



# MATH

STUDENT BOOK

▶ **9th Grade | Unit 3**

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# Math 903

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**LIFEPAC Test is located in the center of the booklet.** Please remove before starting the unit.

**Author:**

James Coe, M.A..

**Editor-In-Chief:**

Richard W. Wheeler, M.A.Ed.

**Editor:**

Robin Hintze Kreutzberg, M.B.A.

**Consulting Editor:**

Robert L. Zenor, M.A., M.S.

**Revision Editor:**

Alan Christopherson, M.S.

**Westover Studios Design Team:**

Phillip Pettet, Creative Lead

Teresa Davis, DTP Lead

Nick Castro

Andi Graham

Jerry Wingo



**804 N. 2nd Ave. E.**

**Rock Rapids, IA 51246-1759**

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# Problem Analysis And Solution

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## INTRODUCTION

A typical and normal question asked by students about algebra is “What good will this subject do me?” or “How will I ever use this course?” This LIFE PAC® contains many good answers to those questions.

We study numbers and their behavior properties and we learn to solve equations all for a major purpose: to solve application problems. Many practical problems are just too complicated to figure out by arithmetic.

Certain words and phrases can easily be translated into symbols and equations. Other word problems involve careful reading and rereading before they can be translated into equations. A four-step procedure can be used for all word problems. Word problems can be problems that deal with certain numbers or they can be about geometry, money, ages, motion, levers, integers, or mixtures.

Although at times textbook problems may not seem realistic, they serve two purposes. They give students an opportunity to learn problem-solving techniques; and they allow perceptive students to see the practical potential in using algebra to solve live problems, whether architectural, engineering, medical, or other science-related problems.

## Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFE PAC. When you have finished this LIFE PAC, you should be able to:

1. Change word problems to symbols.
2. Write equations to solve word problems effectively.
3. Solve problems involving two or more numbers, problems involving geometric concepts, and problems involving money.
4. Solve age problems, motion problems, lever problems, and integer problems.
5. Solve problems with complications beyond the very easy level.

Survey the LIFE PAC. Ask yourself some questions about this study and write your questions here.

A large rectangular area with horizontal red lines, intended for writing questions. The lines are evenly spaced and cover most of the page's width and height.

# 1. WORDS AND SYMBOLS

Before we learn to solve word problems using algebra, we should look at some general procedures. We must be able to

translate certain words and phrases and we must have an identifiable procedure to solve each problem.

## OBJECTIVES

When you have completed this section, you should be able to:

1. Change word problems to symbols.
2. Write equations to solve word problems effectively.

## TRANSLATING WORDS TO SYMBOLS

Too many words are used in word problems to discuss in one lesson, but we can identify some of them. Some words you must be able to identify and use for solving word problems are these: *sum, difference, product, quotient, exceeds, by, excess, diminished by, more than, greater than, fewer than, less than, less, twice, doubled, tripled, consecutive, divided by, divided into, quotient of, increased by, decreased by, times, plus, minus, subtracted from, multiplied by, square of, square root of, cube of, is equal*

*to, is less than, is greater than, numerator, and denominator.* These words indicate which operation or operations will be used to solve the word problem. In most cases, addition, subtraction, multiplication, or division will be involved in the translation from words into symbols.

The concept used most often is the sum of two numbers. Many times, seemingly, the idea is cleverly disguised, but you should look for it when solving a word problem.

**Models:** Two angles are complementary.

Joe and Mary have 48 books on animals.

Traveling in opposite directions, Tom and Jerry leave Auburn.

**Exceeds by and excess are used to show subtraction or addition.**

**Models:** If Laura exceeds the 55 mph speed limit by  $x$ , then her speed will be  $55 + x$ .

If Don's weight,  $x$ , exceeds 175 pounds, the amount of excess could be written as  $x - 175$ .

If you can identify words that describe certain mathematical symbols, you can improve your reading of word problems.

