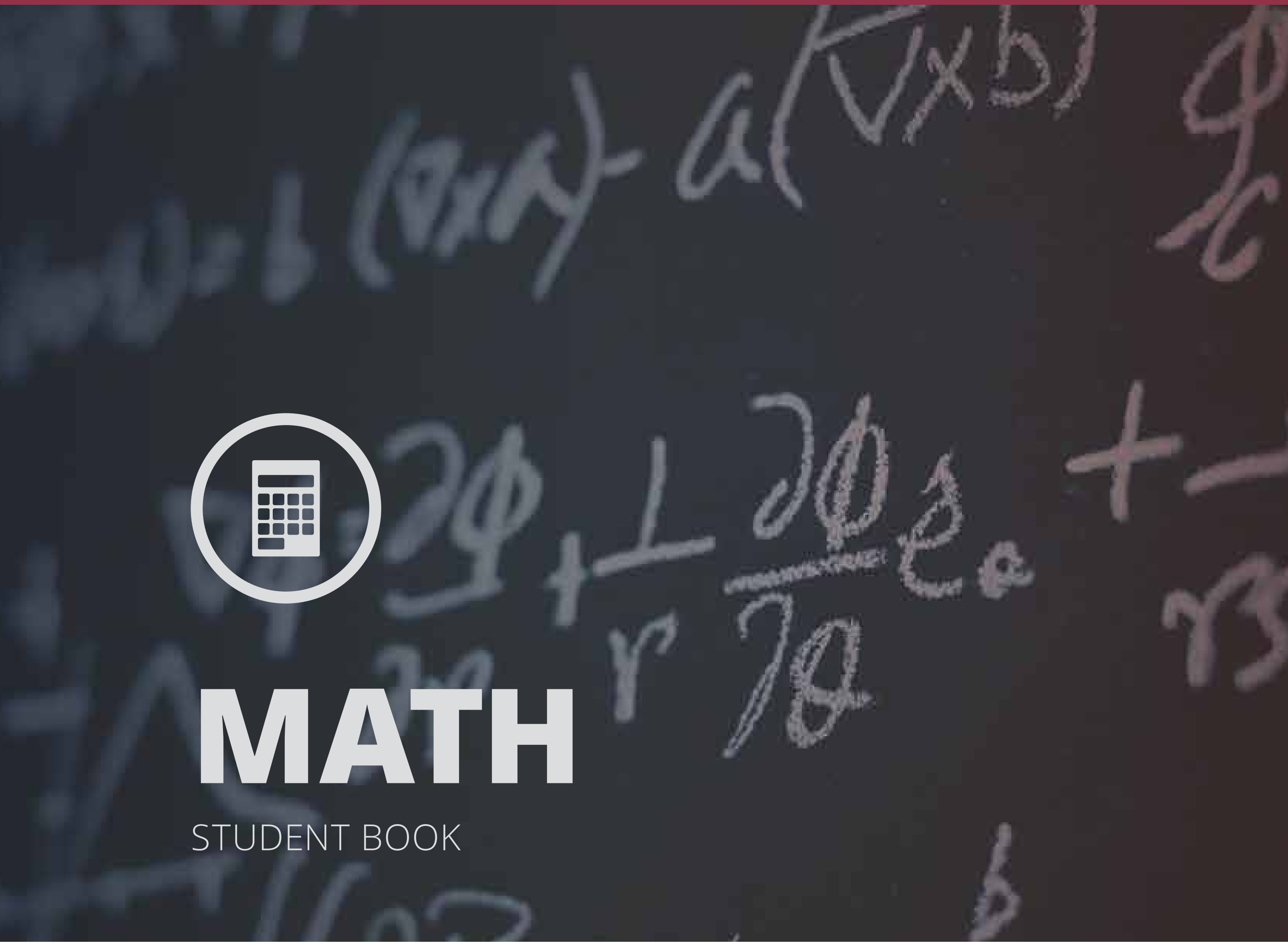
LIFEPAC®



MATH

STUDENT BOOK

9th Grade | Unit 6

Alpha Omega
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Math 906

Algebraic Fractions

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Algebraic Fractions

INTRODUCTION

In this LIFEPAC® you will continue your study in algebra. You will apply what you have learned so far to fractions having polynomial numerators or denominators or both. The factoring techniques that you learned in Mathematics LIFEPAC 905 will be used when performing the basic operations with these fractions. Then you will solve open sentences containing fractions by methods that are quite similar to those you have already used. Finally you will have another opportunity to solve verbal problems, this time in applications that involve fractions.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Determine the excluded value(s) for a fraction.
2. Reduce a fraction to lowest terms.
3. Find sums and differences of fractions.
4. Find products and quotients of fractions.
5. Simplify complex fractions.
6. Solve equations containing fractions.
7. Solve inequalities containing fractions.
8. Change the subject of a formula containing fractions.
9. Solve problems requiring the use of fractions.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. OPERATIONS

As you work through this first section, keep in mind that the basic concepts of reducing, adding, subtracting, multiplying, dividing, and simplifying the fractions of algebra are the same as those used for the fractions of arithmetic.

We will begin by defining algebraic fractions, since we must know what they are in order to be able to work with them.

OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Determine the excluded value(s) for a fraction.
2. Reduce a fraction to lowest terms.
3. Find sums and differences of fractions.
4. Find products and quotients of fractions.
5. Simplify complex fractions.

VOCABULARY

Algebraic fraction—an indicated quotient of two polynomials written in the form $\frac{A}{B}$. *A* is the numerator of the algebraic fraction and *B* is the denominator.

Terms—the numerator and denominator of a fraction.

