



MATH

STUDENT BOOK

▶ **8th Grade** | Unit 8

Math 808

Measures of Solid Figures

Introduction | 3

1. Surface Area 5

Solid Figures | 5

Euler's Formula | 14

Surface Area of Rectangular Prisms | 20

Surface Area of Triangular Prisms | 27

Surface Area of Cylinders | 32

Surface Area of Pyramids, Cones, and Spheres | 37

Surface Area of Composite Figures | 45

SELF TEST 1: Surface Area | 50

2. Volume 53

Volume of Rectangular Prisms | 53

Volume of Triangular Prisms | 58

Volume of Square Pyramids | 62

Volume of Cylinders | 68

Volume of Cones | 73

Volume of Spheres | 78

Changes to Volume | 81

Volume of Composite Figures | 87

SELF TEST 2: Volume | 93

3. Review 97



LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

Author:

Glynlyon Staff

Editor:

Alan Christopherson, M.S.

Westover Studios Design Team:

Phillip Pettet, Creative Lead

Teresa Davis, DTP Lead

Nick Castro

Andi Graham

Jerry Wingo



804 N. 2nd Ave. E.

Rock Rapids, IA 51246-1759

© MMXIV by Alpha Omega Publications a division of Glynlyon, Inc. All rights reserved. LIFEPAK is a registered trademark of Alpha Omega Publications, Inc.

All trademarks and/or service marks referenced in this material are the property of their respective owners. Alpha Omega Publications, Inc. makes no claim of ownership to any trademarks and/or service marks other than their own and their affiliates, and makes no claim of affiliation to any companies whose trademarks may be listed in this material, other than their own.

Some clip art images used in this curriculum are from Corel Corporation, 1600 Carling Avenue, Ottawa, Ontario, Canada K1Z 8R7. These images are specifically for viewing purposes only, to enhance the presentation of this educational material. Any duplication, resyndication, or redistribution for any other purpose is strictly prohibited. Other images in this unit are © 2009 JupiterImages Corporation

Measures of Solid Figures

Introduction

This unit introduces students to geometric solids. Students begin by learning about the characteristics of solids and how to use the characteristics to classify the solids. Next, students learn how to calculate the surface areas and volumes of common solids, as well as for composite figures. Finally, students learn how changes in the dimensions of figures affect volume.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFE PAC. When you have finished this LIFE PAC, you should be able to:

- Identify geometric solids from three-dimensional, pictorial representations.
- Identify geometric solids from net representations.
- Identify the number of faces, bases, lateral faces, edges, and vertices for geometric solids.
- Compute the number of faces, vertices, and edges of a solid with Euler's formula.
- Calculate the surface area of geometric solids and composite figures.
- Calculate the volume of geometric solids and composite figures.

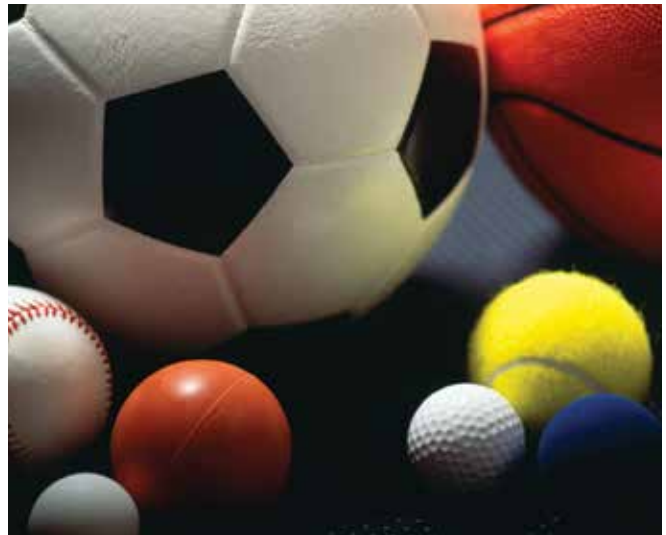
1. Surface Area

SOLID FIGURES

The world around you is full of geometric solids. Just think about the grocery store. The shelves are full of packages in all different shapes. Canned vegetables and pasta are packaged in two different shapes. Now, think about gym class. Most of the

games you play in gym class use a ball that is another shape.

This lesson focuses on the different solids that make up our everyday world and how you can classify them by their characteristics.



Objectives

- Classify a three-dimensional figure by its characteristics.
- Name a three-dimensional figure by its base(s).
- Identify the number of faces, edges, and vertices for a figure.
- Identify the net of a three-dimensional figure.

Vocabulary

base of a solid—a special face of a solid figure

cone—three-dimensional figure with a circular base and lateral surface that tapers to a point

cube—three-dimensional figure made of six congruent squares

cylinder—three-dimensional figure with two parallel, equal circular bases and a curved surface

face—a plane figure that is one side of a solid figure

lateral face—any face that is not a base

net—a two-dimensional representation of a three-dimensional shape when unfolded

plane figure—a two-dimensional shape

prism—three-dimensional figure with two parallel, congruent polygon faces and all other faces are parallelograms

pyramid—three-dimensional figure with a polygon as a base and all other faces are triangles with a common vertex

solid—geometric figure with three dimensions (3D)

sphere—three-dimensional figure where all points are the same distance from the center

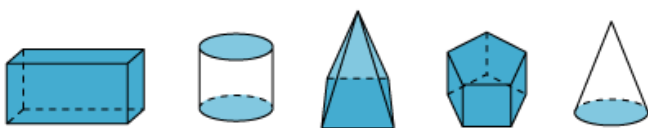
rectangular prism—three-dimensional figure with six faces; has two rectangular parallel, congruent bases and four rectangular lateral faces

volume—the amount of space in a three-dimensional figure

Geometric Solids

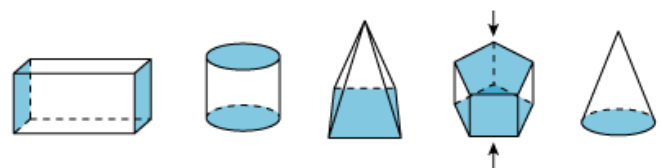
As you know, the objects that you encounter in real life are all three dimensional (3D), meaning that they have a length, width, and height. Each of the shapes we see around us has a specific name. Like anything else, we will use the characteristics of the shape to determine its name.

The *face* of a *solid* is a *plane figure* that's one side of the figure. Some solids have no faces, while others may have 12 faces. Take a look at the following pictures. Each face has been shaded.

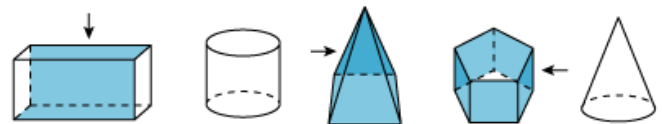


You can classify faces into two categories—*bases* and *lateral faces*. Bases of solids are special faces that help name the solid, while a lateral face is any face that is not a base. Again, some solids may not have a base, while others may have one or two bases. There are never more than two bases on a solid.

In the following pictures, the base(s) has(have) been shaded.



These pictures show the lateral faces.



Make note! Two of the figures do not have lateral faces. A lateral face has to be a flat surface in one plane.

Prisms

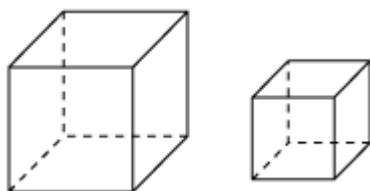
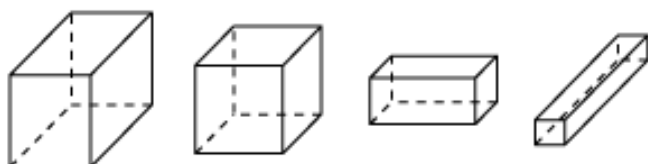
Let's see how faces, more specifically bases, play a key role in identifying the different solids. We'll start with one of the most common 3D shapes, the *prism*.

You see prisms all the time and may not even realize it. Every time you see a box, you're actually looking at a prism. A prism is a 3D figure that has two parallel, congruent bases and has parallelograms for its remaining faces. Here are some pictures of prisms. Note that they don't all have the same-shaped bases.



Just like boxes, prisms come in all different shapes and sizes, as you see illustrated. Let's learn a little more about the different types of prisms, and about how we can distinguish their differences.

A rectangular prism is a prism that has rectangles as its bases. Remember that rectangles include squares! Take a look at some examples of rectangular prisms.

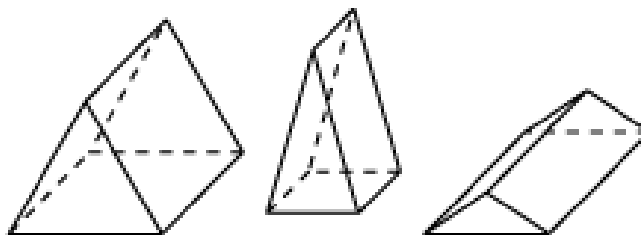


There is a special type of rectangular prism called a *cube*. A cube is a rectangular prism whose six faces are all the same square. A common example of a cube is dice.

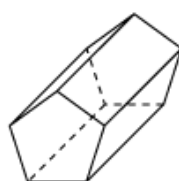


Make note! Dice are not perfect cubes if their sides and corners (vertices) are rounded.

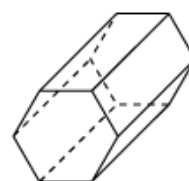
Another type of prism is the triangular prism. This time, the two parallel, congruent bases are triangles. Here are some examples.



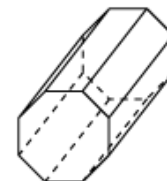
There are other types of prisms. Again, we name them by the shape of their bases. Look at some examples of other prisms.



Pentagonal Prism



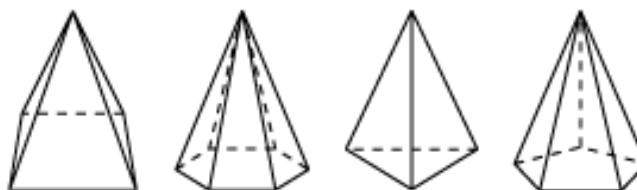
Hexagonal Prism



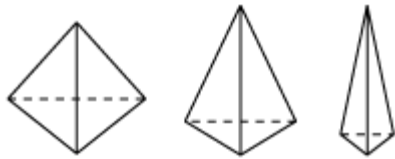
Octagonal Prism

Pyramids

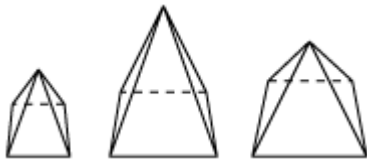
Pyramids differ from prisms because they only have one base, and all of their lateral faces are triangles. Look at the following examples of pyramids. Again, notice that they have different bases, but all of the lateral faces are always triangles.



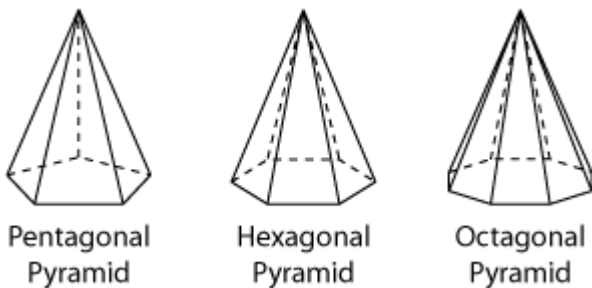
This time, let's begin with the triangular pyramid. A triangular pyramid has all triangular faces, including the base.