**Model:** Frank has some money. George has \$7 more than Frank. How much money do George and Frank have together?

Let  $x$  represent Frank's money. Then  $x + 7$  means George has \$7 more than Frank.  $x + (x + 7)$  represents the amount of money George and Frank have together.



**Complete these activities.**

**1.1 List the operation words given on page 5 into the five groups.**

a. words that involve addition

_____	_____
_____	_____
_____	_____
_____	_____

b. words that involve subtraction

_____	_____
_____	_____
_____	_____
_____	_____

c. words that involve multiplication

_____	_____
_____	_____
_____	_____
_____	_____

d. words that involve division

_____	_____
_____	_____
_____	_____

e. words that are not involved with  $+$ ,  $-$ ,  $\times$ ,  $\div$

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____



Write examples of sentences using one word or a phrase from each of the five groups.

**Model:** Subtraction: George has \$19 less than Tom.

1.2 Addition: \_\_\_\_\_

1.3 Subtraction: \_\_\_\_\_

1.4 Multiplication: \_\_\_\_\_

1.5 Division: \_\_\_\_\_

1.6 Other: \_\_\_\_\_





**Write each of the word expressions in mathematical symbols. Use symbols of grouping when necessary. Do not combine the numbers or simplify the expressions.**

- 1.7 The sum of 8 and 4 \_\_\_\_\_
- 1.8 The difference between six and three \_\_\_\_\_
- 1.9 The product of 9 and 8 \_\_\_\_\_
- 1.10 Twelve divided by six \_\_\_\_\_
- 1.11 4 minus  $x$  \_\_\_\_\_
- 1.12 The square of 6 \_\_\_\_\_
- 1.13 The excess when five exceeds  $x$  \_\_\_\_\_
- 1.14  $x$  exceeded by 3 \_\_\_\_\_
- 1.15 Twice  $x$  decreased by 4 \_\_\_\_\_
- 1.16 Fourteen decreased by three times a number \_\_\_\_\_
- 1.17 Four more than the quotient of  $x$  and 3 \_\_\_\_\_
- 1.18 2 increased by the cube of  $n$  \_\_\_\_\_
- 1.19 12 diminished by 6 times a number \_\_\_\_\_
- 1.20 Three times the difference between  $t$  and  $y$  \_\_\_\_\_
- 1.21 The quotient of 12 and 5 times a number \_\_\_\_\_
- 1.22 Nine times a number diminished by 7 \_\_\_\_\_
- 1.23 The consecutive integer after the integer  $n$  \_\_\_\_\_
- 1.24 The triple of  $x$  \_\_\_\_\_
- 1.25 12 less than 3 times a number \_\_\_\_\_

- 1.26 Two times the sum of 4 and  $y$  \_\_\_\_\_
- 1.27 Jeri is three years younger than Laura, whose age is  $x$ . How old is Jeri? \_\_\_\_\_
- 1.28 In a softball diamond, the distance from first base to third base is 1.4 times the distance from first base to home plate. If  $x$  represents the distance from first base to home plate, how would you represent the distance from first base to third base? \_\_\_\_\_
- 1.29 One side of a square is  $a + 3$  inches. What is the perimeter of the square? \_\_\_\_\_
- 1.30 How many cents are in  $x$  quarters and 25 dimes? \_\_\_\_\_
- 1.31 The width of a rectangle is  $y$  feet long, and it is 4 feet longer than it is wide. What is the area of the rectangle? \_\_\_\_\_
- 1.32 A boy is 6 years older than his sister, whose age is  $x$ . What is the boy's age? \_\_\_\_\_
- 1.33 A room is 15 feet long and  $y$  feet wide. How many square yards are in the floor area?
- 1.34 What is the average of 171 and  $x$ ? \_\_\_\_\_
- 1.35 A house costs \$600 more than 12 times the lot on which it was built. If the lot cost  $x$  dollars, what did the house cost? \_\_\_\_\_
- 1.36 A man walks 10 miles in  $x$  hours. How far does he walk in one hour? \_\_\_\_\_
- 1.37 How many minutes are in  $m$  hours? \_\_\_\_\_
- 1.38 A plane flies  $x$  mph. How far can it go in  $y$  hours? \_\_\_\_\_
- 1.39 If a pipe can fill a tank in  $x$  hours, what fractional part of the tank will be filled in one hour?  
\_\_\_\_\_
- 1.40 Four years ago, Tammy's age was  $x$ . How old is she now? \_\_\_\_\_