Models: $\frac{2}{x+3}$ $\frac{-y^2-3y+1}{5-2y}$ $\frac{a+b+c}{m-n}$ $\frac{k-3}{7}$

Algebraic fractions can be reduced, using similar methods as for reducing arithmetic fractions. Addition, subtraction, multiplication, division, and simplification are also possible with algebraic fractions.

REDUCING FRACTIONS

Algebraic fractions can be reduced by finding the lowest terms. First, however, we need to discuss the circumstances under which algebraic fractions may not even exist!

EXCLUDED VALUES

Since a fraction indicates division ($\frac{A}{B} = A \div B$) and since division by zero is undefined, the denominator of a fraction must be nonzero ($B \neq 0$). If a denominator contains any variables, then a value that would result in zero for that denominator must be *excluded* for the fraction to exist.

In the preceding models, the denominators are $x + 3$, $5 - 2y$, $m - n$, and 7, respectively. The excluded values are $x = -3$ for the first model ($-3 + 3 = 0$), $y = 2.5$ for the second ($5 - 2 \cdot 2.5 = 0$), and $m = n$ for the third ($m - m$ or $n - n = 0$); since the denominator of the fourth fraction is the constant 7 and $7 \neq 0$, that fraction has no excluded values.

In determining the excluded values for the fraction $\frac{x - 3}{x^2 - 4}$, you may be able to see immediately that $2^2 - 4 = 0$; thus, $x = 2$ is an excluded value. However, $(-2)^2 - 4 = 0$ is also true; thus, $x = -2$ is an excluded value as well.

In Mathematics LIFEPAC 905 you learned to factor, and now factoring can be used to find both these excluded values. Since the denominator $x^2 - 4$ is a difference of two squares, it has factors $(x + 2)(x - 2)$. The first factor, $x + 2$, would become zero if $x = -2$; likewise, the second factor, $x - 2$, would become zero if $x = 2$. The excluded values are then $x = 2$ and $x = -2$. In this method we have made use of an important property in mathematics.

Property

If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (or both); if a product of factors is zero, then at least one of the factors must be zero.

Model 1: Find the excluded value(s) for the fraction $\frac{a + 5}{a(b + 3)(c - 2)}$.

Solution: The denominator is already factored, so each of the three factors is set equal to zero.

$a = 0$ $\qquad b + 3 = 0$ $\qquad c - 2 = 0$

$\qquad\qquad b = -3$ $\qquad c = 2$

$\therefore$ The excluded values are $a = 0$, $b = -3$, and $c = 2$.

Model 2: Find the excluded value(s) for the fraction $\frac{7}{d^2 - 5d - 24}$.

Solution: The factors of $d^2 - 5d - 24$ are $(d - 8)(d + 3)$.

$d - 8 = 0$ $\qquad d + 3 = 0$

$d = 8$ $\qquad d = -3$

$\therefore$ The excluded values are $d = 8$ and $d = -3$.

Check:

If $d = 8$, $d^2 - 5d - 24$
$= (8)^2 - 5(8) - 24$
$= 64 - 40 - 24 = 0.$

If $d = -3$, $d^2 - 5d - 24$
$= (-3)^2 - 5(-3) - 24$
$= 9 + 15 - 24 = 0.$

Write the excluded value(s) for each fraction, or none if that is the case.

1.1 $\frac{a}{b - 2}$ ____________

1.2 $\frac{4x + 3}{x}$ ____________

1.3 $\frac{y^2 - y + 5}{y + 4}$ ____________

1.4 $\frac{3}{5n}$ ____________

1.5 $\frac{-2x}{17}$ ____________

1.6 $\frac{a}{3 - 2a}$ ____________

1.7 $\frac{x + 3}{y(z + 5)}$ ____________

1.8 $\frac{k^2 + 5k + 1}{k^2 - 9}$ ____________

1.9 $\frac{7b^3}{b^2 - 7b + 10}$ ____________

1.10 $\frac{x + 11}{3x^2 + 5x - 2}$ ____________

As you work through this LIFEPAC, you are to assume that all fractions do exist; that is, any value(s) that would make a denominator zero are understood to be excluded. However, from time to time (as in the preceding activities), you will be asked to identify these excluded values.

LOWEST TERMS

Now you are ready to begin working with these algebraic fractions. A basic property of fractions will be used in much of this work.

Property

$\frac{A}{B} = \frac{AC}{BC}$ (or $\frac{AC}{BC} = \frac{A}{B}$) for $C \neq 0$; if the numerator and the denominator of a fraction are both multiplied (or divided) by the same nonzero value, then an **equivalent** fraction is obtained.

In arithmetic you learned that the fraction $\frac{1}{2}$ has the same value as the fraction $\frac{5}{10}$, since both the numerator and the denominator of $\frac{1}{2}$ are multiplied by 5. Similarly, the fraction $\frac{12}{18}$ is equivalent to the fraction $\frac{2}{3}$ since both the numerator and the denominator of $\frac{12}{18}$ are divided by 6; this latter procedure is known as *reducing*. An algebraic fraction is reduced to lowest terms when the greatest common factor of its numerator and denominator is 1.

Model 1: Reduce $\frac{24m^2n}{21mp^2}$ to lowest terms.

Solution: The GCF of $24m^2n$ and $21mp^2$ is $3m$. Divide both the numerator and the denominator by $3m$.

$$\frac{24m^2n \div 3m}{21mp^2 \div 3m} = \frac{8mn}{7p^2},$$

the equivalent reduced fraction since the GCF of $8mn$ and $7p^2$ is 1.

Model 2: Reduce $\frac{4y - 20}{12y}$ to lowest terms.

Solution: $4y - 20$ factors into $4(y - 5)$, and $12y$ is $4 \cdot 3y$. Divide both the numerator and the denominator by the common factor 4.

$$\frac{4y - 20}{12y} = \frac{4(y - 5)}{12y}$$
$$= \frac{4(y - 5) \div 4}{12y \div 4}$$
$$= \frac{y - 5}{3y},$$

the equivalent reduced fraction since the GCF of y - 5 and $3y$ is 1.