Rectangular pyramids have a base in the shape of a rectangle, or square, and lateral faces in the shape of triangles.



Like prisms, there are other pyramids named by the shape of their bases. Take a look at some examples of other pyramids.



Cylinders

You probably have some food in a *cylinder* in your kitchen right now. Any time food is packaged in a can, whether its vegetables, tuna, or even tomato sauce, the can is made in the shape of a cylinder. A cylinder is a 3D figure with two parallel, congruent circular bases and a curved surface connecting the two bases.

Make note! Cylinders do not have lateral faces because their sides are curved, rather than flat.



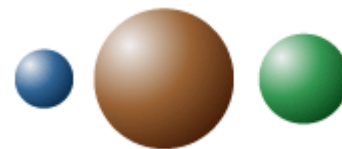
Cones

A *cone* looks like a cross between a cylinder and a pyramid. A cone has one base in the shape of a circle. The base is attached to a curved surface that comes to a point above the base. You often see cones in construction zones or with ice cream in them. Here are some examples of cones that you will see in math.



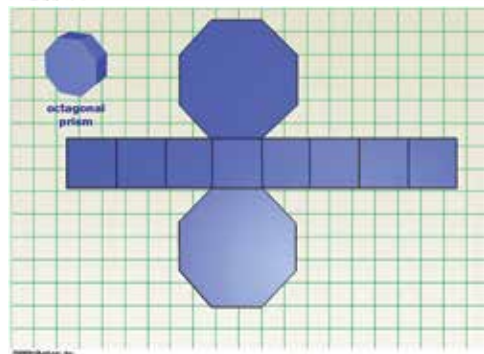
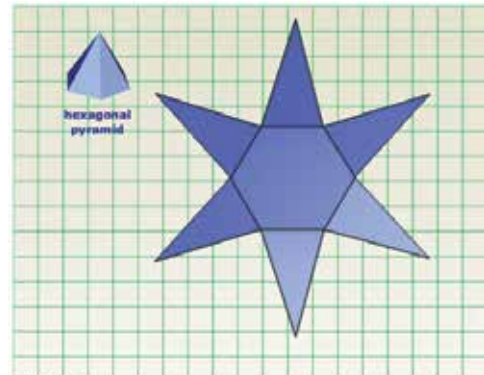
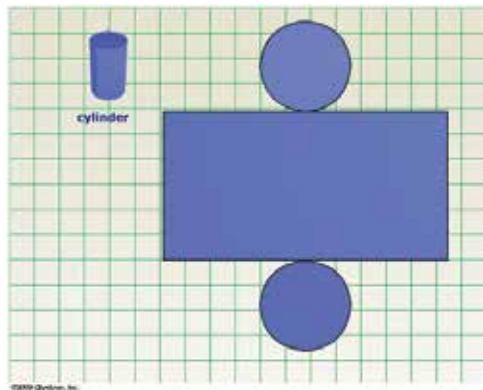
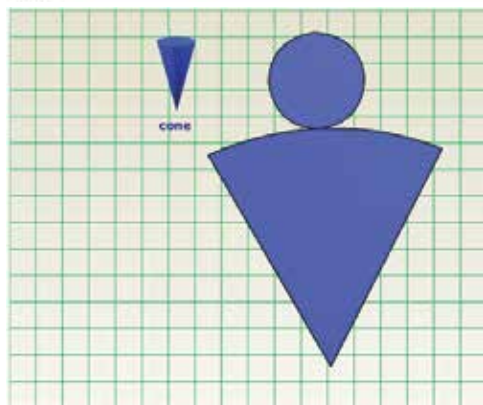
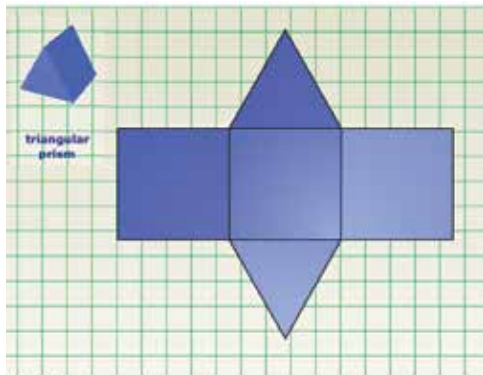
Spheres

In many of the sports we play, we use *spheres*. A sphere is a 3D figure where all of the points are the same distance from the center. We know them better as balls. Think about the games you play, like basketball, baseball, softball, soccer, etc. They all use a sphere-shaped object.



Nets

Sometimes it's helpful to "unfold" a 3D shape. By unfolding the figure and laying it flat, we can see the entire surface of the shape at once. The unfolded figure is called the *net* of the solid. Let's take a look at the nets of some of the more-common geometric solids.



Nets will become more useful as you continue learning about solids.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Each solid can be identified and named by its characteristics.
- The shape of the base names the solid.
- A net is a flat representation of a 3-D solid.



Complete the following activities.

1.1 Match the descriptions to their correct solid name.

- | | | |
|-------|--|---------------------|
| _____ | 3D figure where all points are the same distance from the center | cone |
| _____ | 3D figure made of all triangles | sphere |
| _____ | 3D figure with six congruent squares | triangular prism |
| _____ | 3D figure with two triangle bases and rectangles for the rest of its faces | triangular pyramid |
| _____ | 3D figure with two circular bases | cube |
| _____ | 3D figure with a rectangle as a base and triangles for the rest of its faces | rectangular prism |
| _____ | 3D figure with one circular base and one vertex | rectangular pyramid |
| _____ | 3D figure with rectangles for all of its faces | cylinder |

1.2 A cereal box is an example of a:



- 1.3 A volleyball is an example of
a: _____



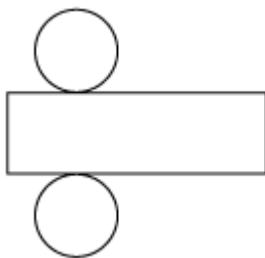
- 1.5 A tent is an example of
a: _____



- 1.4 The Great Pyramid is an example of
a: _____

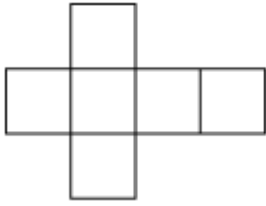


- 1.6 What shape can be created by the given net?



- cone
 prism
 pyramid
 cylinder

1.7 What shape can be created by the given net?



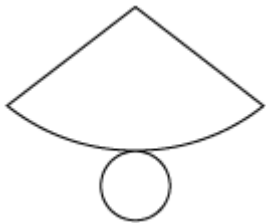
- cube
- square pyramid
- triangular prism
- triangular pyramid

1.8 What shape can be created by the given net?



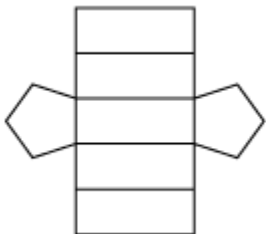
- triangular pyramid
- square pyramid
- triangular prism
- rectangular prism

1.9 What shape can be created by the given net?



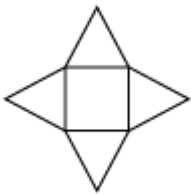
- pyramid
- cylinder
- cone
- cube

1.10 What shape can be created by the given net?



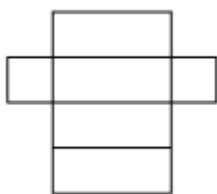
- hexagonal prism
- pentagonal prism
- rectangular prism
- triangular prism

1.11 What shape can be created by the given net?



- triangular prism
- cube
- rectangular prism
- square pyramid

1.12 What shape can be created by the given net?



- triangular prism
- rectangular prism
- square pyramid
- cube

1.13



Describe how the figures are alike.

Describe how the figures are different.

EULER'S FORMULA

An okapi is an African animal native to the Ituri Rainforest in Congo. Take a look at the picture of the okapi. What animal do you think it is closely related to?

At first glance, you might think it's closely related to a zebra, because of the stripes on its legs. The okapi is actually more closely related to the giraffe. It has a similar body shape, but with a much shorter neck, and it also has the same, very long, blue, flexible tongue that it uses to strip leaves and buds from trees.

Sometimes, we have to take a closer look at objects to see the relationships



that exist, as we had to do with the okapi and the giraffe. This lesson will help you take a closer look at solids to identify the relationship that exists among their parts.

Objectives

- Determine the number of faces, lateral faces, edges, and vertices of each geometric solid.
- Identify the relationship that exists among the number of faces, edges, and vertices of a solid figure.

Vocabulary

base of a solid—a special face of a solid figure

edge—line segment where two faces meet

Euler's formula—the relationship among the number of faces, vertices, and edges of a solid; $V + F = E + 2$

face—a plane figure that is one side of a solid figure

lateral face—any face that is not a base

vertices—points where three or more faces intersect

Parts of a Solid

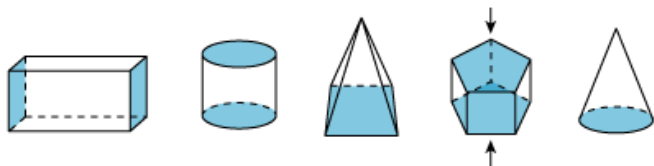
Geometric solids are often composed of plane figures put together to become three-dimensional. We see geometric solids around us everyday, from canned food in our kitchens to boxes used to deliver packages. But, how much do we know

about the relationship that exists among their parts?

The first thing we need to do is identify the different parts of a solid. Solids can have *faces, bases, lateral faces, edges, and vertices*. Not every solid will have every part, but

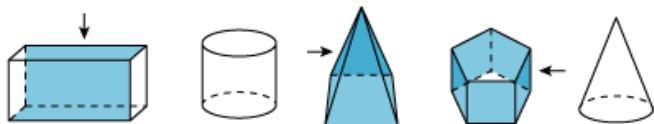
knowing the different parts can help you identify the solid being described.

Each flat side of a solid is called a face. There are two specific types of faces. One type is called the *base of a solid*, which is a special face.



If a solid only has one face, that face is its base. A solid can have, at most, two congruent bases.

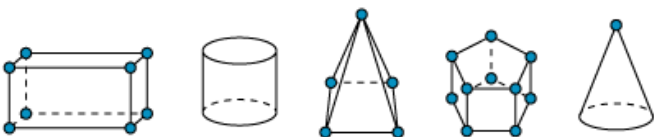
Aside from the base(s), all of the other faces on a solid are called lateral faces.



Where two faces meet, you have an edge.



Where three faces meet at a point, you have a vertex.



You will be counting the total number of bases, edges, and vertices (plural form of vertex) of some solid figures.

Parts of a Rectangular Prism

Let's work on finding out how many faces, edges, and vertices the different solids

have. We'll begin with rectangular prisms. First, let's count the number of faces on a rectangular prism.



The rectangular prism has six faces. Now, let's see how many of them are bases and how many of them are lateral faces.



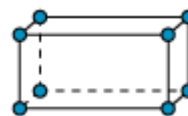
From the picture, you can see that a rectangular prism has two bases and four lateral faces. Remember that all prisms have two parallel, congruent bases.

Now, to determine the number of edges, we need to identify every place where two faces meet.



A rectangular prism will always have 12 edges where two faces meet.

The last characteristic we can count on the rectangular prism is the number of vertices. Vertices are points where three faces meet. How many vertices does the rectangular prism have?



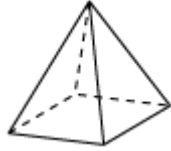
There are eight vertices on a rectangular prism.

