b* and *c* are parallel, and $m\angle 1 = 110°$, find the measures of $\angle 5$, $\angle 6$, and $\angle 8$.



Solution:

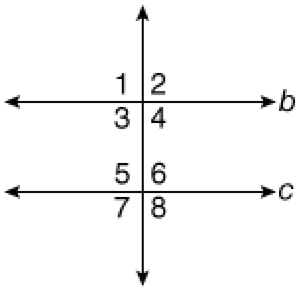
- Look at pairs of corresponding, vertical, and supplementary angles to find the angle measures.
- Since $\angle 1$ and $\angle 5$ are corresponding angles, they are congruent.
- Since $m\angle 1 = 110°$, $m\angle 5 = 110°$.
- Since $m\angle 5 = 110°$ and you can see that $\angle 5$ and $\angle 6$ are supplementary, you can solve for $\angle 6$.

$m\angle 5 + m\angle 6 = 180°$	definition of supplementary angles
$110° + x = 180°$	Substitute in the known value and a variable you can solve for.
$x = 70°$	Subtract 110° from both sides.

- So $m\angle 6 = 70°$.
- You can see that $\angle 5$ and $\angle 8$ are vertical angles. You also know that vertical angles are congruent.
- Since $m\angle 5 = 110°$, $m\angle 8 = 110°$.
- So $m\angle 5 = 110°$, $m\angle 6 = 70°$, and $m\angle 8 = 110°$.

Example:

- If two parallel lines are cut by a perpendicular transversal, what will the measure of each angle be?



Solution:

- This is a unique case because it is the only time supplementary angles are congruent ($90° + 90° = 180°$). The definition of a perpendicular line is that it intersects at 90°. So all of the angles are 90°, and all of the angles are congruent.

Example:

- If two parallel lines are cut by a transversal and the measure of one angle is $90°$, what is the measure of the other angles?

Solution:

- If the measure of one angle is $90°$, the transversal must be perpendicular to the parallel lines. This makes every angle $90°$.

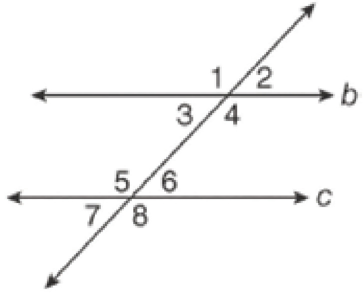
Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Congruent angles have the same measure.
- Complementary angles are two angles whose measures total 90°.
- Supplementary angles are two angles whose measures total 180°.
- Vertical angles and corresponding angles are congruent.

Complete the following activities.

1.14 Select all that apply. Which pairs of angles are supplementary?



- ☐ ∠1 and ∠4
- ☐ ∠5 and ∠6
- ☐ ∠2 and ∠8
- ☐ ∠3 and ∠6

1.15 Select all that apply. Which pairs of angles are vertical angles in the drawing above?

☐ ∠2 and ∠3 ☐ ∠5 and ∠8 ☐ ∠7 and ∠2 ☐ ∠4 and ∠5

1.16 Select all that apply. Which pairs of angles are congruent in the drawing above?

☐ ∠1 and ∠4 ☐ ∠3 and ∠5 ☐ ∠2 and ∠8 ☐ ∠3 and ∠7

1.17 Select all that apply. Which angles are congruent to ∠3 in the drawing above?

☐ ∠2 ☐ ∠7 ☐ ∠6 ☐ ∠4

1.18 ∠*A* and ∠*B* are complementary. If ∠*B* = 26°, what is the measure of ∠*A*?

☐ 26° ☐ 64° ☐ 154° ☐ 74°

1.19 ∠*A* and ∠*B* are supplementary. If m∠*A* = 117°, what is the measure of ∠*B*?

☐ 243° ☐ 83° ☐ 63° ☐ 117°

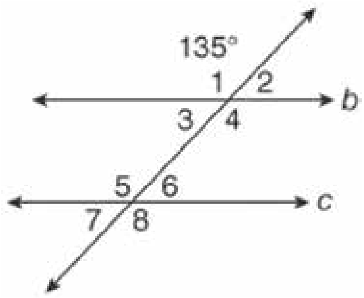
1.20 If two angles are supplementary and congruent, what is the measure of each angle?

☐ 45° ☐ 100° ☐ 90° ☐ 180°

1.21 Select all that apply. If two lines intersect and one angle measures 35°, what are the measures of the other angles?

☐ 35° ☐ 65° ☐ 145° ☐ 45°

1.22 If ∠1 measures 135°, what is the measure of ∠8? (Lines *b* and *c* are parallel.)

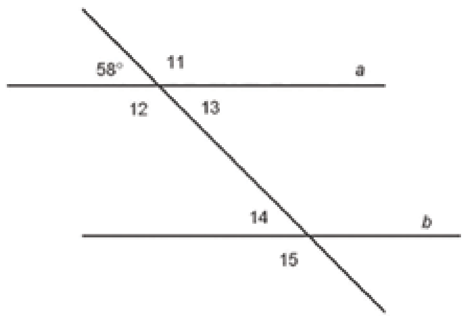


☐ 45°
☐ 135°
☐ 65°
☐ 67.5°

1.23 If ∠1 measures 135°, what is the measure of ∠6 in the drawing above? (Lines *b* and *c* are parallel.)

☐ 135° ☐ 65° ☐ 90° ☐ 45°

Use the diagram to answer the questions below. In the diagram, *a* is parallel to *b*.



