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MATH

STUDENT BOOK

9th Grade | Unit 7

Alpha Omega
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Math 907

Radical Expressions

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Radical Expressions

INTRODUCTION

In this LIFEPAC® you will continue your study in the mathematical system of algebra by learning first about real numbers and then about radical expressions. After becoming familiar with radical expressions, you will learn to simplify them and to perform the four basic operations (addition, subtraction, multiplication, and division) with them. Finally, you will learn to solve equations containing these expressions.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Identify and work with rational numbers.
2. Identify and work with irrational numbers.
3. Draw number-line graphs of open sentences involving real numbers.
4. Simplify radical expressions.
5. Combine (add and subtract) radical expressions.
6. Multiply radical expressions.
7. Divide radical expressions.
8. Solve equations having irrational roots.
9. Solve equations containing radical expressions.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. REAL NUMBERS

In this section, you will study the fundamental set of numbers for beginning algebra and geometry—the *real numbers*. You will learn about two other sets, the *rational numbers* and the *irrational numbers*, that make up the real numbers. You will also learn a new property that applies to no other numbers you have studied so far—*completeness*; this property will be used in graphing the real numbers.

OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Identify and work with rational numbers.
2. Identify and work with irrational numbers.
3. Draw number-line graphs of open sentences involving real numbers.

RATIONAL NUMBERS

You will begin by classifying some numbers that you are quite familiar with already. You will need to discover what numbers are actually included in this classification according to the definition. Conversion between the different forms that a rational number may take should help to understand the classification better. Then you will be ready to graph rational numbers and study their properties.

DEFINITIONS AND CONVERSIONS

The following definition outlines the classification of numbers known as rational numbers.

VOCABULARY

rational number—a number that can be written as a ratio of two integers in the form $\frac{A}{B}$ with $B \neq 0$.

Model 1: $\frac{2}{9}$ is a rational number since it is the ratio of the integers 2 and 9.

Model 2: $4\frac{1}{5}$ is a rational number since it can be written as $\frac{21}{5}$, the ratio of the integers 21 and 5.

Model 3: $-\frac{3}{8}$ is a rational number since it can be written as $\frac{-3}{8}$, the ratio of the integers -3 and 8.

Model 4: 0.283 is a rational number since it can be written as $\frac{283}{1,000}$, the ratio of the integers 283 and 1,000.

Model 5: -81.7 is a rational number since it can be written as $-81\frac{7}{10} = -\frac{817}{10} = \frac{817}{-10}$, the ratio of the integers 817 and -10.

Model 6: 17 is a rational number since it can be written as $\frac{17}{1}$, the ratio of the integers 17 and 1.

Model 7: 0 is a rational number since it can be written as $\frac{0}{1}$, the ratio of the integers 0 and 1.

Model 8: -6 is a rational number since it can be written as $-\frac{6}{1} = \frac{-6}{1}$, the ratio of the integers -6 and 1.

From the models, you can see that the common fractions, mixed numbers, and decimals of arithmetic (as well as their negatives) are included in the rational numbers. Also, you can see that the integers themselves are included in the rational numbers.

You may be wondering which numbers are *not* included in this classification. Such numbers will be considered in detail later in this section, but at the present time you should know that not all decimals are rational and not all fractions are rational. For example, a number that you have probably worked with, π, cannot be written as the ratio of two integers and is not rational; therefore, neither is a fraction such as $\frac{\pi}{6}$ rational. You may have used an *approximation* for π , such as 3.14 or $\frac{22}{7}$, in evaluating formulas. These approximations are themselves rational, but π is not!

A fraction that is rational can be converted to an equivalent decimal form, and a decimal that is rational can be converted to an equivalent fraction form. The two equivalent forms, of course, must have the same sign.

Model 1: Convert $\frac{5}{8}$ and $-\frac{5}{8}$ to decimals.

Solution: $\frac{5}{8} = 5 \div 8 = 0.625$,

a *terminating* decimal.

$\therefore\ \frac{5}{8} = 0.625$ and $-\frac{5}{8} = -0.625$

Model 2: Convert -0.24 to a fraction.

Solution: $-0.24 = -\frac{24}{100} = -\frac{\cancel{4} \cdot 6}{\cancel{4} \cdot 25}$

$\therefore\ -0.24 = -\frac{6}{25}$

Model 3: Convert $\frac{1}{3}$ to a decimal.

Solution: $\frac{1}{3} = 1 \div 3 = 0.3333...$,

a *repeating* decimal.

$\therefore\ \frac{1}{3} = .0\overline{3}$

The decimal $0.\overline{3}$ is said to have a *period* of 1 since one number place continues without end; the decimal -0.363636... = $-0.\overline{36}$ has a period of 2 since two number places continue without end. The line drawn above a repeating decimal is called the *repetend* bar, and it should be over the exact number of places in the period of the decimal.

Models: 0.12341234... = $0.\overline{1234}$ and has a period of 4.

0.12343434... = $0.12\overline{34}$ and has a period of 2.

0.12344444... = $0.123\overline{4}$ and has a period of 1.

We saw that the rational number $\frac{1}{3}$ converts to $0.\overline{3}$, but how does $0.\overline{3}$ convert back to $\frac{1}{3}$? The decimal 0.3 converts to $\frac{3}{10}$, but $\frac{1}{3} \neq \frac{3}{10}$; thus, $0.\overline{3} \neq \frac{3}{10}$ either. The following solutions show a procedure for converting repeating decimals to fractions.

Model 1: Convert $0.\overline{3}$ to a fraction.

Solution: Let $n = 0.\overline{3} = 0.333...$

Then $10n = 10(0.333...) = 3.33...$, since multiplying a decimal by ten moves the decimal point one place to the right.

Now subtract:

$$\begin{array}{rcl} 10n &=& 3.33... \\ 1n &=& 0.33... \\ \hline 9n &=& 3.00... \end{array}$$

or $9n = 3$

and $n = \frac{3}{9}$ or $\frac{\cancel{3} \cdot 1}{\cancel{3} \cdot 3}$

$\therefore\ 0.\overline{3} = \frac{1}{3}$

NOTE: The period of $0.\overline{3}$ is 1, and n is multiplied by $10^1 = 10$.