## SOLVING VERBAL PROBLEMS

Word problems, sometimes called verbal problems, can be very frustrating. If an organized approach to their solution is used, much of the frustration will be avoided. A very simple four-step system can help you organize your problem solving:

1. Identify the number(s) that you are looking for and choose a variable for the unknown number. If you are looking for more than one number make a careful choice for your variable and define the other significant numbers of the problem in terms of that variable.
2. Write an equation (or inequality if the problem doesn't give an equality relationship). This step requires careful examination of the variables from Step 1 and their relationship to each other or other numbers in the problem. You may need to reread the problem several times.

3. Solve the equation using the techniques learned in the preceding LIFE PAC. Be sure to check your equation.
4. Identify your answer(s) and check the conditions of the problem. Answers should be labeled as age of son, age of father; first number, second number; rate of motor boat, rate of canoe; and so on.

The check you did in Step 3 has very little to do with this check. If your equation is correct, however, you may be doing the same arithmetic to check the problem. The best approach in checking your word problem is to forget Steps 1, 2, and 3 and to check the answer independently of your steps.

Many word problems can be categorized as one type or another. When reading a word problem for the first time, learn to think of it as a geometry problem, an integer problem, a coin problem, an age problem, or whatever kind of problem it is.

### VOCABULARY

**Age problem**—a word problem that involves the ages of people.

**Coin problem**—a word problem that involves nickels, dimes, quarters, or half dollars. Sometimes different denominations of bills are used also (\$1 bill, \$5 bill).

**Geometry problem**—a word problem that involves geometric figures. Usually a sketch is drawn to help with the solution.

**Integer problem**—a word problem that involves integers. The integers are the counting numbers, their opposites, and zero.

**Mixture problem**—a word problem that involves mixing of ingredients, sometimes in liquid form.

In the problems that follow, pay strict attention to the suggested approach and the models that are given for each type of problem. Remember the four steps:

1. Identify each unknown number; choose a variable expression for each number.
2. Write an equation or inequality.
3. Solve the equation or inequality.
4. Identify and label answer(s), and check the conditions stated in the problem.

**Read these word problems carefully.** They will be the basis for the next set of activities.

**Problem 1.** A chess team won 4 times as many matches as it lost. If 16 matches were won, how many games did the team lose?

**Problem 2.** The length of a rectangle is 12 inches more than its width. What is the width of the rectangle if the perimeter is 42 inches? (Remember that the perimeter of a rectangle equals length + length + width + width.)

**Problem 3.** A man traveled a certain number of miles by automobile and then nine times as far by airplane. His total trip was 600 miles in length. How far did he travel by plane?

**Problem 4.** If Sarah is 24 years younger than her mother and if the sum of their ages is 68, how old is Sarah?

**Problem 5.** Jane is 8 years older than Tom. Five years ago Jane was 3 times as old as Tom. How old is Tom now?

**Problem 6.** Mark had 3 times as many quarters as nickels. He had \$1.60 in all. How many nickels and how many quarters did Mark have?

**Problem 7.** How many quarts of water must be added to 4 quarts of a 20% water mixture to obtain a 50% mixture?

**Problem 8.** Joe can wash and wax his car in 4 hours. Susan can wash and wax her car in 3 hours. If they worked together to wash their sick friend's car how long would it take them?

**Problem 9.** The tens' digit of a number is twice the ones' digit. The sum of the digits in the number is 12. What is the number?

**Problem 10.** A triangle has a perimeter of 48. If 2 sides are equal and the third side is 6 less than the equal sides, what is its length?



The first step in solving a word problem is to identify the variable. Let  $x$  be a variable used in each of the ten problems. Tell what the variable expressions represent in the problems.