1.24 What is the measure of angle 11?

1.25 What is the measure of angle 12?

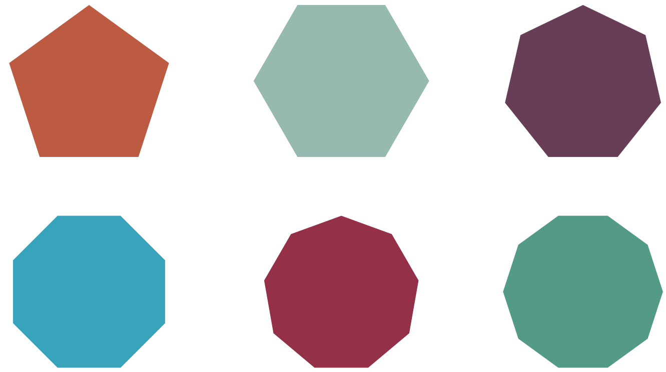
1.26 What is the measure of angle 13?

1.27 What is the measure of angle 14?

1.28 What is the measure of angle 15?

Polygons

Look at these *polygons* carefully. What do you notice? How are they similar? How are they different?



In this lesson, you will explore the attributes of *regular polygons*.

Objectives

- Identify polygons and use correct geometric terminology to describe them.
- Determine the measure of an interior angle of a regular polygon.

VOCABULARY

decagon—a polygon with ten sides
diagonal—an interior line that connects two vertices that are not already connected
dodecagon—a polygon with twelve sides
equiangular—having equal angles
equilateral—having sides of equal length
equilateral triangle—a triangle whose sides all are the same length
heptagon—a polygon with seven sides
hexagon—a polygon with six sides
interior angles—inside angles
octagon—a polygon with eight sides
pentagon—a polygon with five sides
polygon—a closed figure made up of line segments
quadrilateral—a polygon with four sides
regular polygon—a polygon whose sides are all the same length and whose angles are all the same measure

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Did you notice that all the sides in a regular polygon are of equal length? Did you also notice that all the *interior angles* in a regular polygon are congruent? In other words, regular polygons are *equilateral* and *equiangular*.

This might help! *Equilateral* comes from the root words ***aequi***, meaning "equal," and ***latus***, meaning "side." So ***equilateral*** means "equal sides." *Equiangular* also comes from the root word ***aequi*** and the root word ***angulus***, meaning "angle," so ***equiangular*** means "equal angles."

Each of the polygons shown in the introduction has a special name based on the number of sides. The start of each name refers to the number of sides. For example *octa-* means "eight," so an *octagon* has eight sides. While all octagons have eight sides, only a *regular* octagon has eight equal sides and eight congruent angles. Take a look at the following chart of common polygons:

Polygon	Number of Sides
pentagon	5
hexagon	6
heptagon	7
octagon	8
decagon	10
dodecagon	12

If a polygon has more than 12 sides, the number of sides is used as the start of the name. For example, if there are 23 sides in a polygon, you can call it a 23-gon. If it has 100 sides, you can call it a 100-gon, and so on.

Connections! Notice that as the number of sides in the regular polygons increases, the shape gets closer to a circle. Archimedes, an ancient Greek mathematician, found a close approximation for the area of a circle by finding the area of regular polygons with more and more sides.

Sum of Interior Angles

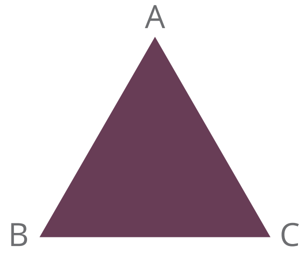
As the number of sides in a polygon increases, the number of angles increases also. So if you add up the angle measures in a polygon, the sum should increase as the number of sides increases.

What would the sum of the measures of the angles in any polygon be? What would the measure of one angle in any regular polygon be?

To answer these questions, start by looking at the simplest polygon, a triangle. The sum of the angle measures for any triangle is 180°. If the triangle is a regular polygon, it is called an *equilateral triangle*.

Example:

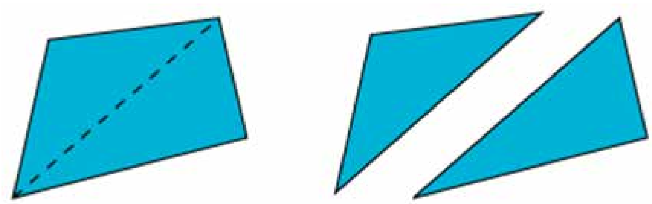
- What is the angle measure of each angle in an equilateral triangle?