Model 2: Convert $-0.\overline{36}$ to a fraction.

Solution: Since the given decimal is negative, its equivalent fraction will be negative also.

Let $n = 0.\overline{36} = 0.363636...$

Then $100n = 100(0.363636...) = 36.3636...$ since multiplying a decimal by one hundred moves the decimal point two places to the right.

Now subtract:

$$\begin{aligned} 100n &= 36.3636... \\ 1n &= 0.3636... \\ \hline 99n &= 36.0000... \end{aligned}$$

$$\text{or } 99n = 36$$

$$\text{and } n = \frac{36}{99} \text{ or } \frac{\cancel{9} \cdot 4}{\cancel{9} \cdot 11}$$

$\therefore$ Since $0.\overline{36} = \frac{4}{11}$, then $-0.36 = -\frac{4}{11}$.

NOTE: The period of $0.\overline{36}$ is 2, and n is multiplied by $10^2 = 100$.

Some other results of this procedure are shown in the following models. Try to find a relationship between the repeating decimal, its period, and its equivalent unreduced fraction.

Models:

$0.\overline{1} = \frac{1}{9}$ $\quad$ $0.\overline{01} = \frac{1}{99}$ $\quad$ $0.\overline{001} = \frac{1}{999}$

$0.\overline{5} = \frac{5}{9}$ $\quad$ $0.\overline{53} = \frac{53}{99}$ $\quad$ $0.\overline{531} = \frac{531}{999} = \frac{59}{111}$

You have seen that a repeating decimal as well as a terminating decimal can be written as the ratio of two integers in the form $\frac{A}{B}$; both types then are rational numbers. Actually, a terminating decimal is just a special type of repeating decimal—one that repeats zero; for example, 0.815 = 0.815000... and -5.3 = -5.3000.... Keeping this fact in mind, consider the following alternate definition and see how it applies to the models from the beginning of this section.

VOCABULARY

rational number—a number that can be written as a repeating decimal.

Models:

$\frac{2}{9} = 0.\overline{2}$

$4\frac{1}{5} = 4.2\overline{0}$

$-\frac{3}{8} = -0.375\overline{0}$

$0.283 = 0.283\overline{0}$

$-81.7 = -81.7\overline{0}$

$17 = 17.\overline{0}$

$0 = 0.\overline{0}$

$-6 = -6.\overline{0}$

Write each rational number as the ratio of two integers in the form $\frac{A}{B}$ and then as a repeating decimal.

Model: $4.3 = \frac{43}{10} = 4.3\overline{0}$

1.1 $\frac{3}{4}$ = ________ = ________

1.2 $-7\frac{1}{3}$ = ________ = ________

1.3 $8\frac{1}{2}$ = ________ = ________

1.4 -6.59 = ________ = ________

1.5 10 = ________ = ________

Convert each fraction to its equivalent decimal form.

1.6 $\frac{3}{5}$ = ________

1.7 $\frac{33}{50}$ = ________

1.8 $\frac{2}{3}$ = ________

1.9 $\frac{1}{15}$ = ________

1.10 $\frac{20}{33}$ = ________

1.11 $-\frac{83}{200}$ = ________

1.12 $-\frac{52}{125}$ = ________

1.13 $-\frac{5}{12}$ = ________

1.14 $-\frac{12}{5}$ = ________

1.15 $-\frac{21}{37}$ = ________

Convert each decimal to its equivalent reduced fraction form.

1.16 0.7 = ________

1.17 $0.\overline{7}$ = ________

1.18 -0.8 = ________

1.19 $-0.\overline{8}$ = ________

1.20 0.25 = ________

1.21 $0.\overline{25}$ = ________

1.22 -0.63 = ________

1.23 $-0.\overline{63}$ = ________

1.24 0.135 = ________

1.25 $0.\overline{135}$ = ________

1.26 0.9 = ________

1.27 $0.\overline{9}$ = ________

Model: $.2\overline{5}$

Solution: Let $n = 0.2\overline{5} = 0.25555...$

Then $10n = 2.55...$ and $100n = 25.55...$

Now subtract:

$$\begin{aligned} 100n &= 25.55... \\ 10n &= 2.55... \\ \hline 90n &= 23.00... \end{aligned}$$

or $90n = 23$

and $n = \frac{23}{90}$

$\therefore 0.2\overline{5} = \frac{23}{90}$

Convert each decimal to its equivalent reduced fraction form. Show your work as in the preceding model.

1.28 $0.8\overline{7}$

1.29 $-0.3\overline{8}$

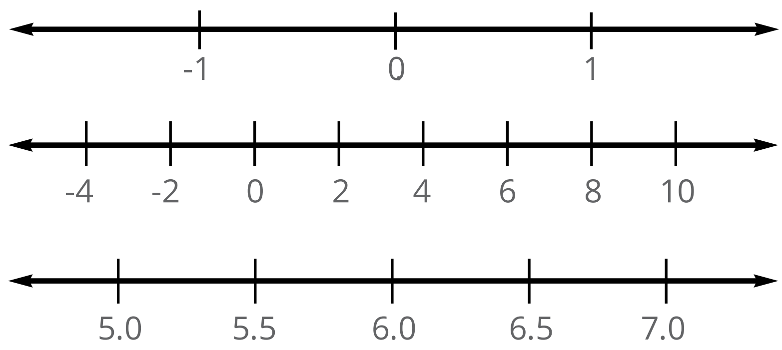
1.30 $0.0\overline{9}$

GRAPHS AND ORDER

Now that rational numbers have been explained, you should be able to graph them on the number line. First, however, a review of some of the basic ideas of graphing may be helpful.

The small vertical line segments drawn on the number line are only reference marks (not graphed points); and the spacing between them, as well as the numbers written below them, may be changed for convenience in graphing.