**Model:**

Problem 1  $x$  represents the number of games the chess team lost  
 $4x$  represents the number of games the chess team won

**1.41** Problem 2  $x$  represents \_\_\_\_\_  
 $x + 12$  represents \_\_\_\_\_

**1.42** Problem 3  $x$  represents \_\_\_\_\_  
 $9x$  represents \_\_\_\_\_

**1.43** Problem 4  $x$  represents \_\_\_\_\_  
 $x - 24$  represents \_\_\_\_\_

**Model:**

Problem 5  $x$  represents Tom's age  
 $x + 8$  represents Jane's age

**1.44** Problem 6  $x$  represents \_\_\_\_\_  
 $3x$  represents \_\_\_\_\_

**1.45** Problem 7  $x$  represents \_\_\_\_\_  
 $x + 4$  represents \_\_\_\_\_

**Model:**

Problem 8  $t$  represents the time it takes to wash and wax a car together  
 1 car/ 4 hours represents Joe's rate  
 1 car/ 3 hours represents Susan's rate

**1.46** Problem 9  $x$  represents \_\_\_\_\_  
 $2x$  represents \_\_\_\_\_

**1.47** Problem 10  $x$  represents \_\_\_\_\_  
 $x - 6$  represents \_\_\_\_\_



The second solution step is to write an equation for the problem. Write an equation for each of the word problems.

**Model:**

Problem 1  $4x = 16$

**1.48** Problem 2 \_\_\_\_\_

**1.49** Problem 3 \_\_\_\_\_

**1.50** Problem 4 \_\_\_\_\_

**1.51** Problem 5 \_\_\_\_\_

**1.52** Problem 6 \_\_\_\_\_

**1.53** Problem 7 \_\_\_\_\_

**Model:**

Problem 8  $1/4(t) + 1/3(t) = 1$  car (rate x time = work)

**1.54** Problem 9 \_\_\_\_\_

**1.55** Problem 10 \_\_\_\_\_

**Write the problem numbers beside the correct description.**

**1.56** geometry problem(s) \_\_\_\_\_

**1.57** age problem(s) \_\_\_\_\_

**1.58** digit problem(s) \_\_\_\_\_

**1.59** work problem(s) \_\_\_\_\_

**1.60** coin problem(s) \_\_\_\_\_

**1.61** mixture problem(s) \_\_\_\_\_



**Write each of these statements as an equation. Choose your own variable if one is not given.**

- 1.62** Three pounds of butter at  $\$n$  per pound cost  $\$3.85$ . \_\_\_\_\_
- 1.63** One-half of a certain number is 95. \_\_\_\_\_
- 1.64** The perimeter of a rectangle is 68 in. The perimeter equals twice the length of  $l$  in., plus twice the width of 9 in. \_\_\_\_\_
- 1.65** Mary Lou has 2 more nickels than pennies, and she has 30 coins all together. \_\_\_\_\_
- 1.66** Jim weighs 30 lb. less than Tom, and together they weigh 210 lb. \_\_\_\_\_
- 1.67** A class of 19 pupils has 5 more girls than boys. \_\_\_\_\_
- 1.68** One side of a triangle is  $x$  in., another side is  $2x$  in., a third side is  $2x$  in., and the perimeter is 36 in. \_\_\_\_\_
- 1.69** John has three times as many marbles as Bill, and together they have 100 marbles. \_\_\_\_\_
- 1.70** Mr. Randall is three times as old as his son John. Nine years ago he was nine times as old as John. \_\_\_\_\_



**Review the material in this section in preparation for the Self Test.** The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific area where restudy is needed for mastery.

# SELF TEST 1

Choose the correct letter to identify the right multiple choice answer (each answer, 2 points).