Solution:

- The sum of the angle measures is 180°, and the angles are congruent because the triangle is a regular polygon. To find the measure of each angle, divide the sum of the angle measures (180°) by the number of angles (3):
- 180° ÷ 3 = 60°
- So each angle in an equilateral triangle measures 60°.

A triangle has three sides, so the next polygon to look at is a *quadrilateral*, which has four sides. To find the sum of the angle measures, you can draw a *diagonal*, connecting two opposite vertices. Notice that the diagonal divides the quadrilateral into two triangles.

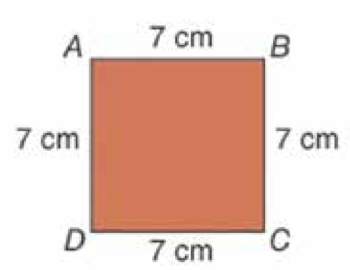


You already know that the sum of the angle measures in a triangle is 180°. Since the quadrilateral is made up of two triangles, the sum of the angle measures is 180° · 2, or 360°.

If the quadrilateral were a regular polygon, what would each angle measure? Work through this next example to find out.

Example:

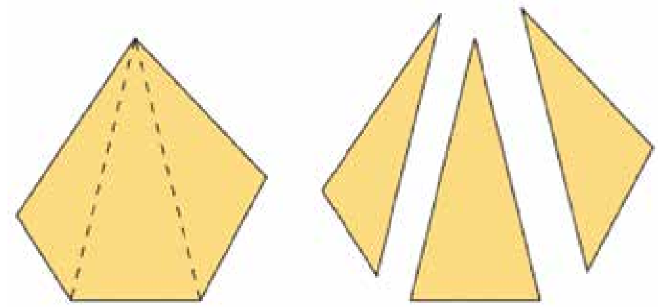
- What is the measure of each angle of a square?



Solution:

- A square is a regular quadrilateral because the side lengths are equal, and the angle measures are congruent. To find the measure of each angle, divide the sum of the angle measures (360°) by the number of angles (4):
- 360° ÷ 4 = 90°
- So each angle in a square measures 90°.

Now take a look at a pentagon, which has five sides. Again, you can divide the polygon into triangles. This time, you get three triangles.



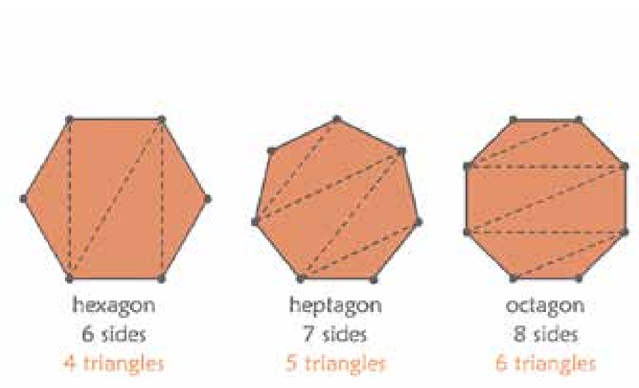
You know that the sum of the angle measures in a triangle is 180°. Since there are three triangles in the pentagon, the sum of the angle measures in a pentagon is 180° · 3, or 540°.

To find the measure of an angle in a regular pentagon, apply the same method you used for finding the measure of each angle in the equilateral triangle and the square. Divide the angle sum (540°) by the number of angles (5):

- 540° ÷ 5 = 108°

So each angle of a regular pentagon measures 108°.

You can continue to apply this same method of dividing each polygon into triangles to find the sum of the angle measures. Is there a pattern? Take a look at the following illustration.



Everything you've learned about angle measures and polygons can be summarized in a table. Take a look at the following table to see the sum of the angle measures in each of the common polygons and the measure of each angle in the regular polygons.

Sides	Number of △'s	Angle Sum	Regular Polygon Angle
3	1	1(180°) = 180°	180° ÷ 3 = 60°
4	2	2(180°) = 360°	360° ÷ 4 = 90°
5	3	3(180°) = 540°	540° ÷ 5 = 108°
6	4	4(180°) = 720°	720° ÷ 6 = 120°
7	5	5(180°) = 900°	900° ÷ 7 = 128.6°
8	6	6(180°) = 1080°	1080° ÷ 8 = 135°

Did you notice that the number of triangles that the polygon can be divided into is always two less than the number of sides in the polygon? That number is multiplied by 180° because the sum of the angle measures in any triangle is 180°.

You can make a generalized expression from this information. If *n* is the number of sides in the polygon, then the sum of the angle measures is the difference of *n* and two, multiplied by 180°:

- sum of angle measures in a polygon = $(n - 2)180°$

Example:

- What is the sum of the angle measures in a hexagon?

Solution:

- A hexagon has 6 sides, so $n = 6$.

$$(n-2)180° =$$
$$(6-2)180° =$$
$$(4)180° = 720°$$

To find the measure of an angle in a regular polygon, divide the angle sum by the number of angles (or sides) in the regular polygon because they are all congruent.