NOTE: The y's cannot be reduced since y is a term (not a factor) of the numerator $y - 5$. Only common factors can be reduced!

Model 3: Reduce $\frac{r^2 - 3r + 2}{r^2 - 1}$ to lowest terms.

Solution: Since r^2 is a term (not a factor) of both the numerator and denominator, to try to reduce this fraction by dividing by r^2 would be wrong, even though very tempting. You must avoid this type of mistake that so many beginning students make.

Factor the trinomial numerator and the binomial denominator; then divide by the common factor. (This reducing is often shown by drawing lines through these factors.)

$$\frac{r^2 - 3r + 2}{r^2 - 1} = \frac{(r - 2)(r - 1)}{(r + 1)(r - 1)}$$
$$= \frac{(r - 2)\cancel{(r - 1)}}{(r + 1)\cancel{(r - 1)}}$$
$$= \frac{r - 2}{r + 1}$$

Model 4: Reduce $\frac{6m + 6n}{9n + 9m}$ to lowest terms.

Solution:
$$\frac{6m + 6n}{9n + 9m} = \frac{6(m + n)}{9(n + m)}$$
$$= \frac{2 \cdot \cancel{3}\cancel{(m + n)}}{3 \cdot \cancel{3}\cancel{(n + m)}}$$
$$= \frac{2}{3}$$

In Model 4, the binomials $m + n$ and $n + m$ are equal and reduce as part of the GCF $3(m + n)$. If, however, the binomials had been $m - n$ and $n - m$, they would not have reduced in quite the same way since they are opposites. $\frac{A}{-A} = -1$; if two expressions are opposites, they divide (or reduce) to negative one.

Model 5: Reduce $\frac{6m - 6n}{9n - 9m}$ to lowest terms.

Solution: $\frac{6m - 6n}{9n - 9m} = \frac{6(m - n)}{9(n - m)}$

$$= \frac{\overset{2}{\cancel{6}}\overset{(-1)}{\cancel{(m - n)}}}{\underset{3}{\cancel{9}}\cancel{(n - m)}}$$

$$= -\frac{2}{3}$$

Note: The (-1) is included in the answer as a minus sign before the fraction.

Model 6: Reduce $\frac{16 - a^2}{a^2 + 20 - 9a}$ to lowest terms.

Solution: $\frac{16 - a^2}{a^2 + 20 - 9a} = \frac{16 - a^2}{a^2 - 9a + 20}$

$$= \frac{(4 + a)\overset{(-1)}{\cancel{(4 - a)}}}{(a - 5)\cancel{(a - 4)}}$$

$$= -\frac{4 + a}{a - 5}$$

Model 7: Reduce $\frac{a + 3b + c}{a^2 - 9b^2}$ to lowest terms.

Solution: $\frac{a + 3b + c}{a^2 - 9b^2} = \frac{a + 3b + c}{(a + 3b)(a - 3b)}$,

but nothing can be reduced since $a + 3b$ is not a factor of the numerator.

$\therefore \frac{a + 3b + c}{a^2 - 9b^2}$ is in lowest terms.

Reduce each fraction to lowest terms.

1.11 $\frac{75a^2b}{25ab^2}$ ____________

1.15 $\frac{38x^2yz^2}{-19xy^2z^3}$ ____________

1.12 $\frac{12m^4}{28m^3}$ ____________

1.16 $\frac{6x + 2}{8}$ ____________

1.13 $\frac{-5jk}{35j^2k^2}$ ____________

1.17 $\frac{x^3 - x^2}{x^4}$ ____________

1.14 $\frac{84y^3}{36y^4}$ ____________

1.18 $\frac{27a}{ab + ac}$ ____________

1.19 $\frac{y + 5}{2y + 10}$ ________________

1.20 $\frac{n + 2}{n^2 - 4}$ ________________

1.21 $\frac{5r - 5s}{5r + 5s}$ ________________

1.22 $\frac{8a + 8b}{12c + 12d}$ ________________

1.23 $\frac{x^2 - y^2}{8x - 8y}$ ________________

1.24 $\frac{a + 5}{a^2 - 25}$ ________________

1.25 $\frac{7 - y}{y - 7}$ ________________

1.26 $\frac{x^2 - 4x - 12}{36 - x^2}$ ________________

1.27 $\frac{m^2}{m^2 - n^2}$ ________________

1.28 $\frac{-5k + 15}{k^2 - 9}$ ________________

1.29 $\frac{(x - 1)^2}{1 - x^2}$

____

1.30 $\frac{12 - 3z^2}{2z^2 - 3z - 2}$

____

ADDING AND SUBTRACTING FRACTIONS

The procedure used for finding sums and differences of algebraic fractions is like that used in arithmetic. First, each fraction is expressed with the same denominator; next, the numerators are combined into a single numerator and written over that denominator; and finally, the resulting fraction is reduced if possible.

COMMON DENOMINATORS

To start with, consider fractions that already have a common denominator. Study the following models carefully before doing the activities.