You now know that all rectangular prisms, no matter the shape or size, will have six

faces (two bases and four lateral faces), 12 edges, and eight vertices.

Parts of a Square Pyramid

Let's take a look at what changes when we have a square pyramid.



Again, we'll begin by counting the total number of faces and, more specifically, the number of bases and lateral faces.



A square pyramid has five faces. The five faces can be broken down into one base and four lateral faces. The shape of the base, in this case a square, determines what type of pyramid you have.

Now, let's determine the number of edges on a square pyramid.



There are eight edges, or eight places, where two faces meet on a square pyramid.

How many vertices are on a square pyramid?



There are five vertices on the square pyramid. You can describe square pyramids as having five faces (one base and four lateral faces), eight edges, and five vertices.

Parts of a Cylinder

So far, we have used solids that have all straight sides. What do you think happens when we have a solid that has a curved side or surface? Let's take a look at a cylinder. Let's again start with faces, then move to edges and, finally, to vertices.



A cylinder has two faces—the two circular bases. The curved surface between the two circles isn't a face. Both of the faces on a cylinder are bases; therefore, a cylinder has no lateral faces. A cylinder also has no edges, since there isn't a place where two faces meet. In addition, a cylinder has no vertices, because there isn't a place where three faces meet.

Euler's Formula

Leonhard Euler is a famous mathematician who lived in the 1700's. He is one of the most prolific mathematicians to ever live. One of his important discoveries is about the relationship that exists among the number of faces, vertices, and edges of a solid. To understand his discovery, you do not need to determine the type of faces, only their quantity.



Euler's formula says that the sum of faces and vertices is equal to the number of edges plus two, or $V + F = E + 2$.

Let's go back to the rectangular prism.



All rectangular prisms have six faces, 12 edges, and eight vertices. We can check and see if we are correct by plugging the values into Euler's formula.

- $V + F = E + 2$
- $8 + 6 = 12 + 2$
- $14 = 14$

Earlier in the lesson, we also found the number of faces, edges, and vertices of a square pyramid and a cylinder. Do you think the same relationship exists? Let's find out.

The square pyramid had five faces, eight edges, and five vertices.



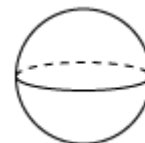
- $V + F = E + 2$
- $5 + 5 = 8 + 2$
- $10 = 10$

The cylinder had two faces, zero edges, and zero vertices.



- $V + F = E + 2$
- $0 + 2 = 0 + 2$
- $2 = 2$

As you can see, the relationship among the number of faces, edges, and vertices holds true for most solids. The only exception to the rule is the geometric solid sphere.



A sphere has zero faces, zero edges, and zero vertices. When we plug these values into the formula, we get:

- $V + F = E + 2$
- $0 + 0 = 0 + 2$
- $0 \neq 2$

Although Euler's formula works for most solids, people most often use it to determine missing quantities of faces, sides, or vertices for prisms and pyramids.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Euler's formula, $V + F = E + 2$, is an easy way to remember the number of faces, edges, and vertices.
 - There are two types of faces, both of which should be included in the total of faces.
-

**Complete the following activities.**

- 1.14** Which solid has five faces, four lateral faces, one base, five vertices, and eight edges?
- cube triangular prism
 square pyramid triangular pyramid
- 1.15** Which solid has six faces, four lateral faces, two bases, eight vertices, and 12 edges?
- square pyramid rectangular prism
 triangular prism triangular pyramid
- 1.16** Which solid has zero faces, zero bases, zero vertices, and zero edges?
- sphere cylinder cone cube
- 1.17** Which solid has two faces, zero lateral faces, two bases, zero vertices, and zero edges?
- cube sphere cylinder cone
- 1.18** Which solid has one face, zero lateral faces, one base, one vertex, and zero edges?
- cube sphere cylinder cone
- 1.19** Which statement correctly shows the relationship found in Euler's formula?
- $V + F = E + 2$ $V = F + E + 2$ $V + F = E - 2$ $V = F + E - 2$

1.20 Use Euler's formula to determine the missing quantity.

Geometric Solid	Number of Faces	Number of Edges	Number of Vertices
triangular prism	5		6
rectangular prism		12	8
pentagonal prism	7	15	
hexagonal prism		18	12
octagonal prism	10		16
triangular pyramid	4	6	
rectangular pyramid		8	5
pentagonal pyramid	6		6

SURFACE AREA OF RECTANGULAR PRISMS

There are a number of different formulas in the study of geometry, making it difficult to keep them all straight, see Figure 1.

Have you ever wondered if you had enough wrapping paper to make it around a gift?

Or, if you bought enough paint to finish painting your bedroom? Knowing how to find the surface area of a rectangular prism will help you answer questions similar to these.



Objectives

- Calculate the surface area of rectangular prisms using a net.
- Calculate the surface area of rectangular prisms using its surface area formula.
- Find a missing measure given the surface area.

Vocabulary

base of a solid—a special face of a solid figure

face—a plane figure that is one side of a solid figure

lateral face—any face that is not a base

surface area—the total area of all the faces

Name	Total Surface Area	Volume	Key
Rectangular Prism	$2lw + 2lh + 2wh$	lwh	l = length w = width h = height B = area of base P = perimeter of base ℓ = slant height r = radius
Triangular Prism	$2B + Ph$	Bh	
Pyramid	$2\ell + B$	$\frac{1}{3} Bh$	
Cylinders	$2\pi rh + 2\pi r^2$	$\pi r^2 h$ or Bh	
Cone	$\pi r \ell + \pi r^2$	$\frac{1}{3} \pi r^2 h$	
Sphere	$4\pi r^2$	$\frac{4}{3} \pi r^3$	

Figure 1 | Formulas

Surface Area of a Net

Finding the *surface area* of a *rectangular prism* is as simple as finding the area of the outside of a box. This measure is useful in many practical situations. It can tell you how much fabric you would need to re-cover a cushioned foot stool, how much paint you should buy to repaint a room, or how much paper you need to wrap a present.

Surface area is also a key measurement if you work in the production of goods. Companies often have to determine what is the most economical option, or cheapest, to package their product. To determine the most economical option, they calculate the surface area of different packages to find which one requires the least amount of packaging.

The surface area of a solid is the total area of all of the *faces*. Let's see how this works by looking at the net of a rectangular prism.

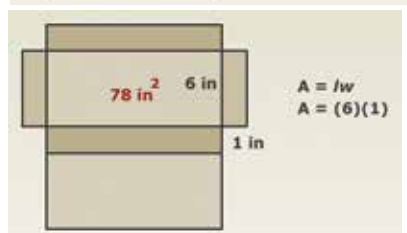
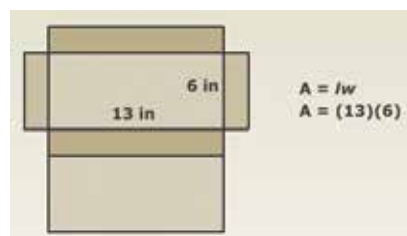
Example:

- ▶ What is the surface area of a rectangular prism that is 6 in. by 13 in. by 1 in.?

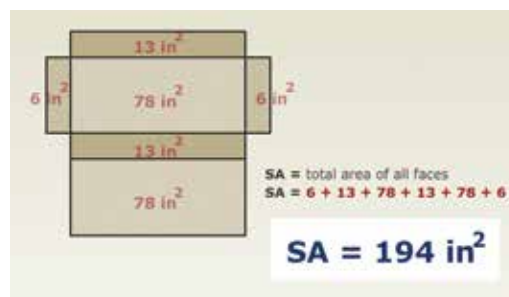


Solution:

- ▶ You can start with finding the area of any face. Then find the area of another.



- ▶ Find the area of the remaining faces and add them together.

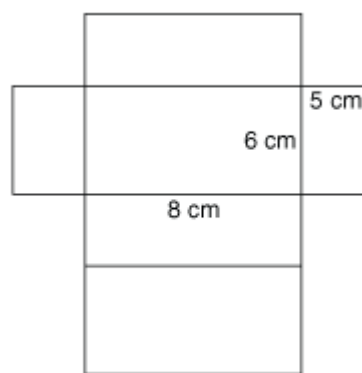


- ▶ The surface area of the rectangular prism is 194 square inches.

Let's look at another example.

Example:

- ▶ Find the surface area of this rectangular prism.



Solution:

- ▶ The first step is to find the area of each rectangle. This gives us the following six measures: 30 cm^2 , 40 cm^2 , 48 cm^2 , 40 cm^2 , 48 cm^2 , and 30 cm^2 .

Reminder! The area of a rectangle is length times width, or $A = lw$.

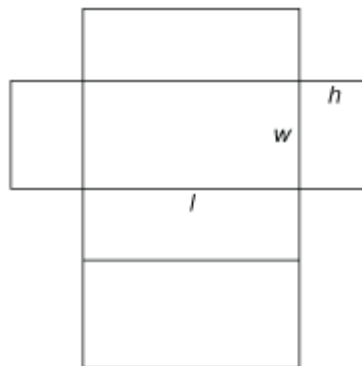
- ▶ To find the surface area, which is the total area, all we have to do is add them all together.
- ▶ $SA = 30\text{ cm}^2 + 40\text{ cm}^2 + 48\text{ cm}^2 + 40\text{ cm}^2 + 48\text{ cm}^2 + 30\text{ cm}^2$
- ▶ $SA = 236\text{ cm}^2$
- ▶ The surface area of the rectangular prism is 236 square centimeters.

Be Careful! Because we are adding the measures together, we don't change the exponent on our units. Surface area, as with all measures of area, uses square units for the answer.

Surface Area by Formula

You can also find the surface area of a rectangular prism by using a formula. Using the formula or finding the area of each face is the same thing. Take a look at how we can go from finding the area of each face to write the formula.

Let's use the following net to help us see how the formula is derived. Notice that instead of numbers, we have l for length, w for width, and h for height.



If we were to find the area of each face, we would have the following measures: $lw + lh + wh + lh + wh + lw$. The next step is to combine like terms. In order to do so, we must have the exact same variables. Let's reorganize the terms to put similar expressions next to one another.

Reminder! Multiplication is commutative; so, lw is the same as wl .

$$\begin{array}{ll}
 lw + lh + wh + lh + wh + lw & \text{Original expression.} \\
 lw + lw + lh + lh + wh + wh & \text{Place like terms} \\
 & \text{together.} \\
 2lw + 2lh + 2wh & \text{Combine like terms.}
 \end{array}$$

We can see that to find the surface area of a rectangular prism, we can use the formula:

$$SA = 2lw + 2lh + 2wh$$

Let's see how if the formula works by using a set of numbers we have already used.

Example:

- ▶ What is the surface area of a rectangular prism that has a length of 5 centimeters, a width of 6 centimeters, and a height of 8 centimeters?

Solution:

- ▶ The first step is to plug the measures into the formula.

$$SA = 2lw + 2lh + 2wh$$

Equation for surface area.

$$SA = 2(5 \text{ cm})(6 \text{ cm}) + 2(5 \text{ cm})(8 \text{ cm}) + 2(6 \text{ cm})(8 \text{ cm})$$

Substitute in the known values.

$$SA = 60 \text{ cm}^2 + 80 \text{ cm}^2 + 96 \text{ cm}^2$$

Multiply.

$$SA = 236 \text{ cm}^2$$

Add.