You can make a generalized expression from this as well. If *n* is the number of sides in the regular polygon, then the angle measure is the angle sum (the first expression) divided by *n*, or the number of sides:

- measure of an interior angle of a regular polygon = $\frac{(n-2)180°}{n}$

See if you can apply these equations to find the sum of the angle measures for other polygons.

Example:

- What is the sum of the angle measures in a decagon? What is the measure of an angle in a regular decagon?

Solution:

- Use the first angle measure expression to answer the first question. There are 10 sides in a decagon, so multiply 2 less than 10 by 180°:

$(n - 2)180°$	original expression
$= (10 - 2)180°$	Substitute 10 into the expression.
$= (8)180°$	Subtract inside the parentheses.
$= 1{,}440°$	Multiply.

- So the sum of the angles in a decagon is 1,440°.
- To answer the second question, substitute the sum of the angles into the numerator of the second angle measure expression:

$$\frac{(n-2)180°}{n}$$ original expression

$$= \frac{1{,}440°}{10}$$ Substitute 1,440 into the numerator.

$$= 144°$$ Divide by 10.

- So the measure of an angle in a regular decagon is 144°.

Example:

- What does an angle in a regular 32-gon measure?

Solution:

- Use the second angle measure expression to solve the problem:

$$\frac{(n-2)180°}{n}$$ original expression

$$= \frac{(32-2)180°}{32}$$ Substitute 32 into the expression.

$$= \frac{(30)180°}{32}$$ Subtract inside the parentheses.

$$= \frac{5{,}400°}{32}$$ Multiply in the numerator.

$$= 168.75°$$ Divide by 32.

- So each angle of a regular 32-gon measures 168.75°.

Did you know! As the number of sides in the polygon increases, the sum of the angles increases. For regular polygons, each angle measure approaches 180° and the polygon approaches a circle.

Regular Polygon Sides	Angle Measure
3	60°
6	120°
10	144°
100	176.40°
1,000	179.64°
10,000	179.97°

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- A regular polygon has equal sides and congruent angles.
- The sum of the angle measures for any polygon with n sides is $(n - 2)180°$.
- The measure of any angle in a regular polygon with n sides is $\frac{(n-2)180°}{n}$.

Complete the following activities.

1.29 Match each polygon with the number of sides it has.

________	octagon	7
________	hexagon	12
________	decagon	6
________	pentagon	5
________	dodecagon	8
________	heptagon	10

1.30 Select all that apply. Which of the following are true of a regular pentagon?

☐ Each angle measure is 108°.
☐ It has six sides.
☐ The angle sum is 720°.
☐ It is equilateral.

1.31 What is the sum of the angle measures in a dodecagon?

☐ 1,440° ☐ 2,160° ☐ 1,800° ☐ 150°

1.32 What is the measure of an angle in a regular dodecagon?

☐ 144° ☐ 1,800° ☐ 154.3° ☐ 150°

1.33 What is the sum of the angle measures in a 16-gon?

☐ 2,880° ☐ 2,520° ☐ 2,160° ☐ 157.5°

1.34 A polygon has an angle sum of 360°, and each angle measures 90°. What is the polygon?

☐ triangle ☐ regular octagon ☐ pentagon ☐ square

1.35 Which regular polygon will have the largest angle measure?

☐ octagon ☐ square ☐ pentagon ☐ hexagon

1.36 Each angle measure of a regular polygon is 60°. What is the polygon?

☐ square
☐ equilateral triangle
☐ quadrilateral
☐ hexagon

Complete the following activities.

1.37 What is the sum of the interior angles of a 15-gon?

1.38 What is the measure of each interior angle of a 15-gon?

1.39 What is the sum of the interior angles of an 18-gon?

1.40 What is the measure of each interior angle of an 18-gon?

1.41 The sum of the interior angles of a regular polygon is $1260°$. How many sides does the polygon have?

CIRCLES

This is the London Eye. At the time it was built in 1999, it was the tallest Ferris wheel in the world. Its main shape is a *circle*; it also contains several parts of a circle.

Circles are all around you. You see them in everyday life and in man-made objects like the London Eye. In this lesson, you will learn about circles and their parts.

Objectives

- Identify parts of a circle.
- Use circle properties to find missing measures.