- 1.01** John's weight exceeds 100 lbs. by  $x$ . John's weight would be represented by \_\_\_\_\_ .  
 a.  $100 + x$       b.  $x - 100$       c.  $100 - x$       d. none of these
- 1.02** The sum of two numbers is  $x$ . If one of the numbers is 12, what is the other?  
 \_\_\_\_\_  
 a.  $12 - x$       b.  $x - 12$       c.  $12 + x$       d.  $12 \cdot x$
- 1.03** The difference of two numbers is 6. The smaller number is  $x$ . Represent the other number. \_\_\_\_\_  
 a.  $6 - x$       b.  $x \div 6$       c.  $6 - (x - 6)$       d.  $6 + x$
- 1.04** A certain number is decreased by 28. Represent the second number if  $x$  is the original number. \_\_\_\_\_  
 a.  $28 - x$       b.  $x - 28$       c.  $x + 28$       d. none of these
- 1.05** Twice a certain number is tripled. The resulting number is \_\_\_\_\_ .  
 a.  $2x + 3$       b.  $2x - 3$       c.  $(2x)3$       d.  $\frac{1}{3}(2x)$
- 1.06** After John worked at a job for 10 years, his salary doubled. What is his salary after 10 years if he started at  $x$ ? \_\_\_\_\_  
 a.  $\frac{1}{2}x$       b.  $x + 2$       c.  $x - 2$       d.  $2x$
- 1.07** 12 is divided by  $x$ . Which does not represent the quotient? \_\_\_\_\_  
 a.  $\frac{12}{x}$       b.  $12 \div x$       c.  $x \div 12$       d.  $x$  divided into 12
- 1.08** The quotient of a number  $x + 4$  and 12 is \_\_\_\_\_ .  
 a.  $\frac{x}{12} + 4$       b.  $x + \frac{1}{3}$       c.  $\frac{x+4}{12}$       d.  $\frac{x}{4} + 12$
- 1.09** The square root of 144 is best described as \_\_\_\_\_ .  
 a.  $144^2$       b.  $\sqrt{144}$       c.  $16 \cdot 9$       d.  $12 \cdot 12$



**1.010** If the numerator of the fraction  $\frac{x}{6}$  is increased by 10, the result will be

- \_\_\_\_\_.
- a.  $\frac{x}{16}$       b.  $\frac{x-10}{16}$       c.  $x + \frac{5}{3}$       d.  $\frac{x+10}{6}$

**Write each word expression in mathematical symbols** (each answer, 3 points).

- 1.011** The difference of 6 and  $x$  \_\_\_\_\_
- 1.012** The quotient of  $y$  and 6 increased by 4 \_\_\_\_\_
- 1.013** One side of a square is  $3x + 2$ . What is the perimeter? \_\_\_\_\_
- 1.014** How many hours are in  $x$  days? \_\_\_\_\_
- 1.015** How old will Bill be in 10 years if he is  $x + 2$  years old now? \_\_\_\_\_
- 1.016** 17 decreased by the square of  $x$  \_\_\_\_\_
- 1.017** If Susie is 14, what was her age  $x$  years ago? \_\_\_\_\_
- 1.018** 4 more than  $x$  tripled \_\_\_\_\_
- 1.019** The value in cents of 4 quarters and  $x$  dimes \_\_\_\_\_
- 1.020** Joe can paint  $x$  houses in a five-day work week.  
How many can he paint in one day? \_\_\_\_\_

**Complete these items** (each answer, 3 points).

- 1.021** Name the four steps for solving a word problem.
1. \_\_\_\_\_
  2. \_\_\_\_\_
  3. \_\_\_\_\_
  4. \_\_\_\_\_
- 1.022** Dwayne’s garden is triangle-shaped with two equal sides and a third side that is 4 ft. more than the length of an equal side. If the perimeter is 49 ft., how long is each side? This problem is \_\_\_\_\_.
- a. an age problem      c. a geometry problem  
b. a distance problem      d. a coin problem

<div style="border: 1px solid black; padding: 5px; display: inline-block;">                 52  <hr style="width: 50%; margin: 0;"/>                 65             </div>	<b>SCORE</b> _____	<b>TEACHER</b> _____ <small>initials                      date</small>
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804 N. 2nd Ave. E.  
Rock Rapids, IA 51246-1759

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