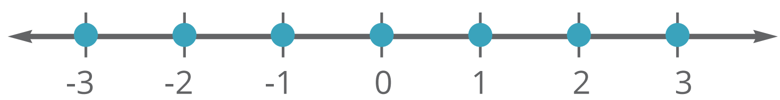
Models:



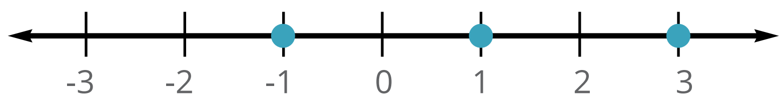
All three lines shown can be thought of as the same number line, but with different reference marks; no points are graphed on these lines.

A point is graphed on the number line by placing a heavy dot on (or between) the appropriate reference mark(s). A darkened arrowhead is used at the end(s) of the line represented on the paper to show a continuation of points.

Model 1: The graph of the integers is shown.



Model 2: The graph of the odd integers larger than -2 is shown.



You have already learned that all the integers are rational numbers since each can be written as the ratio of itself to 1 and as a decimal that repeats zero. From the graph, the order of the integers can be seen to be ...-3 < -2 < -1 < 0 < 1 < 2 < 3...; that is, an integer is less than another integer if it is to the left of the other integer on the number line.

Although infinitely many integers are in the rational numbers, infinitely many nonintegers are also rational, such as $1\frac{1}{4}$ and $-0.\overline{56}$. A nonintegral rational number that would be between two reference marks on the number line is graphed by placing a heavy dot on the approximate corresponding point and writing the number above the dot. The *order* of both integral and nonintegral rational numbers can be determined from the relative positions of their points on the line.

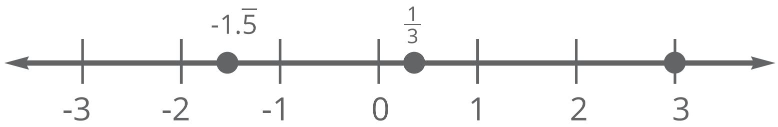
DEFINITION

A* < *B means that *A* is to the left of *B* on the number line;

A* < *B is the *order* of *A* and *B*.

Model 1: Graph the rational numbers $\frac{1}{3}$, $-1.\overline{5}$, and 3; then give their order.

Solution:



$-1.\overline{5} < \frac{1}{3} < 3$ is their order.

Model 2: Graph the rational numbers $1\frac{7}{10}$, 0, $-\frac{1}{2}$, and $1\frac{5}{8}$; then give their order.

Solution: You can easily graph 0 and $-\frac{1}{2}$ and determine their order; you can also see that they are less than the other two numbers, which lie between 1 and 2. But which of the two positive numbers is the smaller?

If the fractions are written with a positive common denominator, then the numerators can be compared. Since $\frac{7}{10}\left(\frac{4}{4}\right) = \frac{28}{40}$ and $\frac{5}{8}\left(\frac{5}{5}\right) = \frac{25}{40}$, you can see that $1\frac{5}{8}$ is smaller than $1\frac{7}{10}$. Thus, the numbers can be graphed in order.

$-\frac{1}{2} < 0 < 1\frac{5}{8} < 1\frac{7}{10}$ is their order.

In many cases, fractions may be more quickly converted to their equivalent decimal forms (rather than written with a common denominator) when finding their order. For example, in the preceding model $1\frac{7}{10} = 1.7$, $1\frac{5}{8} = 1.625$, and $1.625 < 1.7$; therefore, $1\frac{5}{8} < 1\frac{7}{10}$.

Model 3: Give the order of the rational numbers $5.\overline{6}$, $\frac{28}{5}$, $5\frac{5}{6}$, and 5.7.

Solution: The decimal forms are $5.\overline{6}$, $\frac{28}{5} = 5.6$, $5\frac{5}{6} = 5.8\overline{3}$, and 5.7, respectively; and $5.6 < 5.\overline{6} < 5.7 < 5.8\overline{3}$.

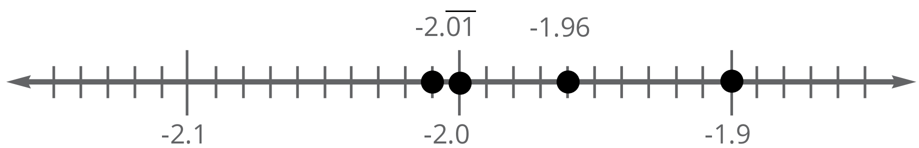
$\therefore$ The order is $\frac{28}{5} < 5.\overline{6} < 5.7 < 5\frac{5}{6}$.

Model 4: Give the order of the rational numbers -2, $-\frac{19}{10}$, $-(1.4)^2$, and $-2\frac{1}{99}$.

Solution: The decimal forms are $-2 = -2.\overline{0}$, $-\frac{19}{10} = -1.9$, $-(1.4)^2 = -1.96$, and $-2\frac{1}{99} = -2.\overline{01}$, respectively; and $-2.\overline{01} < -2.\overline{0} < -1.96 < -1.9$.

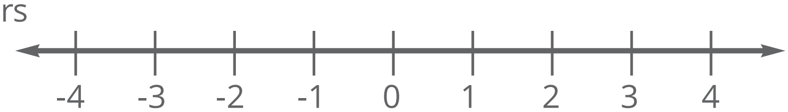
$\therefore$ The order is $-2\frac{1}{99} < -2 < -(1.4)^2 < -\frac{19}{10}$.

Using a number line may be helpful in ordering negative decimal numbers such as the ones in this model.



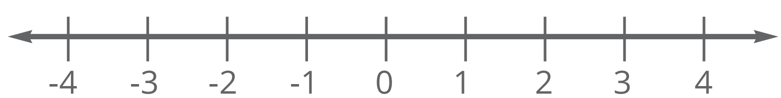
Graph the rational numbers indicated and give their order (smallest to largest) in each case.