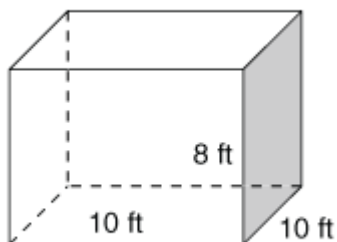
- ▶ The surface area is 236 square centimeters.

Notice that we got the same answer as in the previous example. This means that you can choose either the surface area formula or you can find the area of each face and then add them all together.

Let's try another example using the formula for surface area.

Example:

- ▶ What is the surface area of the following rectangular prism?

**Solution:**

- ▶ The length is 10 feet, the width is 10 feet, and the height is 8 feet. Now we can plug them into our formula.

$$SA = 2lw + 2lh + 2wh$$

Equation for surface area.

$$SA = 2(10 \text{ ft})(10 \text{ ft}) + 2(10 \text{ ft})(8 \text{ ft}) + 2(10 \text{ ft})(8 \text{ ft})$$

Substitute in the known values.

$$SA = 200 \text{ ft}^2 + 160 \text{ ft}^2 + 160 \text{ ft}^2$$

Multiply.

$$SA = 520 \text{ ft}^2$$

Add.

- ▶ The surface area is 520 square feet.

Finding Missing Values using the Surface Area Equation

You can also figure out a missing measure if you've been given the surface area and the other two measures. Let's look at an example of how to find a missing measure.

Example:

- ▶ What is the length of a rectangular prism that has a surface area of 190 square millimeters? The width is 3 millimeters, and the height is 5 millimeters.

Solution:

- ▶ The first step is to plug what we know into the surface area formula.

$$SA = 2lw + 2lh + 2wh$$

Equation for surface area.

$$190 \text{ mm}^2 = 2(l)(3 \text{ mm}) + 2(l)(5 \text{ mm}) + 2(3 \text{ mm})(5 \text{ mm})$$

Substitute in the known values.

$$190 \text{ mm}^2 = (6 \text{ mm})(l) + (10 \text{ mm})(l) + 30 \text{ mm}^2$$

Multiply.

$$190 \text{ mm}^2 = (16 \text{ mm})(l) + 30 \text{ mm}^2$$

Add the like terms on the right.

$$160 \text{ mm}^2 = (16 \text{ mm})(l)$$

Subtract 30 mm^2 from both sides.

$$10 \text{ mm} = l$$

Divide both sides by 16 mm.

- ▶ The length of the prism is 10 millimeters.

Example:

- ▶ A cube has a surface area of 150 square inches. What is the length of its sides?

Solution:

- ▶ Your first thought might be that you haven't been given enough information to solve this problem. However, you have. A cube is a special kind of rectangular prism. It's special because its length, width, and height are all the same measure. This means that we can square the length and then multiply it by six to find the surface area, $SA = 6l^2$.

- ▶ Since we have the surface area, we will be working backward.

$$SA = 6l^2 \text{ Surface area equation.}$$

$$150 \text{ in.}^2 = 6l^2 \text{ Substitute in the known value.}$$

$$25 \text{ in.}^2 = l^2 \text{ Divide both sides by 6.}$$

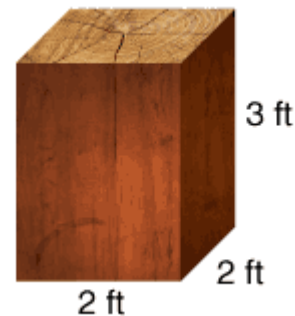
$$5 \text{ in.} = l \text{ Because we had } l^2, \text{ we take the square root of each side.}$$

- ▶ The length of the sides of the cube is 5 inches.

The beginning of the lesson mentioned practical applications of surface area. Let's look at how surface area can help us outside of math class.

Example:

- ▶ A sculptor built a platform in the shape of a rectangular prism for his latest creation. He wants to paint the top and the four sides of the platform.



- ▶ What is the surface area he will need to paint?

Solution:

- ▶ We can use either the formula or the sum of the areas of each face. Let's use the formula, but we need to make a small adjustment to it. We only need to paint one *base*. This means that instead of using $2lw$, we need to use just lw . Here's what our new formula looks like:
- ▶ $SA = lw + 2lh + 2wh$
- ▶ We are now ready to find the surface area of paintable surfaces for the sculptor. The platform has a length of 2 feet, a width of 2 feet, and a height of 3 feet.

$$SA = lw + 2lh + 2wh$$

Equation for surface area with one base.

$$SA = (2 \text{ ft})(2 \text{ ft}) + 2(2 \text{ ft})(3 \text{ ft}) + 2(2 \text{ ft})(3 \text{ ft})$$

Substitute in the known values.

$$SA = 4 \text{ ft}^2 + 12 \text{ ft}^2 + 12 \text{ ft}^2$$

Multiply.

$$SA = 28 \text{ ft}^2$$

Add.

- ▶ The sculptor needs to paint 28 square feet of surface area.

Surface area is a useful measure if you need the total outside measure of a figure.

Be careful not to confuse the surface area of an object with the capacity, or volume, of the object.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

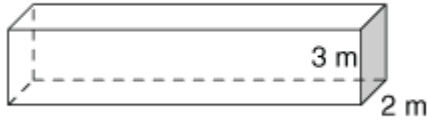
- One way to find the surface area of a rectangular prism is to find the sum of the area of each side of the prism.
- Another way to find the surface area of a rectangular prism is to use the formula.



Complete the following activities.

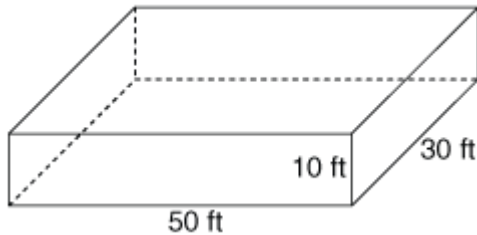
- 1.21** You multiply the length times the width times the height to find the surface area of a rectangular prism.
- True
- False
- 1.22** The surface area is the amount a solid can hold.
- True
- False
- 1.23** A rectangular prism has a length of 5 centimeters, a width of 5 centimeters, and a height of 10 centimeters. The surface area of this prism is 250 square centimeters.
- True
- False
- 1.24** Surface area is measured in units cubed.
- True
- False

1.25 What is the surface area of the rectangular prism, if its length is 17 meters?



- 114 m²
- 114 m³
- 182 m²
- 182 m³

1.26 ABC paint company needs to paint the outside of the building shown. They will also need to paint the roof. What is the surface area that needs to be painted?



- 3,100 ft²
- 4,600 ft²
- 4,100 ft²
- 4,300 ft²

1.27 What is the length of each edge of a cube if its surface area is 486 square units?

- 81 units
- 18 units
- 9 units
- 3 units

1.28 Casey wants to wrap a gift that measures 8 inches by 6 inches by 10 inches. What is the surface area that he needs to wrap?

- 416 in.²
- 376 in.²
- 384 in.²
- 396 in.²

1.29 The surface area of a rectangular prism is 442 square feet. The length is 11 feet, and the height is 8 feet. What is the width of the prism?

- 5 ft
- 8 ft
- 2 ft
- 7 ft

SURFACE AREA OF TRIANGULAR PRISMS

The Toblerone® chocolate bar was invented in 1908 by Theodor Tobler and his cousin, Emil Baumann. Even though their patented chocolate bar, the only one patented, is a mixture of unusual flavors, it's actually the shape of the chocolate bar and its packaging that it's most known

for. It is thought that the Swiss Alps, more specifically the Matterhorn, inspired the triangular shape of the candy and its package in the shape of a triangular prism.

This lesson will help you understand how to find the surface area of a triangular prism.

Objectives

- Find the surface area of a triangular prism using its net.
- Calculate the surface area of a triangular prism.
- Solve for a missing measure when given the surface area and other dimensions of a triangular prism.

Vocabulary

base of a solid—a special face of a solid figure

face—a plane figure that is one side of a solid figure

lateral face—any face that is not a base

surface area—the total area of all the faces

triangular prism—three-dimensional figure with five faces; has two triangular parallel, congruent bases and three rectangular lateral faces

Surface Area of the Net

Emil Baumann's role was production manager for the Toblerone® chocolate bar. Probably the hardest part of his job was determining how much cardboard was necessary to properly package their *triangular prism*-shaped candy. To find this out, he had to find the *surface area* of the packaging.

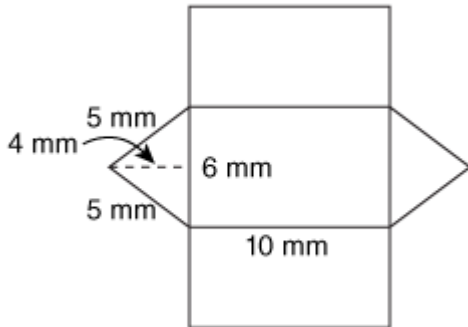
The net of a triangular prism is made up of two triangle-shaped *bases* and three rectangular *lateral faces*.



Since surface area means the total area of the outside of a figure, we need to find the area of each *face* and, then, add them together to get the total area.

Example:

- ▶ What is the surface area of the triangular prism represented?


Solution:

- ▶ Let's start with the triangular bases. The formula for finding the area of a triangle is $\frac{1}{2}bh$ or $\frac{bh}{2}$.
 - $A = \frac{bh}{2}$
 - $A = \frac{(6)(4)}{2}$
 - $A = \frac{24}{2}$
 - $A = 12 \text{ mm}^2$
- ▶ Because the bases of a prism are congruent, we know that each triangle has an area of 12 square millimeters. The two bases would have a total area of 24 square millimeters.
- ▶ Now, let's work on the rectangular faces from top of the net to the bottom of the net.
- ▶ The top rectangle has a length of 10 millimeters. Its width is 5 millimeters, since it is the same measure as the 5 millimeter side of the triangular base. The formula for the area of a rectangle is lw .
 - $A = lw$
 - $A = (10 \text{ mm})(5 \text{ mm})$

- $A = 50 \text{ mm}^2$
 - ▶ The middle rectangle also has a length of 10 millimeters, but it has a width of 6 millimeters.
 - $A = lw$
 - $A = (10 \text{ mm})(6 \text{ mm})$
 - $A = 60 \text{ mm}^2$
 - ▶ The bottom rectangle has a length of 10 millimeters and a width of 5 millimeters.
 - $A = lw$
 - $A = (10 \text{ mm})(5 \text{ mm})$
 - $A = 50 \text{ mm}^2$
 - ▶ The last step is to find the sum of the areas.
 - $SA = 12 \text{ mm}^2 + 12 \text{ mm}^2 + 50 \text{ mm}^2 + 60 \text{ mm}^2 + 50 \text{ mm}^2$
 - $SA = 184 \text{ mm}^2$
 - ▶ The total surface area for this triangular prism is 184 square millimeters.

Surface Area by Formula

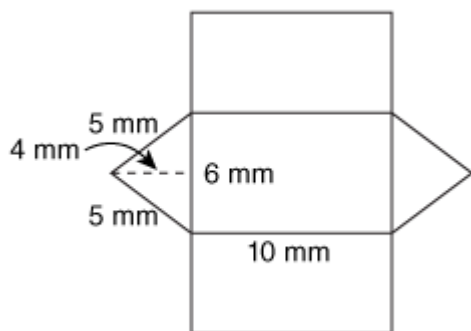
Another approach to finding the surface area of a triangular prism is to use its formula. Let's start by looking at the formula to see where it came from.

$$\blacksquare SA = 2B + Ph$$

The capital letter B stands for the area of the base. The bases on a triangular prism are triangles and, because there are two of them, we multiply the area by two. Thus, we get $2B$.

