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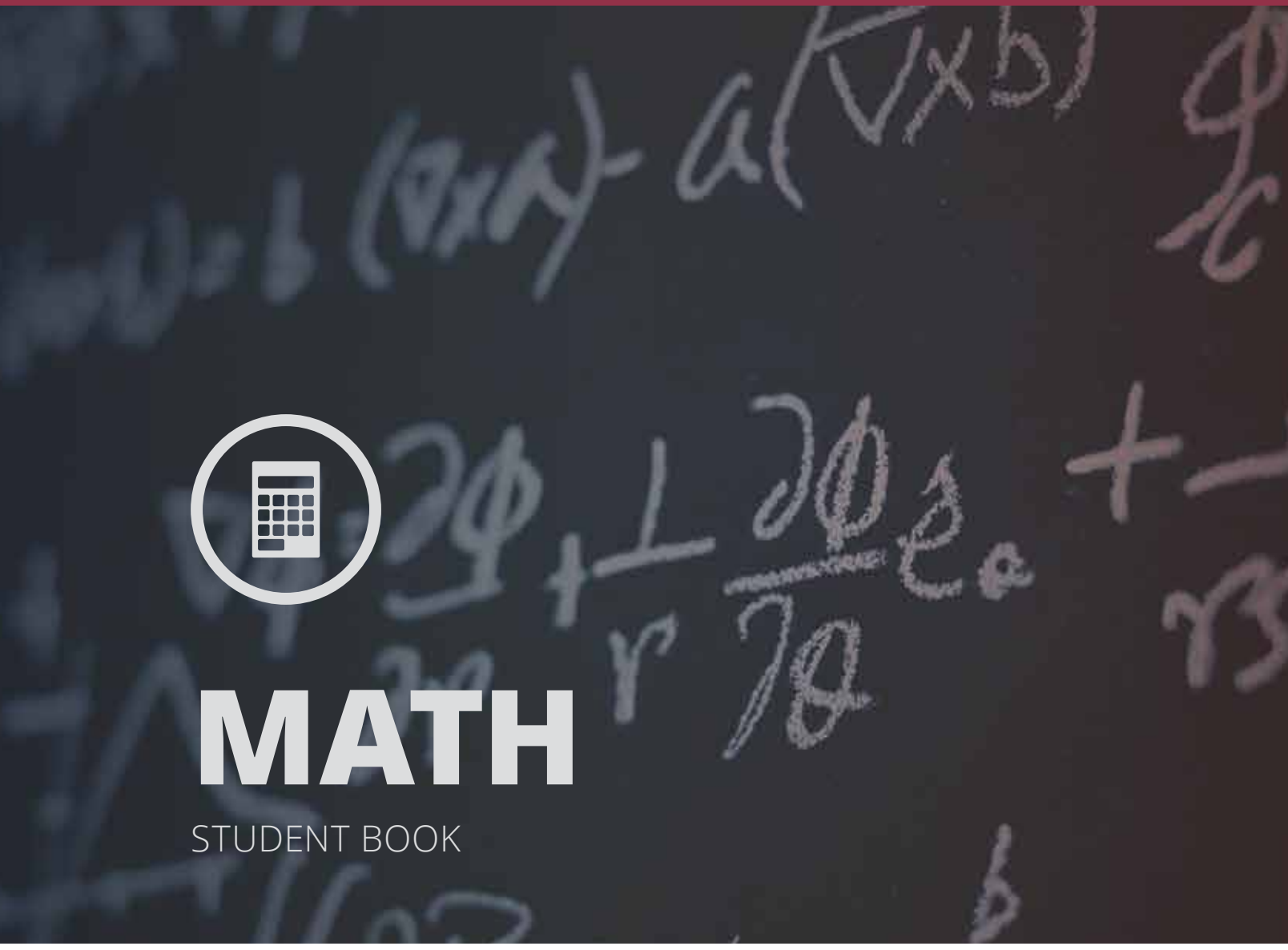

MATH

STUDENT BOOK

8th Grade | Unit 10

Alpha Omega
PUBLICATIONS

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Probability

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Probability

Introduction

This unit addresses outcomes of events and the probabilities of different outcomes. The first section introduces efficient methods for determining outcomes, such as tree diagrams and formulas. It explains the distinction between permutations and combinations. The lessons let students work with the formulas that allow us to compute the number of possible permutations or combinations. Section 2 covers theoretical and experimental probabilities. Lessons in this section show how to find probabilities for disjointed and overlapping events, as well as independent and dependent events.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Determine the number of possible outcomes using tree diagrams and the fundamental counting principle.
- Identify and evaluate permutation and combination problems.
- Find theoretical and experimental probabilities.
- Identify and compute probabilities of independent and dependent events.
- Identify and compute probabilities of overlapping and disjointed events.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. Outcomes

Tree Diagrams and the Counting Principle

Many science classes ask you to perform experiments as part of your assignments. You might even be asked to complete a lab report about your experiment, specifically about your outcome. Outcomes can also happen in experiments in math.

This lesson will help you understand the role that outcomes play in a probability experiment. You will also learn that knowing all possible outcomes can help you make predictions.

Objectives

- Identify all the possible outcomes for a given situation.
- Use tree diagrams to identify probabilities.
- Use the counting principle to identify probabilities.

Vocabulary

counting principle—uses multiplication to find the possible number of outcomes
outcome—a possible event
tree diagram—an organizational tool that uses branches to list choices

Outcomes

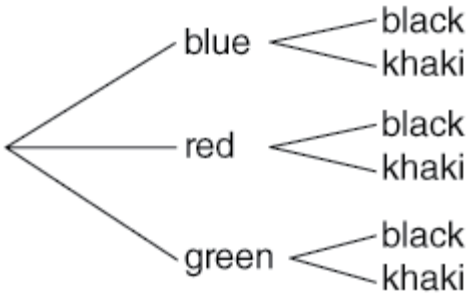
The *outcome* of a science experiment is the final result you get after completing the experiment. Outcomes, in probability experiments, are similar to those of a science experiment. The outcomes of a probability experiment are the possible events that can occur.