Model 1: $\frac{x}{7} - \frac{3x}{7} + \frac{8x}{7}$

$= \frac{x - 3x + 8x}{7}$

$= \frac{6x}{7}$

Model 2: $\frac{7}{a + 2} + \frac{1}{a + 2}$

$= \frac{7 + 1}{a + 2}$

$= \frac{8}{a + 2}$

Model 3: $\frac{4m}{9n^2} - \frac{7m}{9n^2}$

$= \frac{4m - 7m}{9n^2}$

$= \frac{\overset{1}{\cancel{-3}}m}{\underset{3}{\cancel{9}}n^2}$

$= \frac{-m}{3n^2}$ or $-\frac{m}{3n^2}$

Model 4:

$$\frac{y+3}{2y} + \frac{3y-1}{2y} - \frac{4y+7}{2y}$$

$$= \frac{(y+3) + (3y-1) - (4y+7)}{2y}$$

$$= \frac{y + 3 + 3y - 1 - 4y - 7}{2y}$$

$$= \frac{-5}{2y} \text{ or } -\frac{5}{2y}$$

Model 5:

$$\frac{4k}{3-2k} - \frac{6}{3-2k}$$

$$= \frac{4k-6}{3-2k}$$

$$= \frac{2\overset{(-1)}{\cancel{(2k-3)}}}{\underset{1}{\cancel{3-2k}}}$$

$$= -2$$

Find the indicated sums and differences.

1.31 $\frac{y}{3} + \frac{5y}{3} - \frac{4y}{3}$ ______________

1.32 $\frac{8}{a} - \frac{6}{a} + \frac{7}{a}$ ______________

1.33 $\frac{6}{y-z} - \frac{2}{y-z}$ ______________

1.34 $\frac{m}{m+n} + \frac{n}{m+n}$ ______________

1.35 $\frac{7x^2}{12w} - \frac{3x^2}{12w}$ ______________

1.36 $\frac{r^2}{r-s} - \frac{s^2}{r-s}$ ______________

1.37 $\frac{x+2}{10y} + \frac{3x-1}{10y} - \frac{2x+5}{10y}$

____

1.38 $\frac{2k}{4k^2-9} + \frac{3}{4k^2-9}$

____

1.39 $\frac{x-y}{x+y} - \frac{y-x}{x+y}$

____

1.40 $\frac{11}{b-7} - \frac{2b-3}{b-7}$

____

LEAST COMMON DENOMINATORS

To combine fractions having different denominators, you must first express them with a common denominator, preferably with the smallest such denominator.

VOCABULARY

Least common denominator (LCD)—for a group of fractions, the smallest value that has all the denominators as factors.

Model 1: Find the LCD for $\frac{3}{14x^3} + \frac{7}{12x^2}$.

Solution: $14x^3 = 2^1 \cdot 7^1 \cdot x^3$ and
$12x^2 = 2^2 \cdot 3^1 \cdot x^2$.

The LCD will be the product of every factor that appears, including the largest exponent of each factor.

$\therefore$ The LCD is $2^2 \cdot 3^1 \cdot 7^1 \cdot x^3$ or $84x^3$.

Model 2: Find the LCD for $\frac{n}{n+4} - \frac{5}{n-2}$.

Solution: $n + 4$ and $n - 2$ are both prime, and the LCD will be their product.

$\therefore$ The LCD is $(n + 4)(n - 2)$ or $n^2 + 2n - 8$.

Model 3: Find the LCD for $\frac{y+5}{6y^2-6} + \frac{7}{8y+8}$.

Solution:

$$6y^2 - 6 = 6(y^2 - 1) = 2 \cdot 3(y + 1)(y - 1)$$

$$8y + 8 = 8(y + 1) = 2^3(y + 1)$$

$\therefore$ The LCD is $2^3 \cdot 3^1(y + 1)(y - 1)$ or $24(y + 1)(y - 1)$.

To express each fraction as an equivalent one having the desired LCD, we will again use the basic property of fractions: $\frac{A}{B} = \frac{AC}{BC}$ for $C \neq 0$. The value of the multiplier C is determined from the factor or factors needed by a denominator to become the LCD. Once this value has been found, both the numerator and the denominator are multiplied by it.

Model 4: Combine $\frac{3}{14x^3} + \frac{7}{12x^2}$. (See Model 1)

Solution: The LCD is $84x^3$. The multiplier must be chosen to make each denominator equal $84x^3$.

$$= \frac{3}{14x^3}\left[\frac{6}{6}\right] + \frac{7}{12x^2}\left[\frac{7x}{7x}\right]$$

$$= \frac{18}{84x^3} + \frac{49x}{84x^3}$$

$$= \frac{18 + 49x}{84x^3},$$

which does not reduce since $18 + 49x$ is prime.

Model 5: Combine $\frac{n}{n+4} - \frac{5}{n-2}$.

Solution: The LCD is $(n + 4)(n - 2)$
or $n^2 + 2n - 8$.

$$= \frac{n}{n+4}\left[\frac{n-2}{n-2}\right] - \frac{5}{n-2}\left[\frac{n+4}{n+4}\right]$$

$$= \frac{n^2 - 2n}{(n+4)(n-2)} - \frac{5n + 20}{(n-2)(n+4)}$$

$$= \frac{(n^2 - 2n) - (5n + 20)}{(n+4)(n-2)}$$

$$= \frac{n^2 - 2n - 5n - 20}{(n+4)(n-2)}$$

$$= \frac{n^2 - 7n - 20}{(n+4)(n-2)}$$, which does not reduce.

NOTE: The answer is usually left as shown, but it may also be written as $\frac{n^2 - 7n - 20}{n^2 + 2n - 8}$.

Model 6: Combine $\frac{y+5}{6y^2 - 6} + \frac{7}{8y + 8}$. (See Model 3)

Solution: The LCD is $24(y + 1)(y - 1)$.

$$\frac{y+5}{6(y+1)(y-1)}\left[\frac{4}{4}\right] + \frac{7}{8(y+1)}\left[\frac{3(y-1)}{3(y-1)}\right]$$

$$= \frac{4y + 20}{24(y+1)(y-1)} + \frac{21y - 21}{24(y+1)(y-1)}$$

$$= \frac{4y + 20 + 21y - 21}{24(y+1)(y-1)}$$

$$= \frac{25y - 1}{24(y+1)(y-1)}$$, the answer.