The capital letter P stands for the perimeter of the base, which is the sum of the sides of the triangle. The perimeter times the height will give us the area of all three rectangular faces at one time.

Let's look at the last example to help us understand how the formula works.



$$SA = 2B + Ph$$

$$SA = 2\left(\frac{6(4)}{2}\right) + (6 \text{ mm} + 5 \text{ mm} + 5 \text{ mm})(10 \text{ mm})$$

$$SA = 2(12 \text{ mm}^2) + (16 \text{ mm})(10 \text{ mm})$$

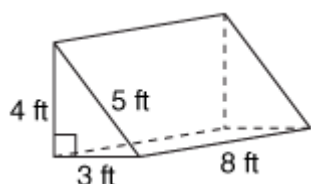
$$SA = 24 \text{ mm}^2 + 160 \text{ mm}^2$$

$$SA = 184 \text{ mm}^2$$

Notice that we got the same answer. Because we got the same answer, we know that we can use the formula or find the sum of the areas of each face. Let's look at another example using the formula.

Example:

- ▶ Find the surface area of the following figure.



Solution:

- ▶ $SA = 2B + Ph$
- ▶ $SA = 2\left(\frac{3(4)}{2}\right) + (3 \text{ ft} + 4 \text{ ft} + 5 \text{ ft})(8 \text{ ft})$
- ▶ $SA = 2(6 \text{ ft}^2) + (12 \text{ ft})(8 \text{ ft})$

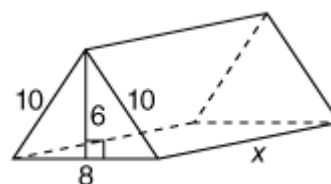
- ▶ $SA = 12 \text{ ft}^2 + 96 \text{ ft}^2$
- ▶ $SA = 108 \text{ ft}^2$
- ▶ The total surface area for this triangular prism is 108 square feet.

Finding Missing Measures for Triangular Prisms

Sometimes you will be asked to find a missing measure of a figure. The best way to solve a problem similar to this is to plug the information you know into the surface area formula and solve for the unknown. Take a look at an example.

Example:

- ▶ What is the height of this figure if it has a surface area of 524 square centimeters?



Solution:

- ▶ $SA = 2B + Ph$
- ▶ $524 \text{ cm}^2 = 2\left(\frac{8(6)}{2}\right) + (8 \text{ cm} + 10 \text{ cm} + 10 \text{ cm})(x)$
- ▶ $524 \text{ cm}^2 = 2(24 \text{ cm}^2) + (28 \text{ cm})(x)$
- ▶ $524 \text{ cm}^2 = 48 \text{ cm}^2 + (28 \text{ cm})x$
- ▶ $476 \text{ cm}^2 = (28 \text{ cm})x$
- ▶ $17 \text{ cm} = x$
- ▶ The height of the prism is 17 centimeters.

When solving for unknown measures, set up the problem using an equation, and solve for the variable. Remember that you have to keep the equation balanced at all times, so whatever you do to one side of

the equal sign, be sure you do the same to the other side.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- The surface area of a triangular prism can be found by using the formula:

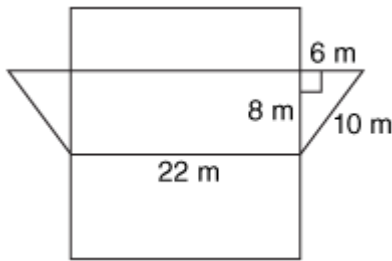
$SA = 2B + Ph$. The capital letter B stands for the area of the base. The capital letter P stands for the perimeter of the base, which is the sum of the sides of the triangle. The h is the height of the prism.

- Another way to find the surface area is by finding the sum of the areas of each face. If you use this method, be careful to include each of the five faces.

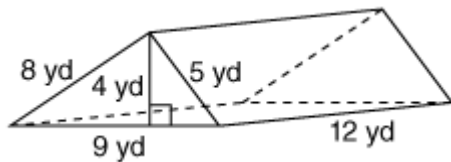


Complete the following activities.

1.30 The surface area of the triangular prism is:



1.31 The surface area of the prism is:



1.32 Which is the correct formula to find the surface area of a triangular prism?

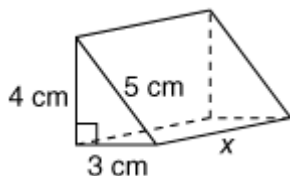
$\frac{B}{2} + Ph$

$B + Ph$

$2B + Ph$

$\frac{2B}{2} + Ph$

1.33 The surface area of the triangular prism pictured is 204 square centimeters. What is the height of the prism?



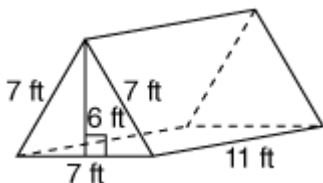
9 cm

16 cm

12 cm

22 cm

1.34 How much fabric is needed to make a tent in the shape of a triangular prism with the following measures?



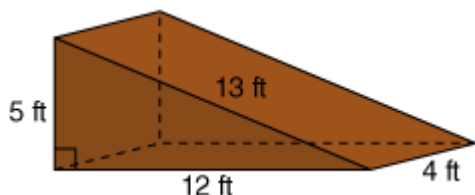
273 ft²

329 ft²

294 ft²

266 ft²

1.35 You built a skateboard ramp in the shape of a triangular prism. If you want to paint the entire ramp, even the bottom, what is the total area you will paint?



150 ft²

210 ft²

240 ft²

180 ft²

SURFACE AREA OF CYLINDERS

There are a number of different formulas in the study of geometry, making it difficult to keep them all straight. As formulas are presented, add them to a formula sheet to help you when completing the lessons.

Objectives

- Determine the surface area of a net of a cylinder.
- Calculate the surface area of a cylinder using its formula.
- Find the length of the curved surface of a cylinder.
- Understand the derivation of the surface area formula for a cylinder.



Vocabulary

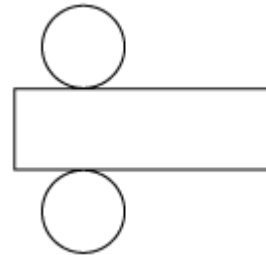
cylinder—three-dimensional figure with two parallel, equal circular bases and a curved surface

surface area—total area of the outside of a figure

Surface Area of a Net

Cylinders are a very common shape. Canned vegetables, pipes, and even containers of tennis balls are all typically cylinders. Anytime a company produces a good in the shape of a cylinder, they need to know its *surface area* to determine how much material they need for packaging.

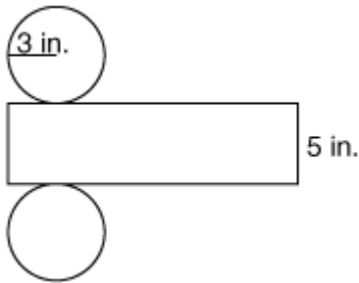
Remember that when you have the net of a cylinder, you have two congruent, circular bases and a curved surface. The curved surface, when laid flat, resembles a rectangle.



The surface area of a cylinder is the total area of its net; in this case, it's the area of the two circular bases as well as the area of the curved surface. We know that the area of a circle is found by using the formula πr^2 . The curved surface is a rectangle. So, we can use the formula lw to find its area. Let's find the surface area of a cylinder.

Example:

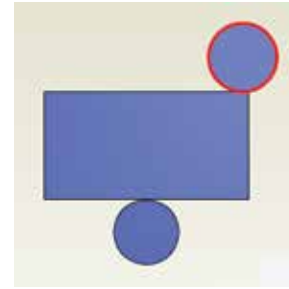
- ▶ Find the surface area of the cylinder represented.

**Solution:**

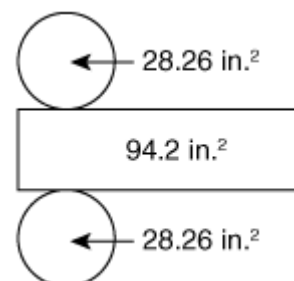
- ▶ Let's start by finding the area of one of the bases. Since they are congruent, we only need to find the area once to take care of two parts of the net.

Be Careful! Make sure you use the order of operations to help you simplify. Always do exponents before you multiply.

- ▶ $A = \pi r^2$
- ▶ $A = (3.14)(3 \text{ in.})^2$
- ▶ $A = (3.14)(9 \text{ in.}^2)$
- ▶ $A = 28.26 \text{ in.}^2$
- ▶ We can fill in the circular bases with 28.26 square inches each.
- ▶ We now have to find the area of the curved surface represented by the rectangle. We are told the height of the surface is 5 inches, but what is its length? We weren't given the length of the surface directly, but we were given enough information to figure it out ourselves.



- ▶ If the circular base is rolled the length of the curved surface you will see that it is the exact same measure as the distance around the outside of the circle. We know that the distance around the outside of a circle is its circumference. The formula for the circumference of a circle is $2\pi r$. By solving this formula, we can find the length of the curved surface.
- ▶ $\text{length} = 2\pi r$
- ▶ $\text{length} = 2(3.14)(3 \text{ in.})$
- ▶ $\text{length} = 18.84 \text{ in.}$
- ▶ We are now ready to find the area of the curved surface. We know that the area of a rectangle is length times width or, in the case of this curved surface, we can use circumference times height.
- ▶ $A = lw = Ch$
- ▶ $A = (18.84 \text{ in.})(5 \text{ in.})$
- ▶ $A = 94.2 \text{ in.}^2$
- ▶ Look at the following figure. You can see that we now know the areas of all three surfaces.



- ▶ The last step is to find the sum of the three areas to find the total surface area.
- ▶ $SA = 28.26 \text{ in.}^2 + 28.26 \text{ in.}^2 + 94.2 \text{ in.}^2$
or
- ▶ $SA = 2(28.26 \text{ in.}^2) + 94.2 \text{ in.}^2$
- ▶ $SA = 150.72 \text{ in.}^2$
- ▶ The total surface area of the cylinder is 150.72 square inches.

Surface Area by Formula

You can also find the surface area of a cylinder by using a formula. The formula is $SA = 2\pi rh + 2\pi r^2$.

If you take a closer look at the formula, you can see all the different parts we just used in the previous example. The first part, $2\pi rh$, shows the circumference times the height, which is the area of the curved surface. The second part, $2\pi r^2$, is the area of a circle (the base) multiplied by two, because there are two circular bases on a cylinder.

Let's use the formula this time.

Example:

- ▶ Find the surface area of a cylinder with a radius of 3 inches and a height of 5 inches.

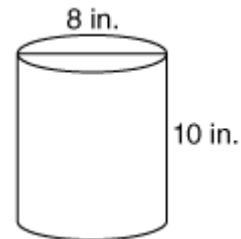
Solution:

- ▶ This problem should look familiar, as we just computed it using the net of the cylinder.
- ▶ $SA = 2\pi rh + 2\pi r^2$
- ▶ $SA = 2(3.14)(3 \text{ in.})(5 \text{ in.}) + 2(3.14)(3 \text{ in.})^2$
- ▶ $SA = 94.2 \text{ in.}^2 + 56.52 \text{ in.}^2$
- ▶ $SA = 150.72 \text{ in.}^2$
- ▶ The total surface area of the cylinder is 150.72 square inches.

Notice we got the same answer, even though this time we used the formula. Let's try a new problem using only the formula this time.