Vocabulary

arc—part of a circle between two points

central angle—an angle that has the center of a circle as its vertex

chord—a line segment that connects two points on a circle

circle—a figure whose points are all the same distance from its center

diameter—a line segment that goes through the center of a circle to connect two points on the circle

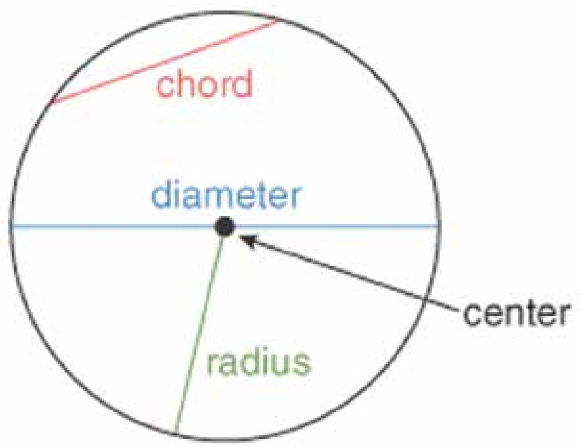
intercepted arc—an arc between the endpoints of two radii

radius—(pl. radii) a line segment that goes from the center of a circle to any point on the circle

semi-circle—an arc with a measure of 180 degrees

Parts of A Circle

A circle is the set of all points that are a given distance from the *center* of the circle. If you held a jump rope and swung it around your head, the end of the rope would form a circle with you as the center.



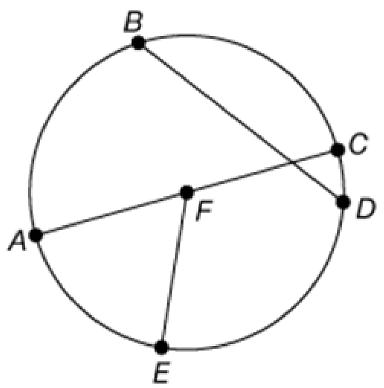
There are also several segments inside the circle. A *chord* is a segment whose endpoints are both on the circle. If the chord goes through the center, it is called a *diameter*. A segment whose endpoints are the center and any point on the circle is a *radius*. All radii in a circle are the same length because the radius is the distance that defines the circle's size. If you look closely at the London Eye, you can see many radii connecting the center to the circle.

Think about it! There are an infinite number of radii and diameters in a circle because there are an infinite number of points on the circle.

You can use the same notation for each of the segments in a circle as you did for line segments. The circle itself can be named for the center point using the symbol $\odot$.

Example:

- Identify each of the following parts of $\odot F$:
  - point *F*
  - $\overline{BD}$
  - $\overline{AC}$
  - $\overline{EF}$
  - $\overline{AF}$
  - $\overline{FC}$



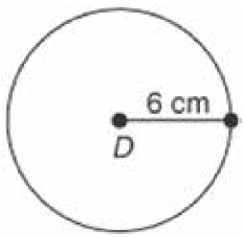
Solution:

- Look at the definition of each part of the circle.
- Point *F* is the *center* of the circle.
- The segment $\overline{BD}$ has its endpoints on the circle, so it is a *chord*.
- The segment $\overline{AC}$ also has its endpoints on the circle, so it is a *chord*. However, it also passes through the center, so it is a *diameter* as well.
- The segments $\overline{EF}$, $\overline{AF}$, and $\overline{FC}$ each have one endpoint as the center and one endpoint on the circle, so these are all *radii*.

Notice in the previous example that $\overline{AF}$ and $\overline{FC}$ make up $\overline{AC}$. Since $\overline{AF}$ and $\overline{FC}$ are both radii, they are the same length. The diameter will always be equal to two lengths of the radius. In other words, the diameter is twice the length of the radius.

Example:

- What is the diameter of $\odot D$?

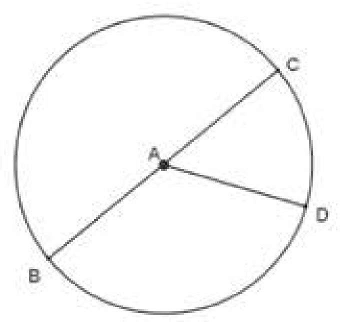


Solution:

- You know that the diameter is twice the length of the radius, and the radius is 6 centimeters.
- 2(6 cm) = 12 cm
- So the diameter is 12 centimeters.

Example:

- What is the length of $\overline{AD}$ in $\odot A$ if $\overline{BC}$ = 14 inches?

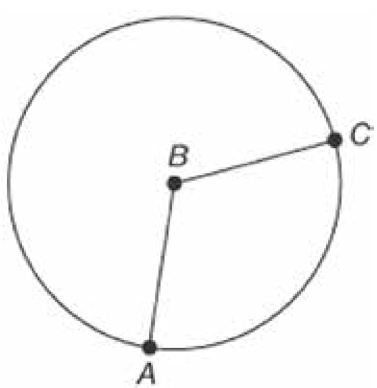


Solution:

- $\overline{AD}$ is a radius and $\overline{BC}$ is a diameter, so $\overline{AD}$ is half the length of $\overline{BC}$.
  $14 \div 2 = 7$ inches.