1.31 The positive even integers



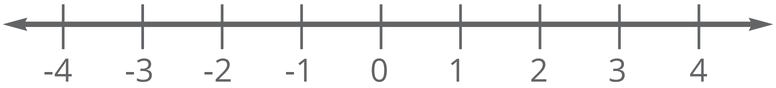
Order: ______________________________

1.32 3.5, -2.3, 0



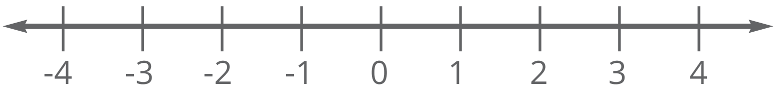
Order: ______________________________

1.33 $-\frac{3}{4}$, -3, $1\frac{1}{2}$, $1\frac{2}{3}$



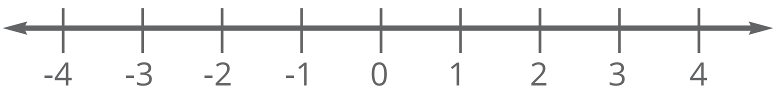
Order: ______________________________

1.34 $2.\overline{8}$, -1.5, 0.7, -0.7



Order: ______________________________

1.35 $\frac{1}{5}$, 3.2, $-1\frac{2}{3}$, -2



Order: ______________________________

Give the order of the rational numbers in each case.

1.36 $2.\overline{5}$, $\frac{5}{2}$, 2.6 ______________________________

1.37 -4.1, $-\frac{103}{25}$, $-(1.9)^2$ ______________________________

1.38 $1\frac{2}{3}$, $(1.3)^2$, $\frac{8}{5}$, $1\frac{13}{20}$ ______________________________

1.39 0, -0.1, 0.1, $-0.\overline{1}$ ______________________________

1.40 $8\frac{3}{11}$, $\frac{25}{3}$, 8.3, $8.\overline{30}$ ______________________________

PROPERTIES

The fact that the rational numbers include non-integers as well as integers results in their having additional properties. For example, the rational numbers are *closed* under defined division, but the integers are not.

Model: The quotient of -4 and 5 is $-4 \div 5 = -\frac{4}{5}$ or -0.8, which is rational even though it is not an integer.

Both the integers and the rational numbers are closed under the operations of addition, subtraction, and multiplication.

PROPERTY
Closure—For rational numbers A and B, the sum $A + B$, the difference $A - B$, the product $A \bullet B$, and the quotient $A \div B$ $(B \neq 0)$ are also rational numbers.

Also, only in some instances can you find an integer between two integers; but in all instances you can find a rational number between two rational numbers. Thus, the rational numbers are *dense*; but the integers are not.

Model: Between the integers -2 and 3, you can find the integers -1, 0, 1, and 2.

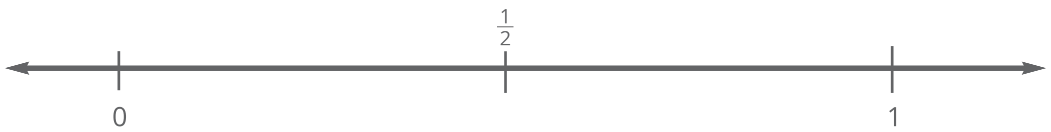
Between the integers 9 and 10, you cannot find another integer.

Between the rational numbers 9 and 10, you can find rational numbers such as 9.001, $9\frac{1}{2}$, $\frac{68}{7}$, and $9.\overline{8}$.

PROPERTY
Density—For rational numbers A and B with $A < B$, another rational number X can always be found so that $A < X < B$.

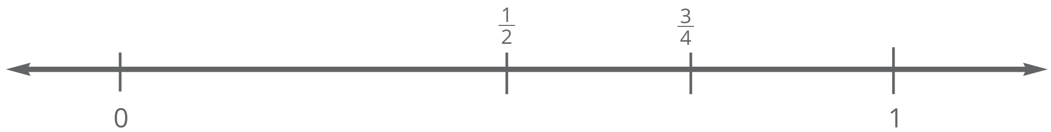
One such rational number X that can easily be found between A and B is their average: $\frac{A+B}{2}$. On the number line, the point that corresponds to $\frac{A+B}{2}$ is midway between the points corresponding to A and B.

Model: Start with the rational numbers 0 and 1; between them is their average, $\frac{0+1}{2}$ or $\frac{1}{2}$.



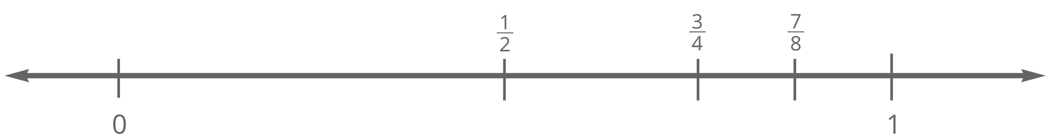
Between $\frac{1}{2}$ and 1 is their average:

$\dfrac{\frac{1}{2}+1}{2}$ or $\dfrac{\frac{3}{2}}{2}$ or $\frac{3}{4}$



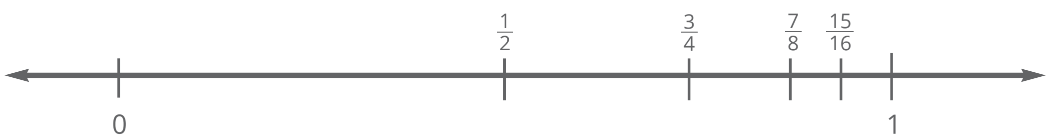
Between $\frac{3}{4}$ and 1 is their average:

$\dfrac{\frac{3}{4}+1}{2}$ or $\dfrac{\frac{7}{4}}{2}$ or $\frac{7}{8}$



Between $\frac{7}{8}$ and 1 is their average:

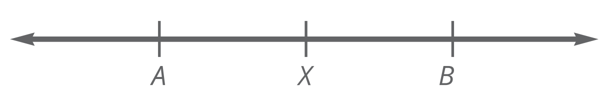
$\dfrac{\frac{7}{8}+1}{2}$ or $\dfrac{\frac{15}{8}}{2}$ or $\frac{15}{16}$. This process can be continued without end.



Thus, infinitely many rational numbers occur between 0 and 1. (Do you think this statement is true for any rational numbers *A* and *B*?)

Of course, other rational numbers are between *A* and *B* beside their average. The following models will show how the number line can be used to help in finding such numbers. You will need to understand one basic idea first: the distance between two points on the number line is the difference between the numbers (larger minus smaller) corresponding to those points.

For example, on the number line shown below, the distance from *A* to *B* is $B - A$; the distance from *A* to *X* is $X - A$; and the distance from *X* to *B* is $B - X$.