Example:

- ▶ Find the surface area of the cylinder shown.



Be Careful! The formula is asking you to use the radius of the circle, not the diameter. The diameter goes across the entire circle, while the radius is halfway.

Solution:

- ▶ $SA = 2\pi rh + 2\pi r^2$
- ▶ $SA = 2(3.14)(4 \text{ in.})(10 \text{ in.}) + 2(3.14)(4 \text{ in.})^2$
- ▶ $SA = 251.2 \text{ in.}^2 + 100.48 \text{ in.}^2$
- ▶ $SA = 351.68 \text{ in.}^2$
- ▶ The surface area is 351.68 square inches.

Example:

- ▶ What is the surface area of a paper towel tube that is open on both ends if it has a diameter of 2 inches and a height of 12 inches?

Solution:

- ▶ Because the two ends are open, we only have to find the surface area of the curved surface.
- ▶ $SA = 2\pi rh$
- ▶ $SA = 2(3.14)(1 \text{ in.})(12 \text{ in.})$

- ▶ $SA = 75.36 \text{ in.}^2$
- ▶ The surface area is 75.36 square inches.

You can find the surface area of a cylinder through two methods. It doesn't matter which method you use; you will get the same answer.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Cylinders are geometric solids composed of two circular bases and a curved surface.
- When the curved surface of a cylinder is laid flat, it resembles a rectangle.
- You can find the surface area of a cylinder by either finding the sum of the areas of the two bases and the curved surface or by using the formula for surface area of cylinders.

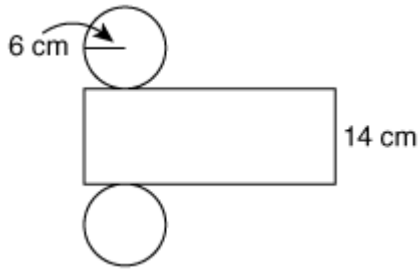
Be careful not to forget the units squared, or your answer will be wrong.



Complete the following activities.

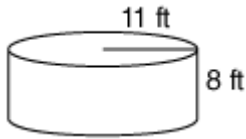
- 1.36** The formula for finding the surface area of a cylinder is $2\pi rh + \pi r^2$.
- True
 - False
- 1.37** A cylinder has a diameter of 10 decimeters. What is the length of the curved surface?
- 1.38** A tube of tennis balls has the following measures: 4-inch diameter and a height of 8 inches. What is the surface area of the tube of tennis balls? (Use 3.14 for π .)
- | | |
|--|--|
| <input type="checkbox"/> 213.52 in. ² | <input type="checkbox"/> 226.08 in. ² |
| <input type="checkbox"/> 301.44 in. ² | <input type="checkbox"/> 125.6 in. ² |

- 1.39 Use the following net to determine the surface area of the cylinder. (Use 3.14 for π .)



- 640.56 cm^2
 339.12 cm^2
 753.6 cm^2
 512.56 cm^2

- 1.40 What is the surface area of the cylinder? (Use 3.14 for π .)

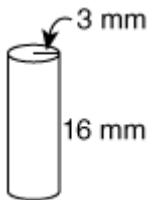


- 552.64 ft^2 759.88 ft^2
 1,312.52 ft^2 1,865.16 ft^2

- 1.41 A can of soup is in the shape of a cylinder. The diameter of the can is 8 centimeters, and the height is 10 centimeters. What is the surface area of the can, rounded to the nearest hundredth square centimeter? (Use 3.14 for π .)

- 351.68 cm^2 251.2 cm^2 502.4 cm^2 100.48 cm^2

- 1.42 What is the approximate surface area of the cylinder shown? (Use 3.14 for π .)



- 301.44 mm^2
 357.96 mm^2
 339.12 mm^2
 370.52 mm^2

- 1.43 What is the surface area of a piece of pipe that is open at both ends, has a radius of 6 inches, and a height of 18 inches? (Use 3.14 for π .)

- 339.12 in.^2 904.32 in.^2 678.24 in.^2 548.96 in.^2

- 1.44 You need to repaint the floor and inside wall of your circular swimming pool. It has a diameter of 16 feet and a depth of 5 feet. What is the surface area that needs to be repainted? (Use 3.14 for π .)

- 1,055.04 ft^2 251.2 ft^2 401.92 ft^2 452.16 ft^2

SURFACE AREA OF PYRAMIDS, CONES, AND SPHERES

The ancient Egyptians have a long, rich history of amazing feats, including the building of the pyramids. The Great Pyramid of Giza took 20 years to build, which is impressive, especially considering they didn't have modern machinery. To understand how truly remarkable this feat is, consider the measures of the pyramid. The Great Pyramid of Giza stood, before erosion, approximately 147 meters tall. The length of each side of the base of the pyramid is approximately 230 meters long. Its size is impressive. Now, imagine if someone wanted to paint it. What do you think the surface area of the Great Pyramid is?



This lesson will show you how to find the surface areas of pyramids, cones, and spheres.

Objectives

- Calculate the surface area of a pyramid using the net of the figure.
- Find the surface area of a pyramid, a cone, and a sphere using formulas.

Vocabulary

base of a solid—a special face of a solid

cone—three-dimensional figure with a circular base and a curved surface that rises to a vertex

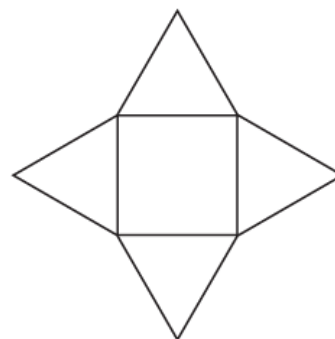
pyramid—three-dimensional figure with a polygon as a base and all other faces are triangles with a common vertex

slant height—height of a lateral surface

surface area—the total area of all the faces

Square Pyramids

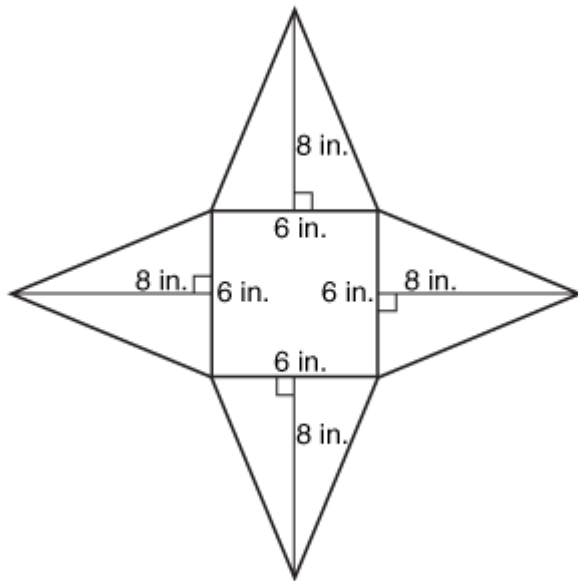
Pyramids can have different bases, but, in this lesson, we are going to focus on square pyramids only. This means that the base of the pyramid will be a square, and it will have four triangular faces. This is a net of a square pyramid.



Surface area is the total area of all of the faces. If we find the area of each face and add them together, we will get the surface area for the pyramid.

Example:

- ▶ Find the surface area of this net.



Solution:

- ▶ Notice that the four triangular faces are equivalent. This means that we only need to find the area of one of the triangles. We can then use that measure for each of the other three triangles.

$$\text{▶ } A = \frac{bh}{2}$$

$$\text{▶ } A = \frac{6(8)}{2}$$

$$\text{▶ } A = \frac{48}{2}$$

$$\text{▶ } A = 24 \text{ in.}^2$$

- ▶ We now know that each of our triangles has an area of 24 square inches. Our next step is to find the area of the square.

$$\text{▶ } A = lw$$

$$\text{▶ } A = (6 \text{ in.})(6 \text{ in.})$$

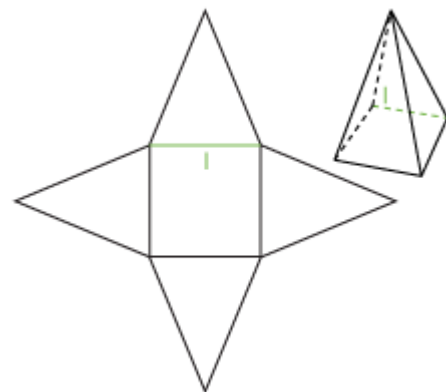
$$\text{▶ } A = 36 \text{ in.}^2$$

- ▶ The final step is to add all the faces together to get the surface area.
- ▶ $SA = 24 \text{ in.}^2 + 24 \text{ in.}^2 + 24 \text{ in.}^2 + 24 \text{ in.}^2 + 36 \text{ in.}^2$
- ▶ $SA = 132 \text{ in.}^2$
- ▶ The surface area of the pyramid is 132 square inches.

Some people find it too tedious to find the area of each face and then add them all together. It is often easier to use a formula. The decision is up to you. Whether you use a formula or find the area of each face, you are still finding the same surface area. Let's look at the use of the formula to find the surface area of a square pyramid.

The formula for finding the surface area of a square pyramid is $SA = 2l\ell + l^2$.

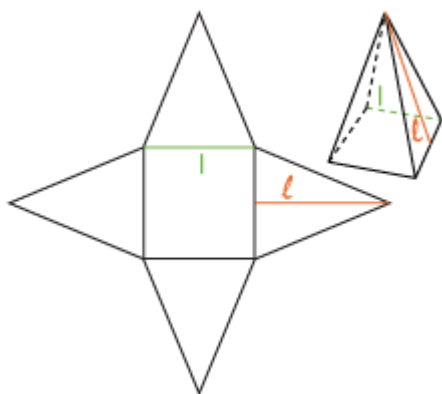
The first thing we need to do is determine what each variable stands for. The l stands for the length of a side of the base. Look at the following figures. The length on both the net and on the solid are highlighted for you.



The other variable you see is the ℓ . The cursive-scripted variable stands for the *slant height*. The slant height is the height of the lateral surface. The slant height goes from the center of a *side* of the base to the vertex

at the top of the lateral face. The following figures show the slant height of a pyramid. Be careful not to confuse the height of the pyramid, which goes straight up from the center of the base, with the slant height of the pyramid.

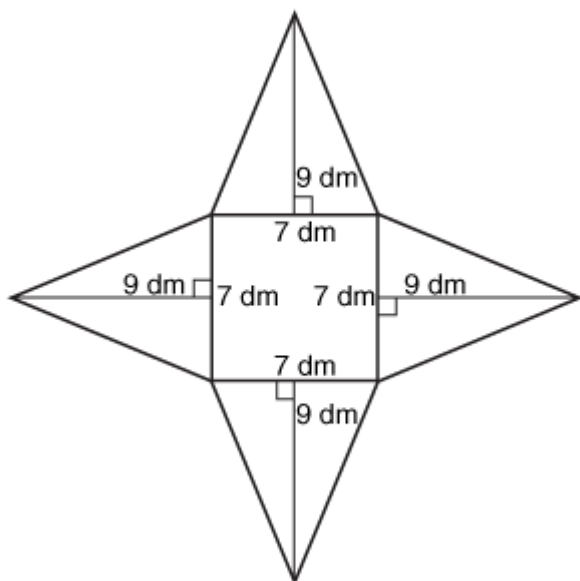
Vocabulary! A lateral face is a face of a solid that is not a base.



Let's try finding the surface area of a pyramid using the formula.