There are a number of ways to determine and list the outcomes for a probability experiment. This lesson is going to focus on using *tree diagrams*. Tree diagrams both help you determine the number of possible outcomes that can occur and allow you to see all the possible outcomes for a specific experiment. This lesson will also introduce you to the *counting principle*, which helps you determine the total number of possible outcomes for an experiment.

Tree Diagrams

Sometimes, it is hard to determine all the possible outcomes for an experiment without using some type of organizational tool. This is where tree diagrams are useful. A tree diagram is an organizational tool that uses branches to list choices/possible combinations. The tree diagram allows you to list all possible outcomes while seeing each possible combination of outcomes.

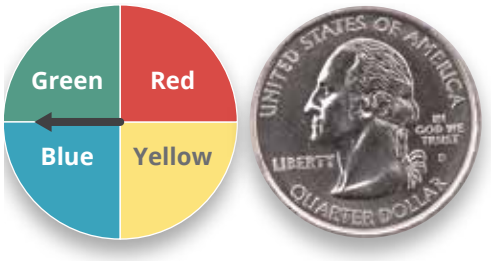
What are all the possible outcomes for combining a shirt with a pair of pants if you have three shirts (one blue, one red, and one green) and two pairs of pants (one black and one khaki)?



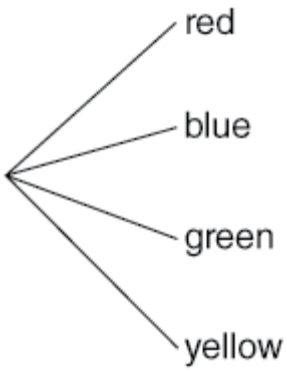
The tree diagram helps us see that there are six possible outcomes. The possible outcomes are:

- blue shirt with black pants
- blue shirt with khaki pants
- red shirt with black pants
- red shirt with khaki pants
- green shirt with black pants
- green shirt with khaki pants

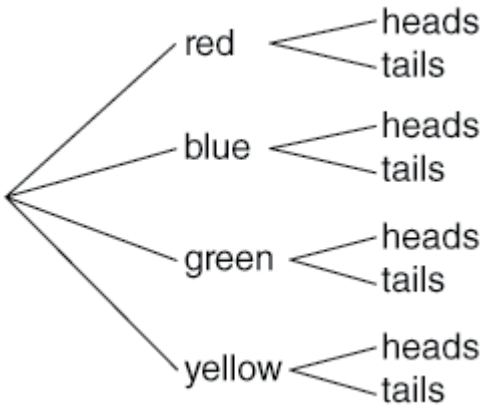
Let's take a look at another experiment. What are all the possible outcomes if you spin the following spinner and then flip the coin?



Let's make a tree diagram of the possible outcomes. The first step is to list all the possible outcomes for the spinner.



The next step is to draw two branches from each color. One branch should be labeled heads, and the other should be labeled tails. This adds eight new branches to our tree, resulting in the following diagram.

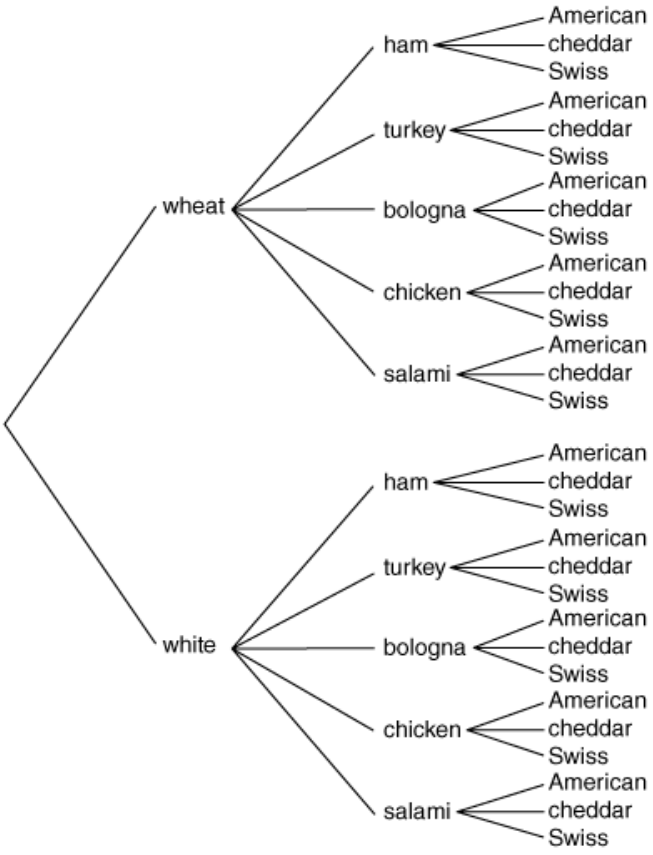


You can now see that there are eight possible outcomes. The possible outcomes are:

- red and heads
- red and tails
- blue and heads
- blue and tails
- green and heads
- green and tails
- yellow and heads
- yellow and tails

Let's take a look at a tree diagram for an experiment that has three different sets of options.

What are all the possible outcomes if you can choose from two different breads (wheat and white), five meats (ham, turkey, bologna, chicken, and salami), and three cheeses (American, cheddar, and Swiss)?