Model: 1: Find the number two-thirds of the way between 10 and 70.

Solution: We are trying to find X on the number line shown.

The distance from 10 to 70 is 70 – 10 or 60.

The distance from 10 to X is $X - 10$.

The distance from 10 to X is $\frac{2}{3}$ of the distance from 10 to 70; thus, the equation is

$$X - 10 = \frac{2}{3} \cdot 60$$

Solve for X: $$X - 10 = \frac{2}{\cancel{3}} \cdot \overset{20}{\cancel{60}}$$

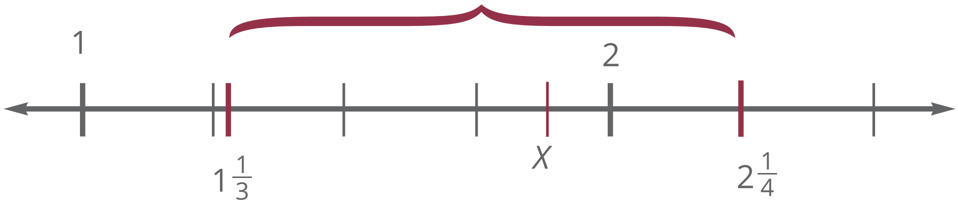
$$X - 10 = 40$$

$$X = 50$$

$\therefore$ 50 is two-thirds of the way between 10 and 70.

Model 2: Find the number three-fifths of the way between $1\frac{1}{3}$ and $2\frac{1}{4}$.

Solution: We are trying to find X on the number line shown.



The distance from $1\frac{1}{3}$ to $2\frac{1}{4}$ is $2\frac{1}{4} - 1\frac{1}{3}$ or $\frac{11}{12}$.

The distance from $1\frac{1}{3}$ to X is $X - 1\frac{1}{3}$ or $X - \frac{4}{3}$.

The distance from $1\frac{1}{3}$ to X is $\frac{3}{5}$ of the distance from $1\frac{1}{3}$ to $2\frac{1}{4}$; thus, the equation is

$$X - \frac{4}{3} = \frac{3}{5} \cdot \frac{11}{12}$$

Solve for X :

$$X - \frac{4}{3} = \frac{\cancel{3}}{5} \cdot \frac{11}{\cancel{12}_4}$$

$$X - \frac{4}{3} = \frac{11}{20}$$

The LCD is 60.

$$60[X - \frac{4}{3}] = \overset{3}{\cancel{60}}\,[\frac{11}{\cancel{20}}]$$

$$60X - 80 = 33$$

$$60X = 113$$

$$X = \frac{113}{60} \text{ or } 1\frac{53}{60}$$

$\therefore$ $1\frac{53}{60}$ is three-fifths of the way between $1\frac{1}{3}$ and $2\frac{1}{4}$.

Model 3: Find the number 0.28 of the way between -4.2 and 1.7.

Solution: We are trying to find X on the number line shown.

The distance from -4.2 to 1.7 is 1.7 – (-4.2) or 5.9.
The distance from -4.2 to X is X – (-4.2) or X + 4.2.
The distance from -4.2 to X is 0.28 of the distance from -4.2 to 1.7; thus, the equation is

$$X + 4.2 = 0.28(5.9)$$

Solve for X :

$$X + 4.2 = 1.652$$

$$X = -2.548$$

$\therefore$ -2.548 is 0.28 of the way between -4.2 and 1.7.

The density property insures that a rational number can be found between each of the following pairs of rational numbers. Find the specific rational number requested in each case. Circle your answer.

1.41 One-fourth of the way between 67 and 83.

1.42 Five-sixths of the way between -12 and 18.

1.43 0.4 of the way between 15 and 50.

1.44 0.61 of the way between -20 and -10.

1.45 One-third of the way between $1\frac{1}{4}$ and $2\frac{3}{4}$.

1.46 Two-sevenths of the way between 3.36 and 3.5.

1.47 0.75 of the way between 0 and 10.

1.48 0.75 of the way between -10 and 0.

1.49 Three-eighths of the way between -3 and 9.

1.50 Midway between 1.79 and 5.33.

IRRATIONAL NUMBERS

You have learned that a rational number has an infinite decimal representation that repeats, and you have worked only with these numbers so far in this LIFEPAC. Now you will observe some numbers that are not included in the rational classification. Square roots often fit this description, as you will see. Graphs of these numbers will also be shown.

DEFINITIONS AND ROUNDING

The following definition outlines the classification of numbers that are *not* rational numbers and are, therefore, called *irrational* numbers.

VOCABULARY

irrational number—a number with an infinite decimal representation that does not repeat.

Model 1: 0.10110111011110... is an irrational number since its infinite decimal does not repeat; it would continue with five 1's and a 0, then six 1's and a 0, and so on.

Model 2: -1.23456789101112... is an irrational number since its infinite decimal does not repeat; it would continue with the digits 13, then 14, and then 15, and so on.

At no time in either of these models could a portion of the decimal representation be written under a repetend bar.

Although the two preceding models may not be very useful irrational numbers, they do illustrate ways in which non-repeating decimals can be formed. And since such decimals could be formed without limit, you should see that infinitely many irrational numbers exist just as infinitely many rational numbers exist.

One very important irrational number, π , was mentioned earlier in this section. The number π is the ratio of the circumference of a circle to the diameter, and it is found in many area and volume formulas. The non-repeating decimal representation of π is 3.1415926..., and this number can be rounded to a rational approximation of any desired accuracy for use in evaluating formulas.

The rules for rounding decimals, whether rational or irrational, are given for you to follow.

1. When rounding to a certain number of places, look at the digit immediately after the last decimal place to be kept.

2. If this digit is 0, 1, 2, 3, or 4, leave the last place to be kept as it is and drop all the remaining digits.

3. If this next digit is 5, 6, 7, 8, or 9, add 1 to the last place to be kept and drop all the remaining digits.

Model: $\pi = 3.1415926...$

To the nearest tenth or to one decimal place, $\pi \doteq 3.1$ since the next digit is 4. (The symbol $\doteq$ means “is approximately equal to.”)