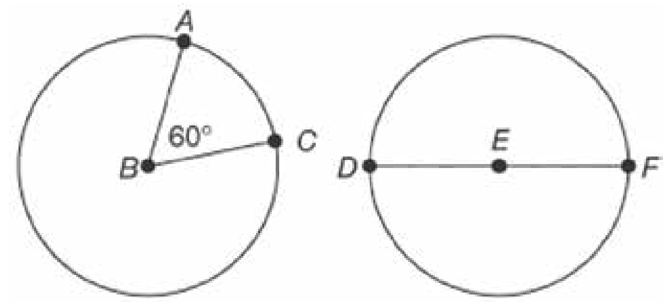
Central Angles

Two radii in a circle will form an angle with the center as the vertex. This is called a *central angle*. The endpoints of the radii that touch the circle become the endpoints of a section of the circle. A section of a circle is called an *arc*. An arc between the endpoints of two radii is called an *intercepted arc*. An arc is named by its endpoints with a short curved segment above it.



$\odot$ *B* has a central angle ∠*ABC* and an intercepted arc $\overset{\frown}{AC}$.

An arc is measured in degrees just like an angle. The measurement of the intercepted arc is the same as its central angle. Use a lowercase m to represent the measure of the arc, just as you did with angles.



For $\odot$ *B*, you would write m∠ *ABC* = m$\overset{\frown}{AC}$ = 60°.

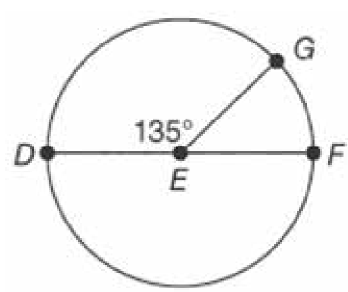
For $\odot$ *E*, you would write m∠ *DEF* = m$\overset{\frown}{DF}$ = 180°.

Notice that $\overline{DF}$ is a diameter and ∠*DEF* is a straight angle that divides the circle in half. This arc is called a *semi-circle*. Any diameter of a circle will intercept an arc of 180°, or a semi-circle.

Try a few examples.

Example:

- If m∠ *DEG* = 135° and $\overline{DF}$ is a diameter of $\odot$ *E*, what is the measure of $\overset{\frown}{GF}$?



Solution:

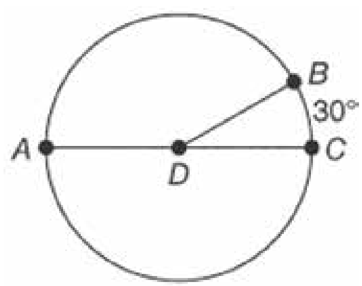
- Since $\overline{DF}$ is a diameter, you know m∠*DEF* = 180°. This means that ∠*DEG* and ∠*GEF* are supplementary angles.
- So you can solve for the measure of ∠*GEF*, which will tell you the measure of the intercepted arc $\overset{\frown}{GF}$.

m∠ *DEG* + m∠GEF = 180°	definition of supplementary angles
135° + x = 180°	Substitute in the known value and a variable you can solve for.
x = 45°	Subtract 135° from both sides.

- ∠*GEF* intercepts $\overset{\frown}{GF}$, so m$\overset{\frown}{GF}$ = 45°.

Example:

- $\overline{AC}$ is a diameter of ⊙ *D* and m$\overset{\frown}{BC}$ = 30°. What is the measure of ∠*ADB*?



Solution:

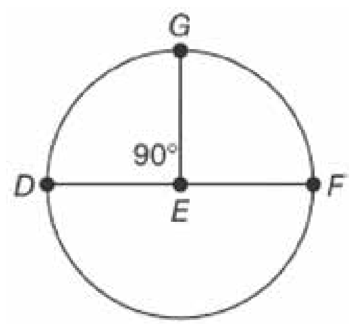
- Since $\overline{AC}$ is a diameter, you know m∠ *ADC* = 180°. This means that ∠*ADB* and ∠*BDC* are supplementary angles.
- m∠ *BDC* = 30° because its intercepted arc, $\overset{\frown}{AC}$, measures 30°.
- Now solve for the measure of ∠*ADB*:

m∠ *ADB* + m∠ *BDC* = 180°	definition of supplementary angles
x + 30° = 180°	Substitute in the known value and a variable you can solve for.
x = 150°	Subtract 30° from both sides.

- So ∠*ADB* measures 150°.

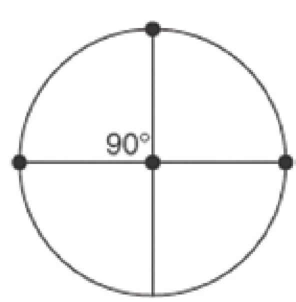
Example:

- If ∠*DEG* = 90° and $\overline{DF}$ is a diameter of ⊙ *E*, what is the measure of $\overset{\frown}{GF}$?



Solution:

- Since $\overline{GE}$ intersects $\overline{DF}$ at a right angle, ∠*DEG* and ∠*GEF* are both 90°.
- 90° + 90° = 180°
- Since m∠ *GEF* = 90°, the intercepted arc $\overset{\frown}{GF}$ = 90°.
- If two diameters are perpendicular, what is the measure of each arc?



- You can see from the drawing that each of the four central angles will measure 90°. Since each central angle is 90°, each intercepted arc also measures 90°. Just as a straight angle divides a circle in half, right angles divide a circle into fourths.