Take a second and count up the number of possible outcomes that exist when you choose one type of bread, one meat choice, and one cheese choice. There are 30 possible outcomes for these choices. A few possible outcomes are:

- wheat bread with turkey and cheddar cheese
- wheat bread with salami and Swiss cheese
- white bread with turkey and American cheese
- white bread with bologna and cheddar cheese
- white bread with chicken and Swiss cheese

Remember, this list isn't all the possible outcomes, just a few of them. If you want to know all the possible outcomes, read across each branch of the tree diagram.

The Counting Principle

As you saw in the last example, sometimes the tree diagrams can become quite large and complicated. Sometimes, you will be asked only to determine the total number of possible outcomes for a given experiment. In this case, you don't need to draw a tree diagram if you can find another way to determine the number of possible outcomes. This is where the counting principle is a useful tool. The counting principle uses multiplication to find the number of possible outcomes.

Let's look back at our last example using a tree diagram. The experiment started with two bread choices, five meat choices, and three cheese choices. Once we drew our tree diagram, we could see that there were 30 possible outcomes. But, what if we didn't care about what the outcomes were, just how many there were? We could use the counting principle to help us determine this.

Number of Breads		Number of Meats		Number of Cheeses		Total number of outcomes
2	•	5	•	3	=	30

The counting principle is easy to use—the only thing that you have to do is multiply the number of items in each category to determine the total possible number of

outcomes. Let's take a look at some other examples.

Example:

- How many combinations are possible if you have four pairs of pants, nine shirts, and five sweaters?

Solution:

- $4 \cdot 9 \cdot 5 = 180$ possible outfits
- Who would have thought that you would be able to create 180 different outfits from just four pairs of pants, nine shirts, and five sweaters! Let's go through a few more examples.

Example:

- A restaurant offers eight appetizers, 12 main courses, and five desserts. How many possible combinations exist on their menu?

Solution:

- $8 \cdot 12 \cdot 5 = 480$ different combinations

Imagine drawing a tree diagram for the last example. There would be eight branches to represent the number of appetizers. Then, from each of those eight branches would be 12 branches to represent the main dishes. Finally, five branches would be drawn from each of the previous branches to represent the desserts. In all, you would have 480 branch stems to follow to see all the different combinations!

Example:

- Your new lock allows you to pick the combination. You want to make sure that there are a number of combinations to pick from. The lock has three dials. The first dial has 10 possible numbers. The second dial has 10 possible numbers, and the third dial has five possible numbers. Determine how many different outcomes you can pick from for your combination.

Solution:

- $10 \cdot 10 \cdot 5 = 500$ different combination possibilities

Example:

- In choosing a license plate, you were offered the chance to design your own combination of letters and digits. How many different license plate combinations can be created if the license plate will have two letters and three digits, in that order, with repetition?

Solution:

- $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 676{,}000$ different combination possibilities

Example:

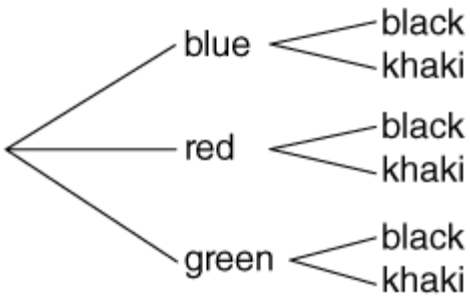
- After signing up for a new email account, you are asked to choose a secure password with three letters and two digits. How many different password combinations are you able to create with three letters and two digits, in any order, without repetition?

Solution:

- $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 = 1{,}404{,}000$ different combination possibilities

Making Predictions

Knowing the number of possible outcomes can help you when you are asked to make predictions about an experiment. Let's look at a tree diagram to see how knowing all possible outcomes can affect your predictions.



The tree diagram shows all the possible combinations of three shirts (blue, red, and green) and two pairs of pants (khaki and black). We see that there are six possible combinations.

If someone asked you to predict the color of pants that would randomly be chosen, you can see from the tree diagram that khaki pants are found in three of the combinations and black pants are found in the other three combinations. This means that there is an equal chance of khaki or black pants being chosen.

What if you were asked to predict what color of shirt would be chosen? Does one shirt color appear in the combinations more than the other two? Looking at the tree diagram, we can see that each shirt color also has an equal chance of being picked.

Not all experiments will have outcomes that all have equal chances of happening. As you learn more about probability, you will see examples where some combinations have a higher—or lesser—chance of happening.

Let's Review

Before moving on to the practice problems, make sure you understand the main points of this lesson.

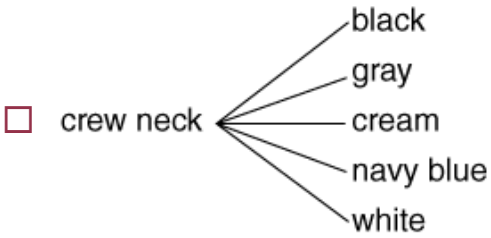
- Outcomes play a key role in any probability experiment.
- A tree diagram helps you see all the possible combinations and if any combination has a better chance of happening than the other combinations.
- The counting principle provides a shortcut for determining the number of combinations that can occur.
- To find the number of possible combinations, multiply the number of options in each category to get the total number of possible outcomes.

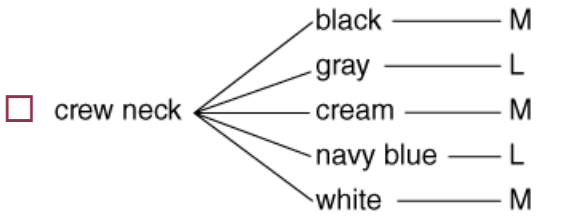
Complete the following activities.

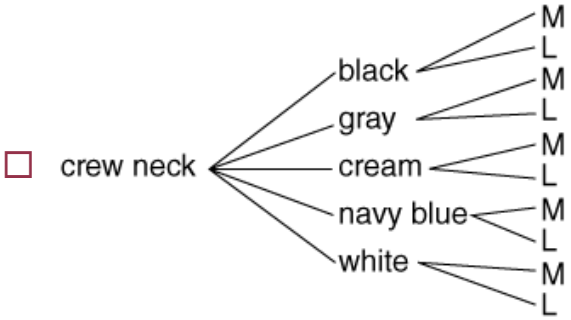
1.1 The counting principle lets you add the number of options per category to find the total number of possible outcomes.