To the nearest hundredth or to two decimal places, $\pi \doteq 3.14$ since the next digit is 1.

To the nearest thousandth or to three decimal places, $\pi \doteq 3.142$ since the next digit is 5.

To the nearest ten-thousandth or to four decimal places, $\pi \doteq 3.1416$ since the next digit is 9.

a. Tell whether each of the following infinite decimals indicates a rational or irrational number. b. If the number is rational, write it using a repetend bar; then round every number to the nearest hundredth.

Models:	-0.783783783...	a. rational	b. $-0.\overline{783} \doteq -0.78$
	51.626626662...	a. irrational	b. $\doteq 51.63$
1.51	4.617617617...	a. ______	b. ______
1.52	-7.6544444...	a. ______	b. ______
1.53	-0.353353335...	a. ______	b. ______
1.54	573.19999...	a. ______	b. ______
1.55	0.2468101214...	a. ______	b. ______
1.56	-13.060060006...	a. ______	b. ______
1.57	-13.60606060...	a. ______	b. ______
1.58	25.2627282930...	a. ______	b. ______
1.59	-3.838838883...	a. ______	b. ______
1.60	99.999099909990...	a. ______	b. ______

SQUARE ROOTS

Suppose that we wish to find the positive number whose square is 30. Since $5^2 = 25$ and $6^2 = 36$, the number must be between 5 and 6. Use a calculator to verify the square of each of the following numbers.

$(5.1)^2 = 26.01$

$(5.2)^2 = 27.04$

$(5.3)^2 = 28.09$

$(5.4)^2 = 29.16$

$(5.5)^2 = 30.25$

Now since $29.16 < 30 < 30.25$, we can conclude that the number must be between 5.4 and 5.5. Continuing to try squares in this way, we can obtain the hundredths place, then the thousandths place, and so on of the desired decimal number, as shown on the next page.

$(5.47)^2 =$ 29.9209

$(5.477)^2$ = 29.997529

$(5.4772)^2$ = 29.99971984

$(5.47722)^2$ = 29.9999389284

This sequence of squares gets closer and closer to 30, but will never reach exactly 30. The number we are looking for is an infinite non-repeating decimal known as the positive *square root* of 30; this irrational number is written as $\sqrt{30} = 5.47722557\ldots$.

Use the $\sqrt{}$ key on a calculator to verify this result.

The negative square root of 30 is written as $-\sqrt{30} = -5.47722557\ldots$;

This statement is true since $(-5.47722557\ldots)^2 = 30$. However, $\sqrt{-30}$ will have no meaning for us at this time since no rational or irrational number will square to yield a negative number.

The following list contains the positive square roots of the first ten counting numbers.

$\sqrt{1}$ =	1	$\sqrt{6}$ =	2.44948974...
$\sqrt{2}$ =	1.41421356...	$\sqrt{7}$ =	2.64575131...
$\sqrt{3}$ =	1.73205080...	$\sqrt{8}$ =	2.82842712...
$\sqrt{4}$ =	2	$\sqrt{9}$ =	3
$\sqrt{5}$ =	2.23606797...	$\sqrt{10}$ =	3.1622776...

From this list you can see that not all indicated square roots are irrational; square roots of numbers having two equal rational factors are themselves rational.

Model 1: $11^2 = 121$;

$\sqrt{121} = 11$ is rational.

Model 2: $(\frac{7}{9})^2 = \frac{49}{81}$;

$\sqrt{\frac{49}{81}} = \frac{7}{9}$ is rational.

Model 3: $(1.04)^2 = 1.0816$;

$\sqrt{1.0816} = 1.04$ is rational.

Model 4: $(6.\overline{3})^2 = 40.\overline{1}$;

$\sqrt{40.\overline{1}} = 6.\overline{3}$ is rational.

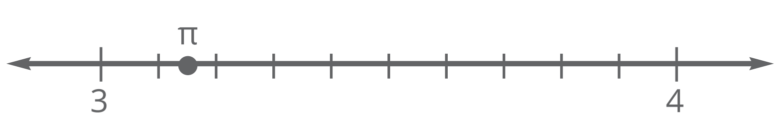
a. Find each of the following square roots, using a calculator when necessary;
b. Then indicate whether each is rational or irrational.

1.61	$\sqrt{25}$	a. ________	b. ________
1.62	$\sqrt{26}$	a. ________	b. ________
1.63	$\sqrt{\frac{4}{9}}$	a. ________	b. ________
1.64	$\sqrt{32.\overline{1}}$	a. ________	b. ________
1.65	$\sqrt{32.1}$	a. ________	b. ________
1.66	$\sqrt{1.522756}$	a. ________	b. ________
1.67	$\sqrt{400}$	a. ________	b. ________
1.68	$\sqrt{4{,}000}$	a. ________	b. ________
1.69	$\sqrt{0.04}$	a. ________	b. ________
1.70	$\sqrt{\frac{1}{64}}$	a. ________	b. ________

GRAPHS AND ORDER

An irrational number can be graphed on the number line by placing a heavy dot on the approximate corresponding point and writing the number above the dot.

Model: The graph of π is shown.



The order of irrational numbers can be determined by observing the relative positions of their points on the number line, or by comparing their decimal representations, or both.

Model: Give the order of the irrational numbers -3.010010001…, $-\sqrt{10}$, and -π.

Solution: The decimal representations are -3.010010001…, $-\sqrt{10}$ = -3.1622776… , and -π = -3.1415926… , respectively; and -3.1622776… < -3.1415926… < -3.010010001… .

On the number line,

∴ The order is $-\sqrt{10} < -\pi <$ -3.010010001…

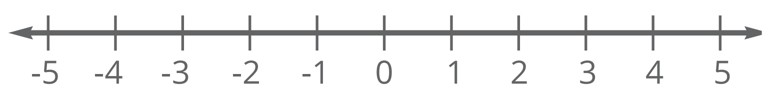
Graph each irrational number on the given number line.

1.71 $\sqrt{17}$

1.72 $-\sqrt{18}$

1.73 $\frac{\pi}{2}$

1.74 -2.818818881…



Give the order of the preceding four irrational numbers (1.71 through 1.74).