We may summarize the procedure for adding or subtracting algebraic fractions in these five steps:

1. Determine the LCD from the prime factorization of the denominator of each fraction;
2. Change each fraction to an equivalent one having the LCD for its denominator;
3. Find all numerator products;
4. Write a single fraction made up of the combined numerators over the LCD; and
5. Reduce, if possible.

Model 7: Combine $\frac{x - 4z}{12} + \frac{3x - 4z}{4} - \frac{2x - y + z}{3}$.

Solution: $12 = 2^2 \cdot 3^1$, $4 = 2^2$, and $3 = 3^1$; thus, the LCD is $2^2 \cdot 3^1$ or 12.

$$\frac{x - 4z}{12}\left[\frac{1}{1}\right] + \frac{3x - 4z}{4}\left[\frac{3}{3}\right] - \frac{2x - y + z}{3}\left[\frac{4}{4}\right]$$

$$= \frac{x - 4z}{12} + \frac{9x - 12z}{12} - \frac{8x - 4y + 4z}{12}$$

$$= \frac{(x - 4z) + (9x - 12z) - (8x - 4y + 4z)}{12}$$

$$= \frac{x - 4z + 9x - 12z - 8x + 4y - 4z}{12}$$

$$= \frac{2x + 4y - 20z}{12}, \text{ which reduces to}$$

$$= \frac{\overset{1}{\cancel{2}}(x + 2y - 10z)}{\cancel{12}_6}$$

$$= \frac{x + 2y - 10z}{6}, \text{ the answer.}$$

Some special types of denominators may simplify your work.

Model 8: Combine $5 + \frac{2}{a - 3} - \frac{3}{3 - a}$.

Solution: The denominator of 5 is understood to be 1 and has no effect on the LCD. The two denominators $a - 3$ and $3 - a$ are just opposites; if either is multiplied by -1, the other results. For example, $-1(3 - a) = -3 + a = a - 3$.

$\therefore$ The LCD is $a - 3$.

$$\frac{5}{1}\left[\frac{a - 3}{a - 3}\right] + \frac{2}{a - 3}\left[\frac{1}{1}\right] - \frac{3}{3 - a}\left[\frac{-1}{-1}\right]$$

$$= \frac{5a - 15}{a - 3} + \frac{2}{a - 3} - \frac{-3}{a - 3}$$

$$= \frac{5a - 15 + 2 + 3}{a - 3}$$

$$= \frac{5a - 10}{a - 3}, \text{ the answer.}$$

Find the indicated sums and differences.

1.41 $\frac{x}{2} + \frac{y}{3} - \frac{z}{4}$

____

1.42 $\frac{5}{a} - \frac{3}{b} + \frac{1}{ab}$

____

1.43 $\frac{4}{7x^2} - \frac{3}{2x^3}$

____

1.44 $\frac{5n+1}{6} + \frac{3n-2}{8}$

____

1.45 $\frac{y}{y-5} + \frac{5}{5-y}$

____

1.46 $\frac{3}{x+5} - \frac{2}{x-3}$

____

1.47 $\frac{x+5}{3} - \frac{x-3}{2}$

____

1.48 $\frac{3}{a-5} + \frac{a+2}{a}$

____

1.49 $\frac{3k}{k-2} + \frac{6}{2-k}$

____

1.52 $\frac{1}{2} + \frac{3}{x+4}$

____

1.50 $\frac{b+1}{b+2} - \frac{b+3}{b+4}$

____

1.53 $\frac{a+2}{4b} - \frac{a-1}{10b} + \frac{a-3}{5b}$

____

1.51 $\frac{y}{y^2-49} - \frac{7}{y+7}$

____

1.54 $\frac{x}{x+1} - \frac{1}{x-1} + \frac{2x}{x^2-1}$

____

1.55 $\frac{1}{m^2+3m+2} + \frac{2}{m^2+4m+3} - \frac{3}{m^2+5m+6}$

____

MULTIPLYING AND DIVIDING FRACTIONS

Products of algebraic fractions are found in the same way as products of arithmetic fractions, according to the following rule.

Product Rule

$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$ ($B \neq 0$, $D \neq 0$); the product of two fractions is the product of the numerators over the product of the denominators; the result is to be in lowest terms.

Model 1:

$$\frac{x}{y} \cdot \frac{2}{3}$$
$$= \frac{x \cdot 2}{y \cdot 3}$$
$$= \frac{2x}{3y}$$

Model 2:

$$-\frac{8}{7m^3} \cdot 28m$$
$$= -\frac{8}{7m^3} \cdot \frac{28m}{1}$$
$$= -\frac{8 \cdot \cancel{28}^{4}\cancel{m}}{\cancel{7}\cancel{m^3}_{2} \cdot 1}$$
$$= -\frac{32}{m^2}$$

Model 3:

$$\frac{a^3b^2c}{d^3} \cdot \frac{ab}{c}$$
$$= \frac{a^3b^2\cancel{c} \cdot ab}{d^3 \cdot \cancel{c}}$$
$$= \frac{a^4b^3}{d^3}$$

Quotients of algebraic fractions are also found in the same way as quotients of arithmetic fractions, according to the following rule.

Quotient Rule

$\frac{A}{B} \div \frac{E}{F} = \frac{A}{B} \cdot \frac{F}{E}$ ($B \neq 0$, $F \neq 0$, $E \neq 0$); the quotient of two fractions is the product of the first fraction and the *reciprocal* of the second fraction; the result is to be in lowest terms.