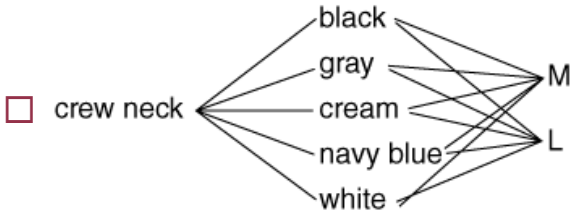
- ○ True
- ○ False

1.2 Santiago wants to buy a new sweater. He can choose the style (crew neck, v-neck, cardigan, or zip up), the color (black, gray, cream, navy blue, or white), and the size (medium or large). Which tree diagram shows all of the possible outcomes for a crew neck sweater.

☐ 

☐ 

☐ 

☐ 

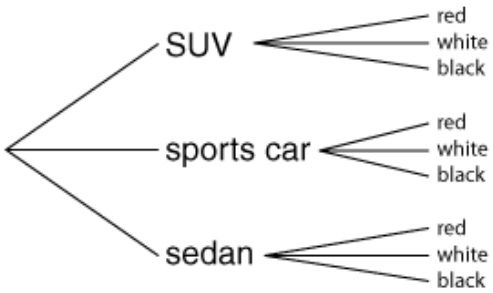
1.3 How many five-digit numbers can be created using the digits 0-9? A number can be repeated for different digits.

1.4 Many states license plates have three letters and three digits (0-9). How many different license plate combinations are available if repetition is allowed?

- ☐ 175,760
- ☐ 1,757,600
- ☐ 17,576,000
- ☐ 175,760,000

1.5 You and your friends are trying to decide on a movie. You can choose from eight movies, five show times, and three locations. How many combinations are possible?

1.6 Which problem matches the tree diagram given?



- ☐ A customer can choose from four car models offered in three colors.
- ☐ A customer can choose from three car models offered in four colors.
- ☐ A customer can choose from four car models offered in four colors.
- ☐ A customer can choose from three car models offered in three colors.

1.7 Alejandria has four shirts and three pairs of pants. She says that she has seven possible combinations.

- ○ True
- ○ False

1.8 A code consists of two letters (they can be repeated) followed by one digit (0-9). The number of possibilities is _____ .

Permutations

Any time you want to open a combination lock, you have to know the correct order of the numbers. It's not good enough to just know the three numbers. If you use them in the wrong order, you will not be able to open the lock. Because you must know the numbers in the correct order, this is a permutation.

This lesson will help you understand permutations and their use in practical applications.

Objectives

- Use permutations to count all possible outcomes.

Vocabulary

counting principle—uses multiplication to find the possible number of outcomes

factorial—the product of a natural number and all of the natural numbers less than itself

permutations—arrangement of objects in which order is important

Permutations

Permutations are arrangements of objects in which order is important, just as in the case of the combination lock. It was not enough to know the three numbers of the combination. You also had to know the correct order of the three numbers. This is a good example of a permutation.

A permutation acknowledges that 1-2-3 is different from 3-2-1, 2-1-3, 2-3-1, 3-1-2, or 1-3-2. Although the same three numbers make up all the different sets of numbers, each is different from the others because of their order, thus making them permutations.

An easy way to find the number of permutations is to use the *counting principle*. The counting principle uses multiplication to find the total number of possible outcomes. You multiply the number of objects per group to find the total number of possible outcomes. Let's look at an example of using the counting principle to help find the number of permutations.

Example:

- Kelly, Cassandra, LaTasha, and Marisol are running in a race. In how many different ways can the four girls finish the race?

Solution:

- 4 girls have the possibility of coming in first. 3 girls would be left to come in second, 2 girls could come in third and 1 girl would be fourth.
  - $4 \cdot 3 \cdot 2 \cdot 1 =$
  - 24
- Using the counting principle, there are 24 different orders in which the four girls could finish the race.

Factorials

In the previous example, we used $4 \cdot 3 \cdot 2 \cdot 1$ to help us determine the number of different ways four girls could finish a race. There is another way for us to write that expression involving the use of *factorials*. We can rewrite $4 \cdot 3 \cdot 2 \cdot 1$ as 4! Anytime you see a number followed by an exclamation point, it means to multiply all of the natural numbers up to, and including, the number. Remember, the natural numbers begin with one.

Connections! Think of 4! as a short cut for saying $4 \cdot 3 \cdot 2 \cdot 1$, similar to how you use g2g, bff, and lol as shortcuts when texting your friends on your cell phone.

Let's evaluate some factorial expressions.

Example:

- Evaluate 6!

Solution:

- 6!
- $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
- 720

Example:

- Evaluate 9!

Solution:

- 9!
- $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
- 362,880
- Let's see if the factorial method works for our original ordering of numbers for a combination lock. A combination lock uses three numbers in a particular order. Three factorial (3!) equals $3 \cdot 2 \cdot 1$, which is six. That is the same number of permutations we found previously!

Permutations using Subsets of a Larger Set

Sometimes, you may not need to use all of the numbers available in the set. If this happens, you will have to apply your knowledge of factorials to a permutation formula.

- $_nP_r = \frac{n!}{(n-r)!}$

In this formula, the *n* stands for the total number of objects. The *r* stands for the number of objects taken at a time. Let's apply this notation to an example.

Example:

- Imagine you have eight books you want to put on a shelf, but your shelf can only hold three books. How many different ways can you organize the eight books into sets of three?