1.75 ________________ < ________________ < ________________ < ________________

COMPLETENESS

Consider the numbers that are between -2 and 3, inclusive. If we want only counting numbers, then the graph is Figure 1; if we want whole numbers, then the graph is Figure 2; and if we want integers, then the graph is Figure 3.

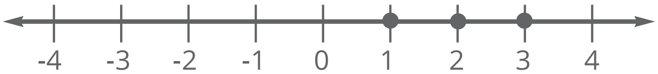


Figure 1

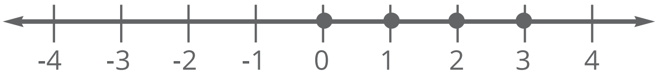


Figure 2

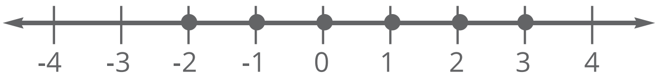


Figure 3

Can we make a graph if we want the rational numbers between -2 and 3, inclusive? We cannot show all of the infinitely many rational numbers between -2 and 3 by individual dots. We cannot use a solid line segment of points, such as Figure 4, because we must allow for "holes" at the points that correspond to irrational numbers such as $-\sqrt{2}$ and 0.313233343... and $\frac{\pi}{3}$. (The same problem arises when we try to graph only the irrational numbers because we will have "holes" at the points that correspond to the rational numbers between -2 and 3.)

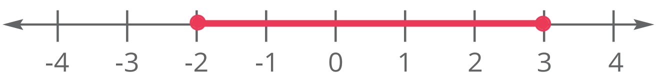


Figure 4

The meaning of the graph in Figure 4, then, is that it shows the points corresponding to both the rational numbers and the irrational numbers *between* -2 and 3, inclusive. If we do not want to include 3, then the graph becomes Figure 5; and if we only want the numbers *between* -2 and 3 (noninclusive), then the graph is Figure 6. The use of a circle instead of a dot in these instances indicates that each number corresponding to a circled point is not included in the result.

Figure 5

Therefore, we cannot make a graph for only the rational numbers between -2 and 3. However, the rational numbers and the irrational numbers together make up the *real numbers*, which have a special property that allows us to make solid graphs such as the preceding ones.

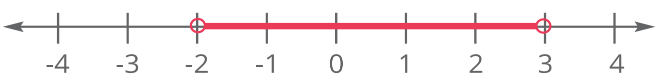


Figure 6

PROPERTY

Completeness—Each real number has one point on the number line, and each point on the number line has one real number.

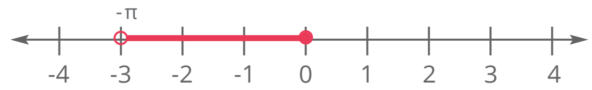
Thus, a more correct name for the number line that you have been using is the real-number line; and every infinite decimal, whether repeating (rational) or non-repeating (irrational) can be represented by a point on that line.

From this time on, you are to assume that variables represent real numbers unless you are specifically told otherwise.

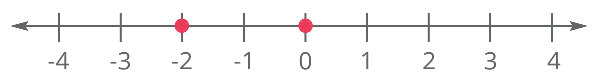
Model 1: The graph of $x \geq 4\frac{1}{2}$ is

Model 2: The graph of $x < 4\frac{1}{2}$ is

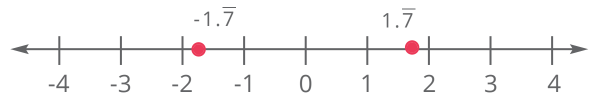
Model 3: The graph of $-\pi < y \leq 0$ is



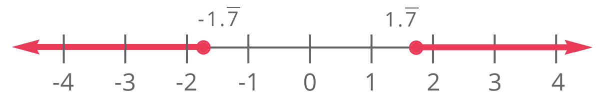
Model 4: The graph of $-\pi < y \leq 0$ for even integers is



Model 5: The graph of $|z| = 1.\overline{7}$ is



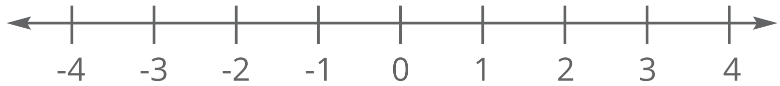
Model 6: The graph of $|z| \geq 1.\overline{7}$ is



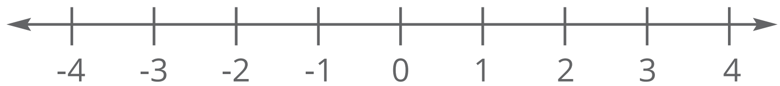
Model 7: The graph of $|z| < 1.\overline{7}$ is

Draw the graph for each of the following conditions.