Model 1: $\frac{x}{y} \div \frac{2}{3}$

$= \frac{x}{y} \cdot \frac{3}{2}$

$= \frac{x \cdot 3}{y \cdot 2}$

$= \frac{3x}{2y}$

Model 2: $-\frac{8}{7m^3} \div 28m$

$= -\frac{8}{7m^3} \cdot \frac{1}{28m}$

$= -\frac{\overset{2}{\cancel{8}} \cdot 1}{7m^3 \cdot \underset{7}{\cancel{28m}}}$

$= -\frac{2}{49m^4}$

Model 3: $\frac{a^3b^2c}{d^3} \div \frac{ab}{c}$

$= \frac{a^3b^2c}{d^3} \cdot \frac{c}{ab}$

$= \frac{\overset{a^2\,b}{\cancel{a^3}\cancel{b^2}}c \cdot c}{d^3 \cdot \cancel{ab}}$

$= \frac{a^2bc^2}{d^3}$

NOTE: No reducing is done in a quotient until the second fraction has been *inverted* to its reciprocal and the product rule has been applied. Then any common factors(s) to the numerator and denominator may be reduced.

The two rules may also be applied to products or quotients of more than two fractions. However, in a mixed product and quotient, only invert the fraction immediately after a division sign when changing to multiplication.

Model 1: $\frac{A}{B} \cdot \frac{C}{D} \cdot \frac{E}{F}$

$= \frac{ACE}{BDF}$

Model 2: $\frac{A}{B} \div \frac{C}{D} \cdot \frac{E}{F}$

$= \frac{A}{B} \cdot \frac{D}{C} \cdot \frac{E}{F}$

$= \frac{ADE}{BCF}$

Model 3:

$$\frac{A}{B} \div \frac{C}{D} \div \frac{E}{F}$$
$$= \frac{A}{B} \bullet \frac{D}{C} \bullet \frac{F}{E}$$
$$= \frac{ADF}{BCE}$$

Model 4:

$$\frac{5x^3z}{yz^2} \div 2xyz \bullet \frac{1}{20x^2} \div \frac{3}{8yz}$$
$$= \frac{5x^3z}{yz^2} \bullet \frac{1}{2xyz} \bullet \frac{1}{20x^2} \bullet \frac{8yz}{3}$$
$$= \frac{5x^3z \bullet 1 \bullet 1 \bullet 8yz}{yz^2 \bullet 2xyz \bullet 20x^2 \bullet 3}$$
$$= \frac{\cancel{40x^3yz^2}}{\cancel{120x^3y^2z^3}}$$
$$3 \quad y \quad z$$
$$= \frac{1}{3yz}$$

Find the indicated products and quotients.

1.56 $\frac{a}{7} \bullet \frac{b}{2}$

1.57 $\frac{11}{d^3} \bullet (-\frac{7}{d^4})$

1.58 $\frac{15ab}{4} \bullet \frac{8}{9a^2b^2}$

1.59 $-\frac{2x^5}{yz^2} \bullet \frac{y^2z}{x^4}$

1.60 $\frac{4m^3}{n^2} \div \frac{2m}{n}$

1.61 $\frac{6a}{8b} \bullet \frac{10c}{12d}$

1.62 $\frac{6a}{8b} \div \frac{10c}{12d}$

____

1.66 $\frac{9z^3}{16xy} \cdot \frac{4x}{27z^3}$

____

1.63 $\frac{50p^3q^2}{3} \div 75p^2q^3$

____

1.67 $(-\frac{1}{3ab}) \div (-3ab)$

____

1.64 $xyz \div \frac{1}{xyz}$

____

1.68 $\frac{3c^2}{ab} \div \frac{5b^2}{ac} \div \frac{bc}{2a^2}$

____

1.65 $-\frac{a^2b}{b^2c} \div \frac{a^2c}{b^2c^2}$

____

1.69 $\frac{5m^3n^2}{2} \div \frac{mn^2}{4} \cdot (-\frac{m^2}{10})$

____

1.70 $\frac{12xy}{z} \div \frac{14yz}{x} \cdot 7xyz \div \frac{6x}{5y}$

____

All of the products and quotients considered so far have had monomials for the numerators and denominators. Now we will look at fractions that also involve polynomials of more than one term. The two rules are still used in the same way, but polynomials of more than one term must be factored before any reducing takes place.

Model 1:

$$\frac{9m + 3p}{7} \cdot \frac{5}{6m + 2p}$$

$$= \frac{3(3m + p)}{7} \cdot \frac{5}{2(3m + p)}$$

$$= \frac{3(\cancel{3m + p}) \cdot 5}{7 \cdot 2\,(\cancel{3m + p})}$$

$$= \frac{15}{14}$$

Model 2:

$$\frac{a^2 + 2a}{8} \div \frac{4 - a^2}{6b}$$

$$= \frac{a^2 + 2a}{8} \cdot \frac{6b}{4 - a^2}$$

$$= \frac{a(\cancel{a + 2})}{\cancel{8}\,4} \cdot \frac{\overset{3}{\cancel{6}}b}{(\cancel{2 + a})(2 - a)}$$

$$= \frac{3ab}{4(2 - a)} \text{ or } \frac{3ab}{8 - 4a}$$

The first of the preceding models shows reducing to a single fraction after the product; the second model shows reducing to a single fraction before the product, which saves writing out one more step in the solution. Just remember that you must have a product (not a quotient) before reducing and that a factor common to any number and any denominator in that product is to be reduced.