Solution:

- In this example, the total number of objects is eight. So, n equals eight. We are taking combinations of three books. So, three is the value of r. Let's substitute those values into the permutation formula.

$$ {}_nP_r = \frac{n!}{(n-r)!} $$ Formula for the number of permutations.

$$ {}_8P_3 = \frac{8!}{(8-3)!} $$ Substitute in the known values.

$$ {}_8P_3 = \frac{8!}{5!} $$ Simplify the expression in the parentheses first.

$$ {}_8P_3 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} $$ Write out the factors for the factorial expressions.

$$ {}_8P_3 = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} $$ Cancel out the common factors in the fraction.

$$ {}_8P_3 = 8 \cdot 7 \cdot 6 $$ Simplify the expression on the right.

$$ {}_8P_3 = 336 $$ Complete the multiplication.

There are 336 different combinations of three books from a set of eight books.

Notice that the division of factorials leaves the factors that are one more than the value of the smaller factorial number. In the example, 8! divided by 5! left the factors 6, 7, and 8, or the factors that begin with one more than five. We can use this as a shortcut when dividing factorials, as in the next example.

Example:

- Twenty students are entered into the school geography bee. If ribbons are awarded to the top three students, how many different combinations of the top three students can there be?

Solution:

$$ {}_nP_r = \frac{n!}{(n-r)!} $$ Formula for the number of permutations.

$$ {}_{20}P_3 = \frac{20!}{(20-3)!} $$ Substitute in the known values.

$$ {}_{20}P_3 = \frac{20!}{17!} $$ Simplify the expression in the parentheses first.

$$ {}_{20}P_3 = 20 \cdot 19 \cdot 18 $$ When dividing 20! by 17!, the factors that remain begin with one more than 17.

$$ {}_{20}P_3 = 6{,}840 $$ Complete the multiplication.

- There are 6,840 possible combinations of the top three finishers out of twenty students.

Let's Review

Before moving on to the practice problems, make sure you understand the main points of this lesson.

- Permutations determine all the possible combinations when the order of the combination is important.
- Permutations use the counting principle, allowing you to multiply the number of options in each group to determine the total number of possible combinations.
- To determine the number of permutations for a whole group, you calculate the factorial for the total number of members of the group.
- When combinations are made using subsets of the total number of options, you can use the formula ${}_nP_r = \frac{n!}{(n-r)!}$

Complete the following activities.

1.9 Order is important in permutations.

- ○ True
- ○ False

1.10 The factorial 4! is equal to _____ .

1.11 Find the value for each factorial.

7! 8!

5! 6!

1.12 How many different ways can five dolls be arranged?

☐ 13 ☐ 12 ☐ 120 ☐ 150

1.13 Evaluate ${}_6P_3$.

☐ 720 ☐ 240 ☐ 540 ☐ 120

1.14 You can remember the three numbers to your combination lock, but you can't remember their order. How many different combinations of the three numbers will you have to try, at most?

1.15 How many combinations of a president, vice-president, secretary, and treasurer can be chosen from a group of 12 students?

☐ 11,880 ☐ 95,040 ☐ 665,280 ☐ 3,991,680

1.16 There are eight runners in the 100-meter dash. In how many ways can the runners finish first, second, and third?

☐ 72 ☐ 6 ☐ 120 ☐ 336

1.17 You have eleven books but only have room for six of them on your shelf. This means that you can arrange the books 39,916,800 ways.

○ True
○ False

1.18 The letters of the word STAR are written on individual cards. If you always choose three cards and then put them back before the next set of three cards, how many different permutations will you have? List all the different permutations.

Combinations

The phrase "locker combination" is actually a very misleading phrase. The phrase leads you to believe that a combination is an arrangement of numbers that must be in a specific order. This is actually not true.

This lesson will help you understand what the term combination really means and how it applies to everyday problems.

Objectives

- Use combinations to count all possible outcomes.

Vocabulary

combination—selection of objects where order is not important

element—one of the objects in a set

Combinations

Combinations are an arrangement of a selection of objects where order is not important. This means that 1-2-3 is considered to be the same as 1-3-2. They are the same because, in combinations, it is the *elements* of the set that are important, not the order. If you look at both sets of numbers, you see that they both contain 1, 2, and 3, just in different orders. This is where combinations differ from permutations.

Let's look at a few examples of combinations.

Example:

- How many combinations are there of the letters m, n, o, and p, if using all four letters?

Solution:

- m-n-o-p
- There is only one combination, since all of the letters are used, and their order doesn't matter.

When you combine all of the elements in a set, you only get one combination, because the order is not important.

Example:

- How many combinations are there of the letters m, n, o, and p using three letters at a time?

Solution:

- m-n-o
- m-n-p
- m-o-p
- n-o-p
- There are four different combinations.

When you combine subsets from a set, you get more combinations that are possible. Let's see what happens as the subset gets smaller.

Example:

- How many combinations are there of the letters m, n, o, and p using two letters at a time?

Solution:

- m-n
- m-o
- m-p
- n-o
- n-p
- o-p
- There are six combinations.

This example shows that, as the subsets get smaller from a fixed set, the number of possible combinations increases!

Combinations Formula

Sometimes you will find that there are too many combinations to easily make a list. In cases where you just need to know the total number of possible combinations, you can use the combinations formula.

- $_nC_r = \frac{n!}{r!(n-r)!}$

The *n* stands for the total number of objects to pick from. The *r* stands for the number of objects in the subset. Let's look at some examples of evaluating combinations using the combination formula.

Example:

- Evaluate $_6C_3$.