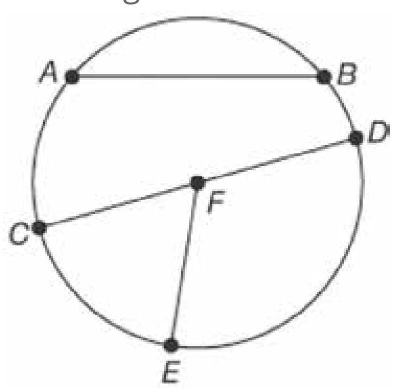
Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- A circle is formed by the set of points that are a given distance (called the radii) from a point called the center.
- Chords, radii, and diameters are segments in a circle.
- Two radii form a central angle that incepts an arc of the circle. The angle measure and the arc measure are the same.

Complete the following activities.

1.42 Which figure is a diameter of $\odot F$?



- ☐ $\overline{AB}$
- ☐ $\overline{CF}$
- ☐ $\overline{CD}$
- ☐ $\overline{CE}$

1.43 Which figure is a radius of $\odot F$?

☐ $\overline{CD}$ ☐ $\overline{AB}$ ☐ $\overline{FD}$ ☐ $\overset{\frown}{CE}$

1.44 Which figure is a chord of $\odot F$?

☐ $\overline{EF}$ ☐ $\overline{AB}$ ☐ $\overline{FC}$ ☐ $\overset{\frown}{AC}$

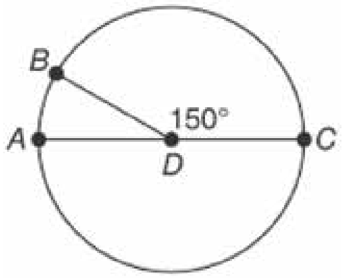
1.45 Which arc measures 180° in $\odot F$?

☐ $\overset{\frown}{AE}$ ☐ $\overset{\frown}{CB}$ ☐ $\overset{\frown}{BD}$ ☐ $\overset{\frown}{DC}$

1.46 A central angle in a circle measures 120°. What is the measure of its intercepted arc?

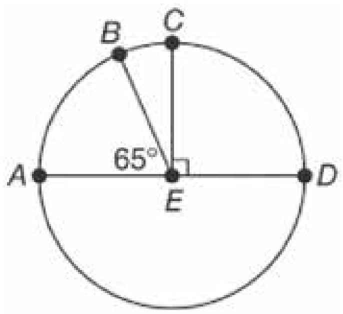
☐ 60° ☐ 120° ☐ 180° ☐ 240°

1.47 $\overline{AC}$ is a diameter of ⊙ *D*. If m∠*BDC* = 150°, what is the measure of $\overset{\frown}{AB}$?



☐ 30°
☐ 150°
☐ 180°
☐ 210°

$\overline{AD}$ **is a diameter of** ⊙ *E*. $\overline{CE}$ is perpendicular to $\overline{AD}$.



1.48 What is the measure of $\overset{\frown}{CD}$ in ⊙ *E*?

☐ 90° ☐ 180° ☐ 115° ☐ 240°

1.49 What is the measure of ∠*BED* in ⊙ *E*?

☐ 90° ☐ 115° ☐ 65° ☐ 25°

1.50 If m∠*AEB* = 65°, what is the measure of $\overset{\frown}{BC}$ in ⊙ *E*?

☐ 35° ☐ 115° ☐ 90° ☐ 25°

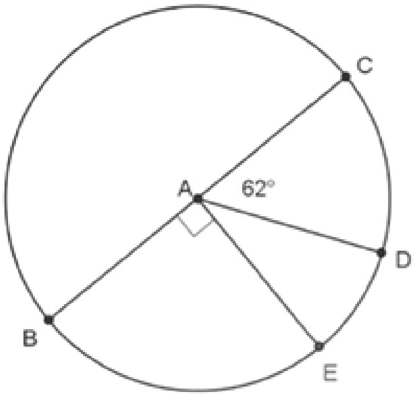
1.51 A radius in a circle is 5 inches. What is the length of the diameter?

☐ 5 in. ☐ 10 in. ☐ 2.5 in. ☐ 20 in.

1.52 A diameter is a chord.

○ True
○ False

Use the diagram of circle *A* to answer the questions below.



1.53 What is the measure of $\overset{\frown}{CD}$?

1.54 What is the measure of $\angle DAE$?

1.55 Name an arc that is equal to $90°$.

1.56 What is the measure of $\overset{\frown}{BC}$?

1.57 What is the measure of $\overset{\frown}{BE}$?

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

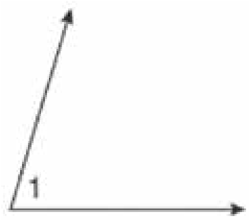
Self Test 1: Basic Geometry

Complete the following activities (5 points, each numbered activity).