Example:

- ▶ Find the surface area of this square pyramid.

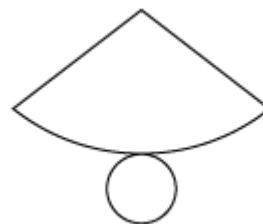


Solution:

- ▶ $SA = 2l + l^2$
- ▶ $SA = 2(7 \text{ dm})(9 \text{ dm}) + (7 \text{ dm})^2$
- ▶ $SA = 126 \text{ dm}^2 + 49 \text{ dm}^2$
- ▶ $SA = 175 \text{ dm}^2$
- ▶ The surface area of the pyramid is 175 square decimeters.

Cones

Cones are a very recognizable geometric solid because many of us have seen them since we were little kids. We know the cone shape from when we eat ice cream in a cone and even from construction sites. But, do you know what the net of a cone looks like? The net includes a circle for the base of the cone and a pie-shaped wedge.

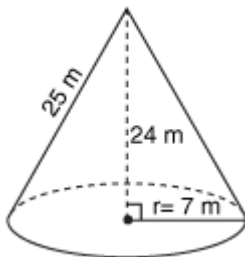


The formula for finding the surface area of a cone is $SA = \pi r l + \pi r^2$. If we take a look at the net of a cone, you can see where each part of the formula comes from. You know that the area of a circle is found by using the formula $A = \pi r^2$, which is the second part of the formula. That means the area of the pie shape is found by using $A = \pi r l$. Because surface area is the total area of all the faces, we know we need to add them. Let's work through a couple of examples of surface area of cones.

Example:

- ▶ Find the surface area of the following figure.

Be Careful! The figure shows two heights. One is the height of the cone, which is measured from the center of the base, and the other is the slant height. We want the slant height, or the height of the lateral surface.



Reminder! The radius is the distance half way across a circle. Be careful to use the radius and not the diameter of a circle.

Solution:

- ▶ $SA = \pi r l + \pi r^2$
- ▶ $SA = (3.14)(7 \text{ m})(25 \text{ m}) + (3.14)(7 \text{ m})^2$
- ▶ $SA = 549.5 \text{ m}^2 + 153.86 \text{ m}^2$
- ▶ $SA = 703.36 \text{ m}^2$
- ▶ The surface area of the cone is 703.36 square meters.

Spheres

A sphere is a solid that has all points that are the same distance from the center of the solid. We know spheres more commonly as balls. Some examples are baseballs, basketballs, volleyballs, bowling balls, and soccer balls. Companies that produce these items need to be able to find the surface area, so that they know how much material they need. Let's take a look at how to find the surface area of a sphere.

The formula for finding the surface area of a sphere is $SA = 4\pi r^2$. It breaks down to four times pi times the radius squared. Be careful to follow the order of operations when simplifying this formula. That means that the first step is to square the radius, and then move on to multiplying. Let's do an example, so that you can see how to find the surface area of a sphere.

Example:

- ▶ Find the surface area of the sphere.


Solution:

- ▶ $SA = 4\pi r^2$
- ▶ $SA = 4(3.14)(11 \text{ ft})^2$
- ▶ $SA = 4(3.14)(121 \text{ ft}^2)$
- ▶ $SA = 1,519.76 \text{ ft}^2$
- ▶ The surface area of the sphere is 1,519.76 square feet.

Example:

- ▶ What is the surface area of a basketball that has a diameter of 8 inches?

Solution:

- ▶ $SA = 4\pi r^2$
- ▶ $SA = 4(3.14)(4 \text{ in.})^2$
- ▶ $SA = 4(3.14)(16 \text{ in.}^2)$
- ▶ $SA = 200.96 \text{ in.}^2$

- ▶ The surface area of the basketball is 200.96 square inches.

Finding Unknown Measures

Sometimes you will be given some of the dimensions of a solid, as well as the surface area, and then be asked to find a missing measure. This is actually a simple thing to do when you have the formula.

Here's an example of finding the slant height of a cone.

Example:

- ▶ What is the slant height of a cone if it has a surface area of 452.16 square meters and a radius of 6 meters?

Solution:

- ▶ $SA = \pi r \ell + \pi r^2$
- ▶ $452.16 \text{ m}^2 = (3.14)(6 \text{ m})(x) + (3.14)(6 \text{ m})^2$
- ▶ $452.16 \text{ m}^2 = (3.14)(6 \text{ m})(x) + (3.14)(36 \text{ m}^2)$
- ▶ $452.16 \text{ m}^2 = (18.84 \text{ m})x + 113.04 \text{ m}^2$
- ▶ $339.12 \text{ m}^2 = (18.84 \text{ m})x$
- ▶ $18 \text{ m} = x$
- ▶ The slant height of the cone is 18 meters.

Even though you can find the surface area by finding the area of each face and then adding them together, you may find it easier to use the formula.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- The formula for finding the surface area of a square pyramid is $SA = 2\ell^2 + l^2$.
- The formula for finding the surface area of a cone is $SA = \pi r \ell + \pi r^2$.
- The formula for finding the surface area of a sphere is $SA = 4\pi r^2$.
- If you know the total surface area of an object, you can use the surface area formulas to solve for unknown dimensions.



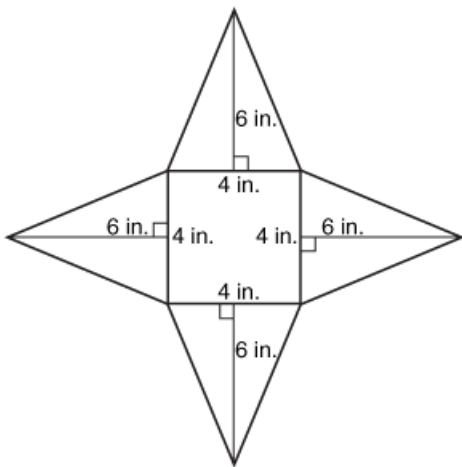
Complete the following activities.

1.45 The formulas for finding the surface area of a cone and a pyramid are the same.

- True
- False

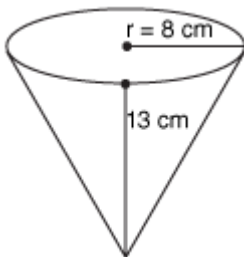
1.46 What is the difference between the height and the slant height of a pyramid?

1.47 Find the surface area of the pyramid.



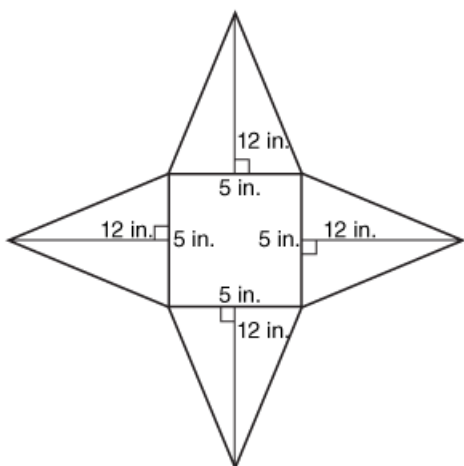
- 64 in.²
- 56 in.²
- 88 in.²
- 118 in.²

1.48 Find the surface area of the cone. (Use 3.14 for π .)



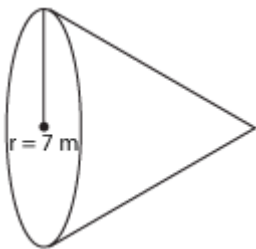
- 376.8 cm²
- 427.04 cm²
- 527.52 cm²
- 628 cm²

1.49 What is the surface area of the pyramid?



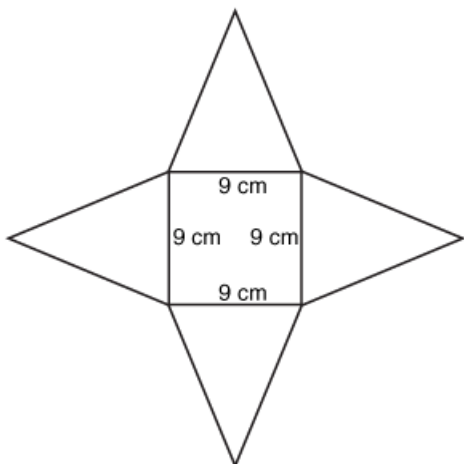
- 130 in.²
- 145 in.²
- 160 in.²
- 175 in.²

1.50 What is the slant height if the surface area is 395.64 square meters? (Use 3.14 for π .)



- 7 m
- 9 m
- 11 m
- 13 m

1.51 What is the slant height if the surface area is 243 square centimeters?



- 8 cm
- 10 cm
- 11 cm
- 9 cm

1.52 The height of a side on a pyramid is 11 inches long, and its square base has sides that are 6 inches long. What is the surface area?

- 233 in.²
- 168 in.²
- 144 in.²
- 207 in.²

- 1.53** A ball has a diameter of 8 inches. What is its surface area? (Use 3.14 for π .)
- 200.96 in.² 803.88 in.² 401.92 in.² 100.48 in.²
- 1.54** Gaelic football is played with a ball with a radius of 5 inches. What is the surface area of the ball used in Gaelic football? (Use 3.14 for π .)
- 157 in.² 125.6 in.² 251.2 in.² 314 in.²

SURFACE AREA OF COMPOSITE FIGURES

Many of the three-dimensional objects we encounter are made up of more than one geometric solid. It is helpful to be able to

identify which solids an object is made up of and to be able to find its surface area.

This lesson will help you understand how to find the surface area of composite figures.

Objectives

- Identify the solids of a composite figure.
- Calculate the surface area of a composite figure.

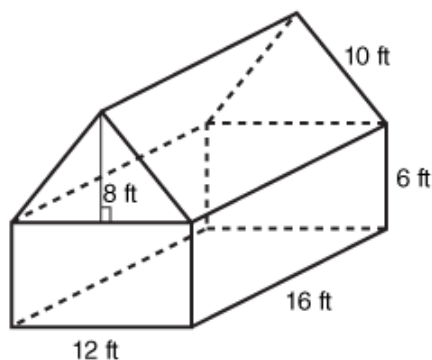
Surface Area of Composite Figures

Because our world includes objects that are made up of more than one geometric solid, it is important for us to understand how to find the surface area of composite figures. Surface area is the total area of the outside of a figure. The surface area can be found through two different methods. The first way is to find the area of each of the outside surfaces of the composite figure. The second way is to use the formula for each of the solids in the composite figure.

When finding the surface area of a composite figure, it is often best to use a combination of the two methods. Let's take a look at some examples.

Example:

- ▶ What is the surface area of the following figure?

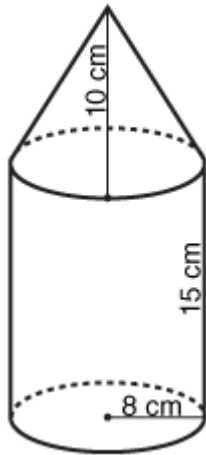


Solution:

- ▶ The first step is to identify that the figure is composed of a rectangular prism, on the bottom, and a triangular prism, on top. Notice that the top of the rectangular prism and the bottom of the triangular prism will not be included in the surface area. This is because they are not outside faces of the composite figure.
- ▶ We need to find the area of each exterior face.
- ▶ $(2)\left(\frac{(12)(8)}{2}\right) + 2(10\text{ ft})(16\text{ ft}) + 2(12\text{ ft})(6\text{ ft}) + 2(16\text{ ft})(6\text{ ft}) + (16\text{ ft})(12\text{ ft})$
- ▶ $96\text{ ft}^2 + 320\text{ ft}^2 + 144\text{ ft}^2 + 192\text{ ft}^2 + 192\text{ ft}^2$
- ▶ 944 ft^2
- ▶ The surface area for the figure is 944 square feet.