Solution:

- The formula tells us that we are picking combinations of three objects from a set of six objects.

$$_nC_r = \frac{n!}{r!(n-r)!}$$ Formula for the number of combinations.

$$_6C_3 = \frac{6!}{3!(6-3)!}$$ Substitute in the known values.

$$_6C_3 = \frac{6!}{3!3!}$$ Simplify the expression in the parentheses first.

$$_6C_3 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$$ Write out the factors for the factorial expressions.

$$_6C_3 = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$ Cancel out the common factors in the fraction.

$$_6C_3 = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}$$ Simplify the expression on the right.

$$_6C_3 = \frac{120}{6}$$ Complete the multiplications on the right.

$_6C_3 = 20$ Complete the division.

- There are 20 possible combinations of three objects taken from a group of six.

Example:

- Evaluate $_{10}C_8$.

Solution:

$$_nC_r = \frac{n!}{r!(n-r)!}$$ Formula for the number of combinations.

$$_{10}C_8 = \frac{10!}{8!(10-8)!}$$ Substitute in the known values.

$$_{10}C_8 = \frac{10!}{8!2!}$$ Simplify the expression in the parentheses first.

$$_{10}C_8 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$$ Write out the factors for the factorial expressions.

$$_{10}C_8 = \frac{10 \cdot 9}{2 \cdot 1}$$ Cancel out the common factors in the fraction.

$$_{10}C_8 = \frac{90}{2}$$ Complete the multiplications on the right.

$_{10}C_8 = 45$ Complete the division.

- There are 45 possible combinations of eight objects taken from a group of 10.

Here is a word problem that involves combinations.

Example:

- The basketball coach has 12 players on his team. How many lineups of five players can he create?

Solution:

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$ Formula for the number of combinations.

$$ {}_{12}C_5 = \frac{12!}{5!(12-5)!} $$ Substitute in the known values.

$$ {}_{12}C_5 = \frac{12!}{5!7!} $$ Simplify the expression in the parentheses first.

$$ {}_{12}C_5 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} $$ Write out the factors for the factorial expressions.

$$ {}_{12}C_5 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} $$ Cancel out the common factors in the fraction.

$$ {}_{12}C_5 = \frac{95,040}{120} $$ Complete the multiplications on the right.

$$ {}_{12}C_5 = 792 $$ Complete the division.

- The coach can make 792 combinations of five players from the group of 12 players.

Let's Review

Before moving on to the practice problems, make sure you understand the main points of this lesson.

- Combinations are similar to permutations because they are an arrangement of objects.
- Combinations are different from permutations because order does not matter in combinations.
- The set of elements of 1-2-3 is considered to be the same as 1-3-2.

Complete the following activities.

1.19 Combinations are the same thing as permutations because they are an arrangement of objects where order is important.

- ○ True
- ○ False

1.20 Evaluate ${}_6C_3$.

- ☐ 720
- ☐ 120
- ☐ 20
- ☐ 6

1.21 Which notation correctly shows seven objects taken four at a time?

- ☐ ${}_4C_7$
- ☐ ${}_7C_4$

1.22 You have a frame that holds three pictures. You pulled out your favorite five photos. How many sets of three are there?
Make a list of all the possible combinations using the numbers 1 - 5 to represent the photos.

1.23 Evaluate ${}_9C_4$

- ☐ 3,024
- ☐ 24
- ☐ 1,008
- ☐ 126

1.24 Li has to read four books from a list of 10 books. How many sets of four books can be made?

1.25 The track team has 11 distance runners but can only enter five in the meet. How many combinations of five runners can the coach create?

☐ 462 ☐ 120 ☐ 55,440 ☐ 3,696

1.26 A pizza place offers nine toppings on their pizzas. They are currently offering a special on pizzas with three toppings. How many different combinations of three toppings can be created?

☐ 504 ☐ 84 ☐ 6 ☐ 108

Mixed Review of Outcomes

Take a look at the two pictures. There are five differences in the pictures. Can you find them?

This lesson uses a similar skill to the one you just used to identify the differences in the pictures. You have to examine word problems carefully to determine if a problem is a permutation or a combination.

Can You Spot the Differences?

Hair color changed, poster added to door, roller blades removed, color of box on floor changed, back suitcase is smaller.

Objectives

- Identify if a problem involves combinations or permutations.
- Use the combination formula to determine the total possible outcomes.
- Use the permutation formula to determine the total possible outcomes.

Vocabulary

combination—selection of objects where order is not important
element—one of the objects in a set
permutations—arrangements of objects in which order is important

Combinations versus Permutations

Combinations and *permutations* are very similar in that they both deal with the arrangement of objects. But, you have to be careful when determining if you should be using a permutation or a combination, because there is a difference between them.

The order of the *elements* in the set is important in permutations but not in combinations. This means that in the set of elements *x-y-z*, *y-x-z* is a different set in a permutation. In a combination, *y-x-z* is the same as *x-y-z*, because order doesn't matter.

Permutations and combinations are easy to compute, especially when given in their formula notations. They can be a little more difficult when presented within a word problem. You will have to read the problem carefully to determine if order is important to the question asked. Let's look at a few examples to help you better understand the slight differences between permutations and combinations.

Example:

- In how many different orders can you watch three movies from a selection of six movies?

Solution:

- Because the question is asking for all the different orders in which you can watch three movies, order is important. That makes this a permutation problem. This means

whether you watch a particular movie first, second, or third will make a difference in the set.

- When picking your first movie to watch, you can choose from six different titles. When picking the second movie, you can choose from the remaining five titles. Finally, the third movie will be chosen from the four movies left. Putting this information into numbers gives you the following:
- $6 \cdot 5 \cdot 4 = 120$
- You can watch three movies from a group of six in 120 different orders.

Example:

- How many different groups of three movies can you make from six movies?