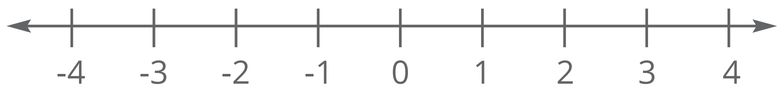
1.76 $x \leq 3\frac{1}{3}$



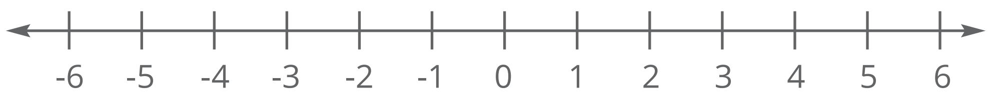
1.77 $x > -2.2$



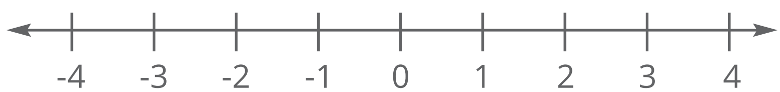
1.78 $-1 < y \leq \pi$



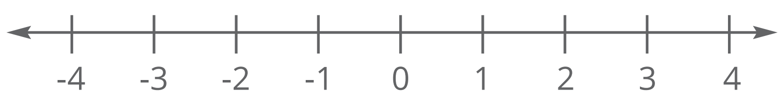
1.79 $|y| = 5\frac{3}{8}$



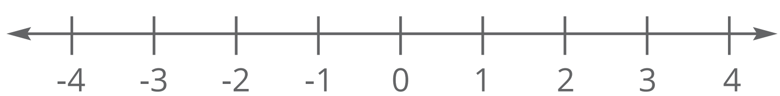
1.80 $|n| \leq 3$



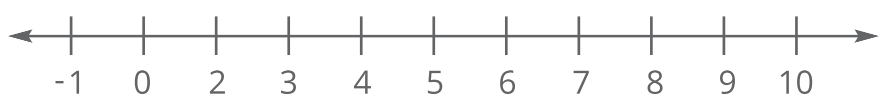
1.81 $|n| \leq 3$ for odd integers



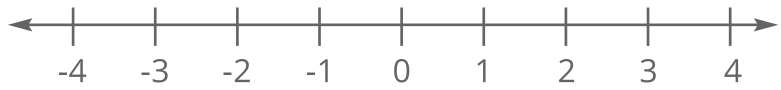
1.82 $|n| > 3$



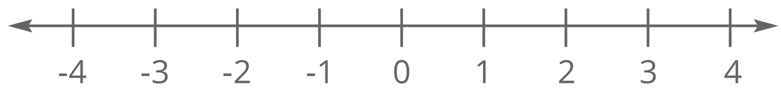
1.83 $a \geq \sqrt{71}$



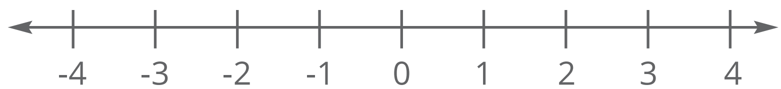
1.84 $-2 \leq t < 2$



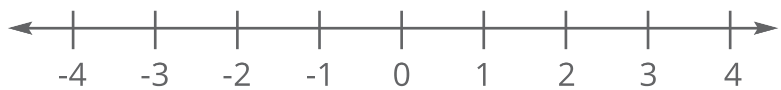
1.85 $-2 \leq t < 2$ for integers



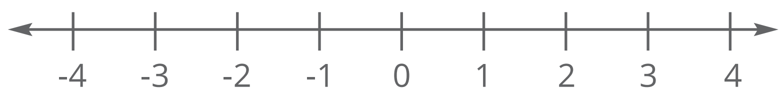
1.86 $1 < |k| < 4$ for integers



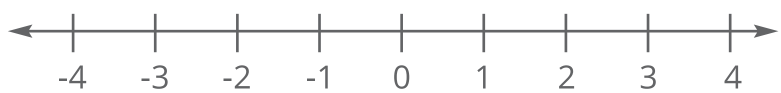
1.87 $1 < |k| < 4$



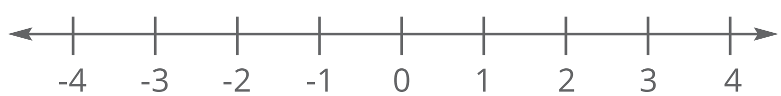
1.88 $|z| = \sqrt{5}$



1.89 $|z| = -\sqrt{5}$



1.90 $-3\frac{1}{2} < b < 0.\overline{6}$



Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific area where restudy is needed for mastery.

SELF TEST 1

Convert each fraction to its equivalent decimal form, (each answer, 3 points).

1.01 $\frac{2}{5}$ = ____________

1.02 $-\frac{16}{33}$ = ____________

1.03 $\frac{19}{16}$ = ____________

Convert each decimal to its equivalent reduced fraction form (each answer, 3 points).

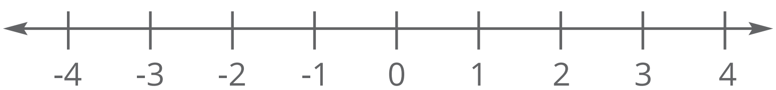
1.04 -0.72 = ____________

1.05 $0.\overline{72}$ = ____________

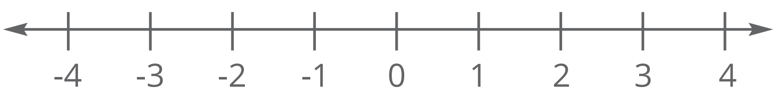
1.06 $0.7\overline{2}$ = ____________

Graph the given rational numbers (each location, 1 point).

1.07 $-3\frac{1}{3}$, 0, 1.6



1.08 2.4, $-\frac{5}{4}$, $3.\overline{8}$



Write the order of the given rational numbers (each numbered item, 3 points).

1.09 $-3, \frac{1}{3}, 0.3$ ____________ < ____________ < ____________

1.010 $5.\overline{40}, (\frac{7}{3})^2, 5\frac{2}{5}$ ____________ < ____________ < ____________

Find the rational number (each answer, 3 points).

1.011 One-sixth of the way between 7 and 16.

1.012 0.8 of the way between -0.5 and $4\frac{1}{2}$.

Tell whether each infinite decimal is rational or irrational (each answer, 2 points).

1.013 -7.234567... ______________________

1.014 -7.232323... ______________________

Round each number to the nearest hundredth (each answer, 2 points).

1.015 $0.\overline{61}$ ____________ **1.016** 0.292292229... ____________

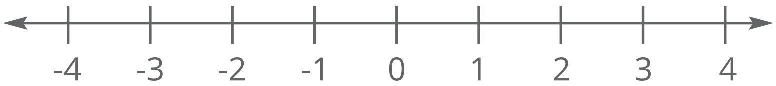
a. Find each square root; b. then tell whether each is rational or irrational (each answer, 3 points).

1.017 $\sqrt{\frac{25}{49}}$ = a.________ b.______________________

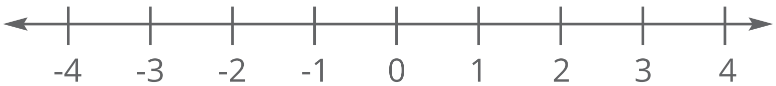
1.018 $\sqrt{32.49}$ = a.________ b.______________________

Draw the graph of each condition (each graph, 3 points).

1.019 $y < 1.\overline{8}$



1.020 $|h| \geq \sqrt{13}$



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