Example:

- ▶ Find the surface area of the following figure.

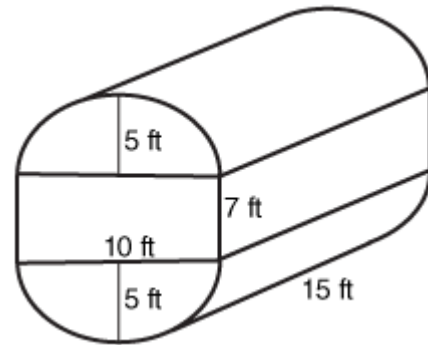

Solution:

- ▶ The composite figure is a cone and a cylinder. Again, we will not include the mutual face of the two solids that is an interior face.
- ▶ We can begin by finding the area of the curved surface of the cone. The formula for the area of the curved surface is $\pi r \ell$.
 - ▶ $\pi r \ell$
 - ▶ $(3.14)(8 \text{ cm})(10 \text{ cm})$
 - ▶ 251.2 cm^2
- ▶ Now we need to find the area of the curved surface of the cylinder, using the formula $2\pi rh$.
 - ▶ $2\pi rh$
 - ▶ $2(3.14)(8 \text{ cm})(15 \text{ cm})$
 - ▶ 753.6 cm^2
- ▶ Finally, we need to find the area of the circular base of the cylinder, using the formula πr^2 .
 - ▶ πr^2
 - ▶ $(3.14)(8 \text{ cm})^2$

- ▶ $(3.14)(64 \text{ cm}^2)$
- ▶ 200.96 cm^2
- ▶ Now that we have the area of all of the exterior faces, all that is left is to find their sum.
- ▶ $251.2 \text{ cm}^2 + 753.6 \text{ cm}^2 + 200.96 \text{ cm}^2$
- ▶ 1205.76 cm^2
- ▶ The surface area for the figure is 1205.76 square centimeters.

Example:

- ▶ What is the surface area of the figure?


Solution:

- ▶ This time, we have half of a cylinder on top of a rectangular prism, and then another half of a cylinder below the rectangular prism. If we look closely at the two halves of cylinders, we see that they are actually one cylinder, because they have the exact same measurements.
- ▶ Because the entire cylinder is on the exterior of the composite figure, we can use the formula for finding the surface area of a cylinder.
 - ▶ $SA = 2\pi rh + 2\pi r^2$
 - ▶ $SA = 2(3.14)(5 \text{ ft})(15 \text{ ft}) + 2(3.14)(5 \text{ ft})^2$
 - ▶ $SA = 471 \text{ ft}^2 + 157 \text{ ft}^2$
 - ▶ $SA = 628 \text{ ft}^2$

- ▶ Now we need to determine which faces we see of the rectangular prism.
- ▶ The front of the composite figure has a face with measures of 7 feet and 10 feet, which can be multiplied to find the area.
- ▶ $A = 7 \text{ ft} \cdot 10 \text{ ft}$
- ▶ $A = 70 \text{ ft}^2$
- ▶ Since we have two faces with that area, we need to multiply it by two, gives us 140 square feet.
- ▶ The side of the composite figure has a face with measures of 7 feet and 15 feet. So its area is $7 \text{ ft} \cdot 15 \text{ ft}$, or 105 square feet. Again, we know the other side of the composite figure will have the same face, so we need to multiply 105 square feet by two to get 210 square feet.
- ▶ The final step is to add all the areas together.
- ▶ $628 \text{ ft}^2 + 140 \text{ ft}^2 + 210 \text{ ft}^2$
- ▶ 978 ft^2
- ▶ The surface area is 978 square feet.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

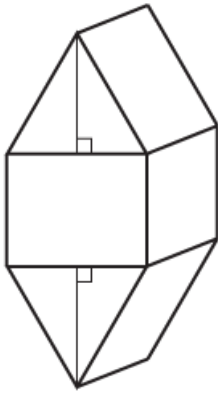
- Composite figures are made up of two or more geometric solids.
- When finding the surface area of a composite figure, you can only use the area of the faces that are exterior faces.
- It is best to use a combination of the two methods of finding surface area, using the formulas as well as finding the sum of the areas of the faces.



Complete the following activities.

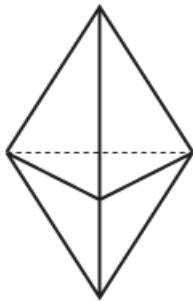
- 1.55** Finding the surface areas of composite figures can only be done using a formula.
- True
 False
- 1.56** Sometimes, you will have only half of a solid used in a composite figure.
- True
 False
- 1.57** The units for surface areas will always be _____.

1.58 What is the other shape in the composite figure besides the rectangular prism?



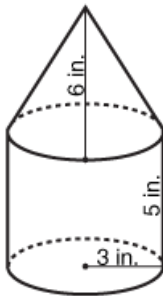
- triangular pyramid
- square pyramid
- triangular prism
- cube

1.59 The composite figure shown is made up of two ____.



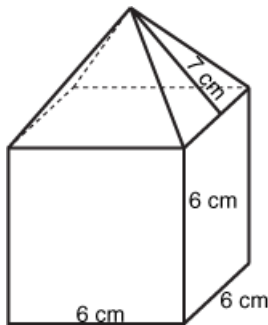
- cones
- triangular pyramids
- square pyramids
- rectangular prisms

1.60 What is the surface area of the figure shown? (Use 3.14 for π .)



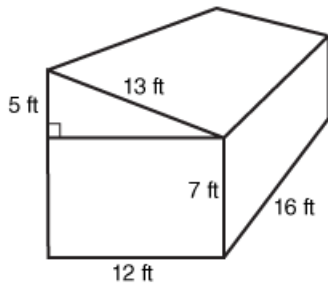
- 178.98 in.²
- 178.98 in.³
- 235.5 in.²
- 235.5 in.³

1.61 What is the surface area of the figure shown?



- 300 cm²
- 336 cm²
- 264 cm²
- 358 cm²

1.62 What is the surface area of the figure shown?



- 740 ft²
- 1,124 ft²
- 1,316 ft²
- 932 ft²



Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific area where restudy is needed for mastery.

SELF TEST 1: Surface Area

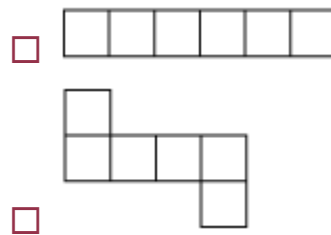
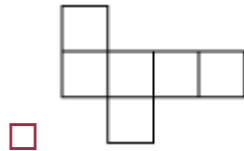
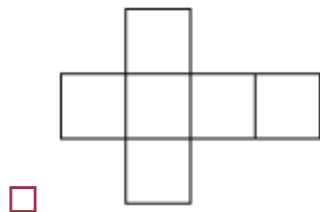
Complete the following activities (6 points, every numbered activity).

Students, please check with your teacher to determine if a formula sheet can be used with this self test.

1.01 Surface area is always measured in square units.

- True
- False

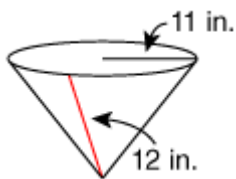
1.02 Which of the following is *not* the net of a cube?



1.03 A cone is a solid that has one circular base and only one face.

- True
- False

1.04 What is the surface area of the cone? (Use 3.14 for π .)

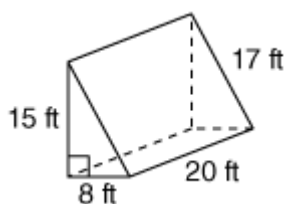


- 794.42 in.²
- 483.56 in.²
- 822.68 in.²
- 414.48 in.²

1.05 The surface area of a geometric solid is the amount of space inside the solid.

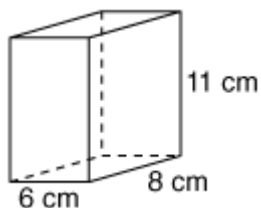
- True
- False

1.06 What is the surface area of the triangular prism shown?



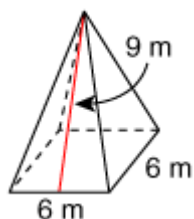
- 860 ft²
 920 ft²
 1,040 ft²
 760 ft²

1.07 What is the surface area of the rectangular prism?



- 528 cm²
 440 cm²
 404 cm²
 484 cm²

1.08 The square pyramid pictured has a surface area of ____.



- 144 m²
 120 m²
 189 m²
 126 m²

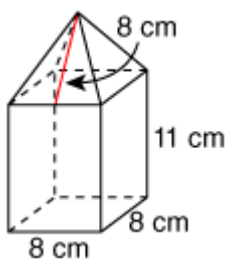
1.09 The net of a solid is the view from above the solid.

- True
 False

1.010 What is the surface area of a sphere with a radius of 7 feet? (Use 3.14 for π .)

- 175.84 ft² 615.44 ft² 351.68 ft² 703.36 ft²

1.011 Identify the two solids that create the composite figure shown.



- cone
 square pyramid
 cube
 rectangular prism

1.012 What is the radius of a sphere if its surface area is 2,122.64 square inches?
(Use 3.14 for π .)

- 11 in. 13 in. 121 in. 169 in.

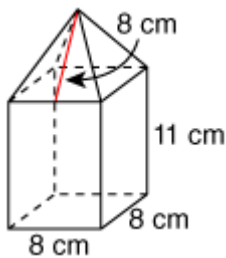
1.013 What is the slant height of a square pyramid that has a surface area of 189 square feet and a side length of 7 feet?

- 14 ft 12 ft 10 ft 8 ft

1.014 What is the height of cylinder with a surface area of 226.08 square meters and a radius of 3 meters? (Use 3.14 for π .)

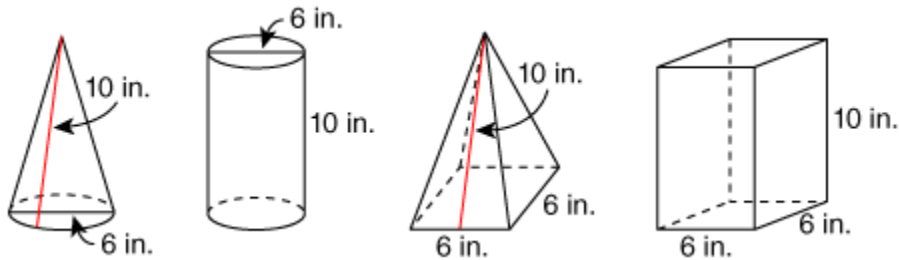
- 27 meters 18 meters 3 meters 9 meters

1.015 What is the surface area of the composite figure?



- 608 cm²
 672 cm²
 570 cm²
 544 cm²

1.016 Which container has the greatest surface area? (Use 3.14 for π .)



- cone square pyramid
 cylinder rectangular prism

<div style="border: 1px solid black; padding: 5px; display: inline-block;"> 76 <hr style="width: 50%; margin: 0;"/> 96 </div>	SCORE _____	TEACHER _____ initials date
--	--------------------	--



MAT0808 - May '14 Printing

ISBN 978-0-7403-3186-2



9 780740 331862



804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759

800-622-3070
www.aop.com