Solution:

- This question is asking about the number of different groups. This means that each group needs to have three different elements as compared to any other group. In this case, order is not important, making this a combination problem.
- This problem can be represented as ${}_6C_3$.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$ Formula for the number of combinations.

$${}_6C_3 = \frac{6!}{3!(6-3)!}$$ Substitute in the known values.

$${}_6C_3 = \frac{6!}{3!3!}$$ Simplify the expression in the parentheses first.

$${}_6C_3 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$$ Write out the factors for the factorial expressions.

$${}_6C_3 = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$ Cancel out the common factors in the fraction.

$${}_6C_3 = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}$$ Simplify the expression on the right.

$${}_6C_3 = \frac{120}{6}$$ Complete the multiplications on the right.

$${}_6C_3 = 20$$ Complete the division.

- You can create 20 different groups of three movies from a selection of six movies.

Looking back at the two examples, you can see both the similarities and the differences. You have to be sure to look closely before completing the problem. It is similar to the opening pictures; you have to look for the little details to help you determine which one it is. Try a few more examples.

Example:

- A teacher wants to select a three-person committee to represent the 26 students in the class. How many different three-person committees can the teacher choose?

Solution:

- Order is not important in choosing the committee. It doesn't matter if someone was chosen first or third. Therefore, this problem involves combinations.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$ Formula for the number of combinations.

$${}_{26}C_3 = \frac{26!}{3!(26-3)!}$$ Substitute in the known values.

$${}_{26}C_3 = \frac{26!}{3!23!}$$ Simplify the expression in the parentheses first.

$${}_{26}C_3 = \frac{26 \cdot 25 \cdot 24}{3 \cdot 2 \cdot 1}$$ 26! divided by 23! leaves the factors greater than 23.

$${}_{26}C_3 = \frac{15,600}{6}$$ Complete the multiplications on the right.

$${}_{26}C_3 = 2,600$$ Complete the division.

- The teacher can create 2,600 different committees.

Example:

- Class elections are being held for the offices of president, vice-president, and secretary. Any of the 26 students in the class can be elected. The president will be the student with the most votes, the vice-president is the student with the second most votes, and secretary has the third most votes. How many different ways can there be class officers?

Solution:

- This problem is a permutation because a student can be selected for any of the positions. Order is important.

$_nP_r = \frac{n!}{(n-r)!}$	Formula for the number of permutations.
$_{26}P_3 = \frac{26!}{(26-3)!}$	Substitute in the known values.
$_{26}P_3 = \frac{26!}{23!}$	Simplify the expression in the parentheses first.
$_{26}P_3 = 26 \cdot 25 \cdot 24$	26! divided by 23! leaves the factors greater than 23.
$_{26}P_3 = 15{,}600$	Complete the multiplication.

- There are 15,600 different ways that the class can select student officers.

Let's Review

Before moving on to the practice problems, make sure you understand the main points of this lesson.

- Combinations and permutations are similar mathematical problems, but they have their differences.
  - When calculating the number of permutations of a group, the order of the objects is important.
  - This means that *a-b-c* is different from *a-c-b*, even though they are composed of the same elements.
  - A combination of a group of objects is an arrangement where the order of the elements is not important.
  - The combination *a-b-c* is the same as *a-c-b*.

Complete the following activities.

1.27 There are 100 runners entered in a marathon. How many different groups of three runners can finish in first, second, and third?

Does this problem involve permutations or combinations?_____

1.28 There are 100 runners entered in a marathon. Trophies are awarded for first place, second place, and third place. In how many different ways could the runners receive trophies?

Does this problem involve permutations or combinations? _____

1.29 Which is greater ${}_4P_3$ or ${}_4C_3$?

☐ ${}_4P_3$ ☐ ${}_4C_3$

1.30 In how many different ways can seven children line up in one row to have their picture taken?

☐ 7 ☐ 49 ☐ 2,520 ☐ 5,040

1.31 How many different groups of three kids can you make from seven kids?

☐ 35 ☐ 210 ☐ 6 ☐ 24

1.32 If 11 runners are participating in a race, how many different ways can there be first and second place runners?

☐ 11 ☐ 39,916,800 ☐ 110 ☐ 126,400

1.33 How many different ways can five students form a line? Determine if the word problem is a permutation or a combination, then evaluate.

1.34 How many groups of four shirts can you choose from six shirts? Determine if the word problem is a permutation or a combination, then evaluate.

1.35 Which question below is a permutation?

- ☐ How many different teams of four runners can a coach choose from a group of seven runners?
- ☐ You and three friends want to line up next to one another for a picture. How many ways can the picture be taken?
- ☐ You want to choose three pairs of shorts from five to go on vacation. How many groups of three pairs of shorts can you choose?
- ☐ How many groups of five movies can be chosen from 11?

1.36 Do you use a mathematical permutation or a mathematical combination to open a combination lock?

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific area where restudy is needed for mastery.

SELF TEST 1: Outcomes

Complete the following activities (6 points, each numbered activity).

1.01 Evaluate ${}_9P_4$.

☐ 15,120 ☐ 36 ☐ 3,024 ☐ 504

1.02 Evaluate ${}_6C_2$.

☐ 30 ☐ 720 ☐ 120 ☐ 15

1.03 Louisa has five shirts (white, gray, black, green, and blue), four pairs of pants (black, khaki, navy, and jeans), and two pairs of shoes (flip flops and sneakers). How many different outfits can she create?

☐ 10 ☐ 20 ☐ 40 ☐ 60

1.04 Which is greater, ${}_7P_5$ or ${}_7C_5$?

☐ ${}_7P_5$ ☐ ${}_7C_5$

1.05 How many ways can six kids line up in a single row to have their picture taken?

1.06 How many sandwiches can be created from three types of bread, six meats, and eight toppings, if you can only use one item from each category?