1.01 Which number shows the measure of an acute angle?

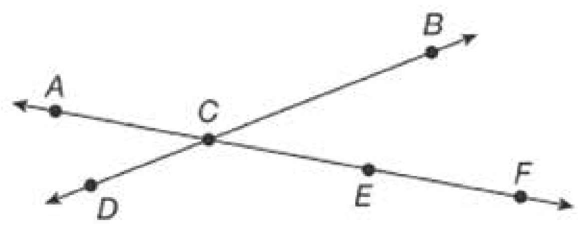
☐ 45° ☐ 90° ☐ 135° ☐ 180°

1.02 *Estimate* the measure of ∠1 .



☐ 90°
☐ 80°
☐ 110°
☐ 45°

1.03 Select all that apply. Which of the following names a ray in the drawing?

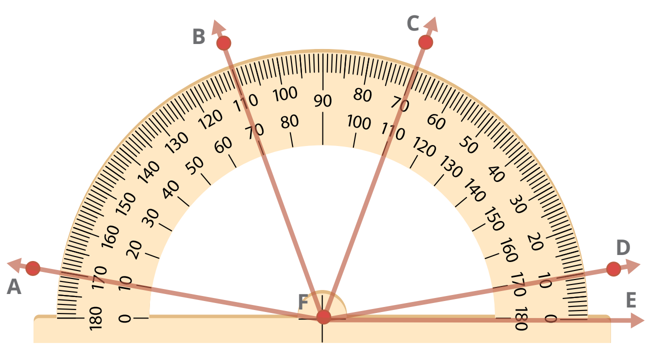


☐ $\overrightarrow{CB}$ ☐ $\overrightarrow{AF}$
☐ $\overrightarrow{BF}$ ☐ $\overrightarrow{AB}$

1.04 Select all that apply. Which of the following names an angle in the drawing used in the previous question?

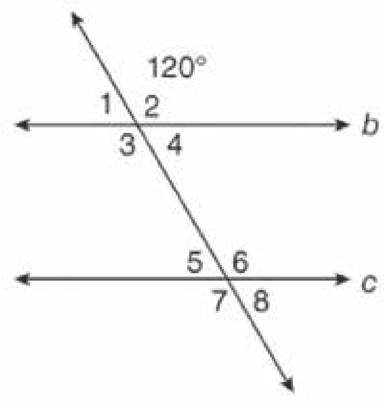
☐ ∠*ACD* ☐ ∠*CBE* ☐ ∠*FBC* ☐ ∠*DCE*

1.05 Which angle measures 70°?



☐ ∠*EFA*
☐ ∠*EFB*
☐ ∠*EFC*
☐ ∠*EFD*

1.06 Select all that apply. Which pairs of angles are supplementary?



☐ ∠1 and ∠8
☐ ∠2 and ∠4
☐ ∠3 and ∠5
☐ ∠6 and ∠7

1.07 Select all that apply. Which angles are congruent to ∠4 in the drawing used in the previous question?

☐ ∠1 ☐ ∠7 ☐ ∠8 ☐ ∠2

1.08 ∠*A* and ∠*B* are complementary and congruent. What is the measure of each of these angles?

☐ 90° ☐ 45° ☐ 50° ☐ 180°

1.09 Two lines intersect and two of the vertical angles measure 37°. What is the measure of the other two vertical angles?

☐ 37° ☐ 74° ☐ 90° ☐ 143°

1.010 What is a polygon with 10 sides called?

☐ dodecagon ☐ octagon ☐ tarragon ☐ decagon

1.011 What is the measure of an angle in a regular hexagon?

☐ 144° ☐ 135° ☐ 120° ☐ 108°

1.012 What is the sum of the angle measures in a heptagon?

☐ 900° ☐ 540° ☐ 360° ☐ 720°

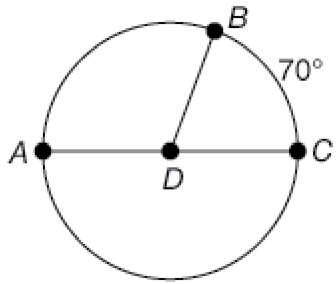
1.013 Which polygon will have the largest angle sum?

☐ octagon ☐ heptagon ☐ pentagon ☐ dodecagon

1.014 A section of a circle has both endpoints on the circle. What is the section of the circle called?

☐ arc ☐ radius ☐ chord ☐ diameter

1.015 $\overline{AC}$ is a diameter of $\odot D$, and $m\angle BC = 70°$. What is the measure of $\angle ADB$?



- ☐ 70°
- ☐ 30°
- ☐ 110°
- ☐ 90°

1.016 What is the sum of the interior angles of a 30-gon?

1.017 What is the measure of each interior angle of a regular 30-gon?

1.018 An angle measures 42°. What is the measure of its complementary angle?

1.019 An angle measures 77°. What is the measure of its supplementary angle?

1.020 A circular swimming pool has a diameter of 18 feet. What is the radius of the pool?

80/100 **SCORE** ________ **TEACHER** ________ ________
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