Model 3:

$$\frac{2x^2 - 5x - 3}{9 - x^2} \div \frac{8x^2 + 2x - 1}{4x^2 + 11x - 3}$$

$$= \frac{2x^2 - 5x - 3}{9 - x^2} \cdot \frac{4x^2 + 11x - 3}{8x^2 + 2x - 1}$$

$$= \frac{(\cancel{2x + 1})\overset{(-1)}{(\cancel{x - 3})}}{(\cancel{3 + x})(\cancel{3 - x})} \cdot \frac{(\cancel{4x - 1})(\cancel{x + 3})}{(\cancel{4x - 1})(\cancel{2x + 1})}$$

$$= -1$$

Model 4: $\frac{n^2 - 25}{n^2 - 15n + 50} \div \frac{(n-5)^2}{n^2 - 5n} \bullet \frac{5n - 50}{n^2 + 5n}$

$= \frac{n^2 - 25}{n^2 - 15n + 50} \bullet \frac{n^2 - 5n}{(n-5)^2} \bullet \frac{5n - 50}{n^2 + 5n}$

$= \frac{\cancel{(n+5)}\cancel{(n-5)}}{\cancel{(n-10)}\cancel{(n-5)}} \bullet \frac{\cancel{n}\,\cancel{(n-5)}}{\cancel{(n-5)}(n-5)} \bullet \frac{5\cancel{(n-10)}}{\cancel{n}\cancel{(n+5)}}$

$= \frac{5}{n-5}$

Find the indicated products and quotients.

1.71 $\frac{3a - 6}{4a^2} \bullet \frac{2a}{5a - 10}$ ________________

1.72 $\frac{x^2 - y^2}{3x} \bullet \frac{3y}{x + y}$ ________________

1.73 $\frac{k^2 - 36}{k} \bullet \frac{k^2}{k - 6}$ ________________

1.74 $\frac{3m - 9}{4m + 8} \bullet \frac{m^2 + 5m + 6}{m^2 - 9}$ ________________

1.75 $\frac{7a - a^2}{2a} \div \frac{49 - a^2}{2a - 14}$ ________________

1.76 $\frac{y^2 + 3y - 10}{3y + 15} \div (10 - 5y)$ ________________

1.77 $\frac{4a - 4b}{12} \div \frac{a^2 - b^2}{3}$ ________________

1.78 $\frac{2x^2 + x - 3}{9} \bullet \frac{(x + 1)^2}{2x^2 + 5x + 3}$ ________________

1.79 $\dfrac{6y^2 + y - 2}{6y + 4} \cdot \dfrac{16y + 8}{4y^2 - 1}$

1.81 $\dfrac{n^2 + 5n + 6}{n^2 + 7n + 12} \cdot \dfrac{n^2 + 9n + 20}{n^2 + 11n + 30}$

1.80 $\dfrac{1}{25 - y^2} \div \dfrac{6 - 3y}{y^2 - 7y + 10}$

1.82 $\dfrac{c^2 + 11cd}{c^2d^2} \div \dfrac{2c + 22d}{11cd^2}$

1.83 $\dfrac{x^2 + 3x - 4}{x^2 - 7x + 6} \div \dfrac{x^3 - 8x^2}{x^2 + 6x} \div \dfrac{x^2 + 10x + 24}{x^2 - 14x + 48}$

1.84 $\dfrac{abc}{a^2 - 3a - 10} \div \dfrac{a^2}{a^2 - 25} \cdot \dfrac{4 - a^2}{9b^2 - 4} \div \dfrac{abc + 5bc}{3ab - 2a}$

1.85 $\dfrac{m^2}{m + n} \cdot \dfrac{n^2}{m - n} \cdot \dfrac{mn}{m^2 + n^2} \div \dfrac{m^2n^2}{m^2 - n^2}$

SIMPLIFYING COMPLEX FRACTIONS

You may remember studying complex arithmetic fractions. Algebraic fractions, too, can be complex.

VOCABULARY

Complex fraction—a fraction that has at least one fraction in its own numerator or denominator.

Models: $\frac{7}{\frac{2}{3}}$ $\frac{\frac{2}{3}+\frac{3}{4}}{\frac{1}{2}}$ $\frac{x-\frac{1}{2}}{3}$

Since any fraction is an indicated quotient ($\frac{A}{B} = A \div B$), the complex fraction $\frac{7}{\frac{2}{3}}$ means $7 \div \frac{2}{3} = \frac{7}{1} \bullet \frac{3}{2} = \frac{21}{2}$; the fraction $\frac{21}{2}$ (or the decimal 10.5) is then the simplified form of the complex fraction $\frac{7}{\frac{2}{3}}$.

Of the two methods for simplifying a complex fraction, the first has just been described: dividing the numerator by the denominator. However, you need to perform any indicated operations in the numerator and in the denominator before finding their quotient.

Model 1: Simplify $\frac{\frac{2}{3}+\frac{3}{4}}{\frac{1}{2}}$.

Solution: $\frac{\frac{2}{3}+\frac{3}{4}}{\frac{1}{2}} = \frac{\frac{8}{12}+\frac{9}{12}}{\frac{1}{2}}$

$= \frac{\frac{17}{12}}{\frac{1}{2}}$

$= \frac{17}{12} \div \frac{1}{2}$

$= \frac{17}{\cancel{12}_{6}} \bullet \frac{\cancel{2}}{1}$

$= \frac{17}{6}$ or $2.8\overline{3}$

Model 2: Simplify $\dfrac{x - \frac{1}{2}}{3}$.

Solution:
$$\frac{x - \frac{1}{2}}{3} = \frac{\frac{2x}{2} - \frac{1}{2}}{3}$$
$$= \frac{\frac{2x - 1}{2}}{3}$$
$$= \frac{2x - 1}{2} \div 3$$
$$= \frac{2x - 1}{2} \bullet \frac{1}{3}$$
$$= \frac{2x - 1}{6}$$

The second method is a shorter procedure for simplifying most complex fractions: Multiply the numerator and the denominator of the complex fraction by the LCD of all the fractions it contains.

Model 3: Simplify $\dfrac{\frac{2}{3} + \frac{3}{4}}{\frac{1}{2}}$.