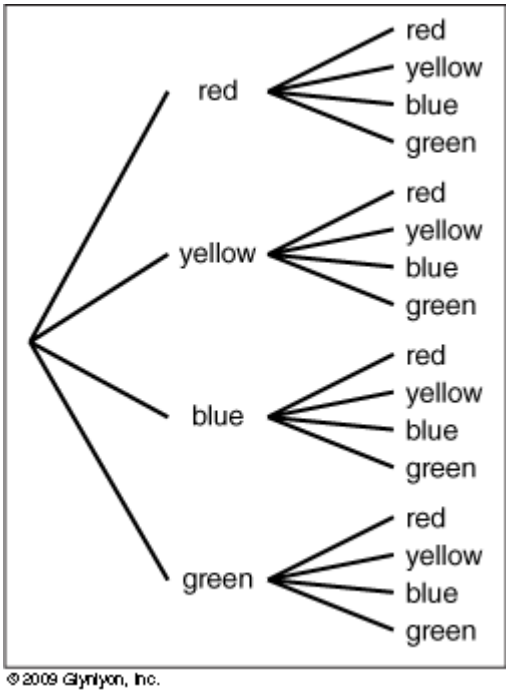
☐ 17 ☐ 144 ☐ 120 ☐ 72

1.07 How many combinations are there of the five letters *j*, *k*, *l*, *m*, and *n* using three letters at a time?

☐ 10 ☐ 120 ☐ 12 ☐ 60

1.08 Evaluate ${}_9P_2$.

1.09 How many different outcomes can happen from spinning a four-section spinner twice, according to the diagram given?



- ☐ 4
- ☐ 8
- ☐ 12
- ☐ 16

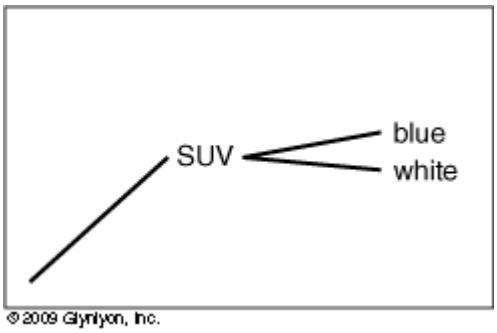
1.010 Your English teacher has given you a list of 10 possible topics for essays. You need to pick four topics from the list to write about over the school year. How many different groups of four essay topics can you choose?

1.011 Which scenario would result in 12 possible outcomes?

- ☐ You roll a number cube twice.
- ☐ You flip a coin twice.
- ☐ You roll a number cube, and then you flip a coin.
- ☐ You roll a number cube once.

1.012 Evaluate ${}_5C_4$.

1.013 A car company offers four different styles (SUV, sedan, convertible, and hatchback). Each style is offered in five colors (blue, white, black, red, and silver). Choose the diagram that shows how many different versions of an SUV can be created.

☐ 

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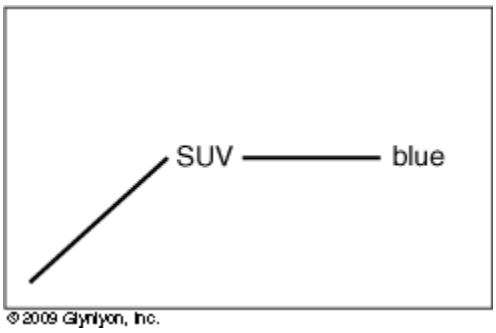
☐ 

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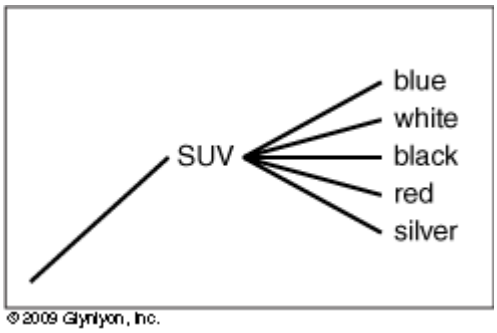
☐ 

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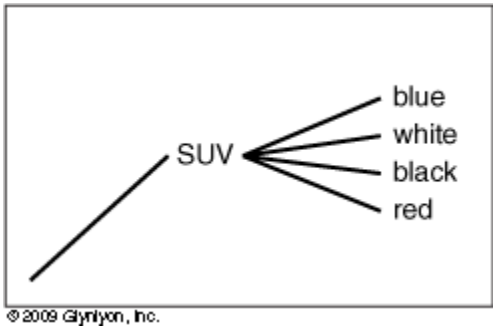
☐ 

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1.014 From a stack of seven movies, you can choose any two that you want to watch. How many different combinations can you choose from?

☐ 2 ☐ 21 ☐ 42 ☐ 5,040

1.015 Use the counting principle to determine the number of possible outfits that can be created using one item from each category.

7 shirts

6 pairs of pants

6 pairs of socks

4 hats

☐ 1,512

☐ 23

☐ 5,040

☐ 1,008

72/90 **SCORE** _____ **TEACHER** _____ _____
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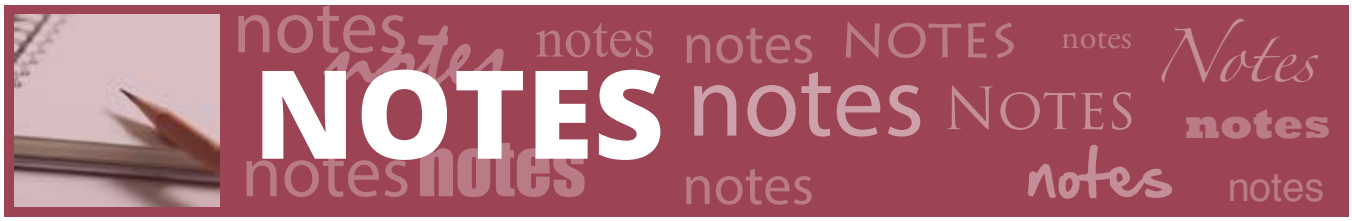
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