Solution: The LCD of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ is 12.
$$\frac{\frac{2}{3} + \frac{3}{4}}{\frac{1}{2}} \left[\frac{12}{12}\right] = \frac{\frac{2}{\not{3}}[\not{12}^{\,4}] + \frac{3}{\not{4}}[\not{12}^{\,3}]}{\frac{1}{\not{2}}[\not{12}]^{\,6}}$$
$$= \frac{8 + 9}{6}$$
$$= \frac{17}{6} \text{ or } 2.8\overline{3}$$

Model 4: Simplify $\dfrac{x - \frac{1}{2}}{3}$.

Solution: The only fraction is $\frac{1}{2}$, so the LCD is 2.
$$\frac{x - \frac{1}{2}}{3} \left[\frac{2}{2}\right] = \frac{x\,[2] - \frac{1}{2}\,[2]}{3\,[2]}$$
$$= \frac{2x - 1}{6}$$

You can see that the results are the same by either method, and you may use the method of your choice. In some cases you will need to reduce the resulting fraction to lowest terms. Compare the two solutions to the following model:

Model 5: Simplify $\frac{\frac{1}{y} + 1}{y - \frac{1}{y}}$.

Solutions: Method 1

$$\frac{\frac{1}{y} + 1}{y - \frac{1}{y}} = \frac{\frac{1}{y} + \frac{y}{y}}{\frac{y^2}{y} - \frac{1}{y}}$$

$$= \frac{\frac{1 + y}{y}}{\frac{y^2 - 1}{y}}$$

$$= \frac{1 + y}{y} \div \frac{y^2 - 1}{y}$$

$$= \frac{1 + y}{y} \bullet \frac{y}{y^2 - 1}$$

$$= \frac{\cancel{1 + y}}{\cancel{y}} \bullet \frac{\cancel{y}}{(\cancel{y + 1})(y - 1)}$$

$$= \frac{1}{y - 1}$$

Method 2

$$\frac{\frac{1}{y} + 1}{y - \frac{1}{y}} \left[\frac{y}{y}\right] = \frac{\frac{1}{\cancel{y}}[\cancel{y}] + 1[y]}{y[y] - \frac{1}{\cancel{y}}[\cancel{y}]}$$

$$= \frac{1 + y}{y^2 - 1}$$

$$= \frac{\cancel{1 + y}}{(\cancel{y + 1})(y - 1)}$$

$$= \frac{1}{y - 1}$$

Simplify each complex fraction.

1.86 $\frac{\frac{1}{4} + x}{\frac{1}{2}}$ ____________________

1.87 $\frac{y - \frac{2}{3}}{y + \frac{1}{5}}$ ____________________

1.88 $\dfrac{\frac{a}{b} + c}{\frac{a}{b} - c}$

____

1.92 $\dfrac{\frac{y}{5} + 1}{\frac{y^2}{25} - 1}$

____

1.89 $\dfrac{2 + \frac{1}{a}}{\frac{2}{a} - a}$

____

1.93 $\dfrac{a - \frac{9}{a}}{1 - \frac{3}{a}}$

____

1.90 $\dfrac{\frac{x}{2} + \frac{x}{3}}{\frac{x}{4}}$

____

1.94 $\dfrac{\frac{1}{3} + \frac{1}{5} - \frac{1}{7}}{\frac{1}{x}}$

____

1.91 $\dfrac{\frac{2}{x} + \frac{3}{x}}{\frac{4}{x}}$

____

1.95 $\dfrac{\frac{n}{12} - \frac{2}{9}}{\frac{n}{6}}$

____

1.96 $\dfrac{m - \frac{1}{m}}{m + \frac{1}{m}}$ ______________

1.98 $\dfrac{\frac{a}{b} + 2 + \frac{b}{a}}{\frac{a}{b} - \frac{b}{a}}$ ______________

1.97 $\dfrac{x + \frac{1}{2}}{2 + \frac{1}{x}}$ ______________

1.99 $\dfrac{\frac{k-3}{4} - \frac{7}{k}}{\frac{k-7}{4}}$ ______________

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific area where restudy is needed for mastery.

SELF TEST 1

Give the excluded value(s) for each fraction (each answer, 3 points).

1.01 $\frac{2}{x\,(x - 3)}$ ____________________

1.02 $\frac{y + 5}{y^2 + 4y - 32}$ ____________________

1.03 $\frac{-7z}{4z + 1}$ ____________________

Reduce each fraction to lowest terms (each answer, 3 points).

1.04 $\frac{6a^2b^3}{8ab^4}$ ____________________

1.05 $\frac{3 - k}{k - 3}$ ____________________

1.06 $\frac{n^2 - 7n - 44}{n^2 - 121}$ ____________________

Perform the indicated operations (each answer, 4 points).

1.07 $\frac{4x}{2x + y} + \frac{2y}{2x + y}$

1.08 $\frac{d + 3}{8d} - \frac{2d + 1}{10d^2}$

1.09 $\frac{3}{n^2 - 9} + \frac{7}{3 - n}$

1.010 $\frac{m}{n} \cdot \frac{n}{p} \div \frac{p}{q}$

1.011 $\frac{4x^2yz^3}{9} \cdot \frac{45y}{8x^5z^3}$

1.012 $\frac{k + 5}{k^2 + 3k - 10} \div \frac{7k + 14}{4 - k^2}$

Simplify each complex fraction (each answer, 3 points).

1.013 $\dfrac{\frac{1}{x}}{\frac{1}{y}}$

1.015 $\dfrac{\frac{5}{a}+1}{\frac{a}{5}-\frac{5}{a}}$

1.014 $\dfrac{\frac{m}{5}-\frac{1}{6}}{\frac{1}{3}}$

41/51 **SCORE** ________ **TEACHER** ________ ________
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