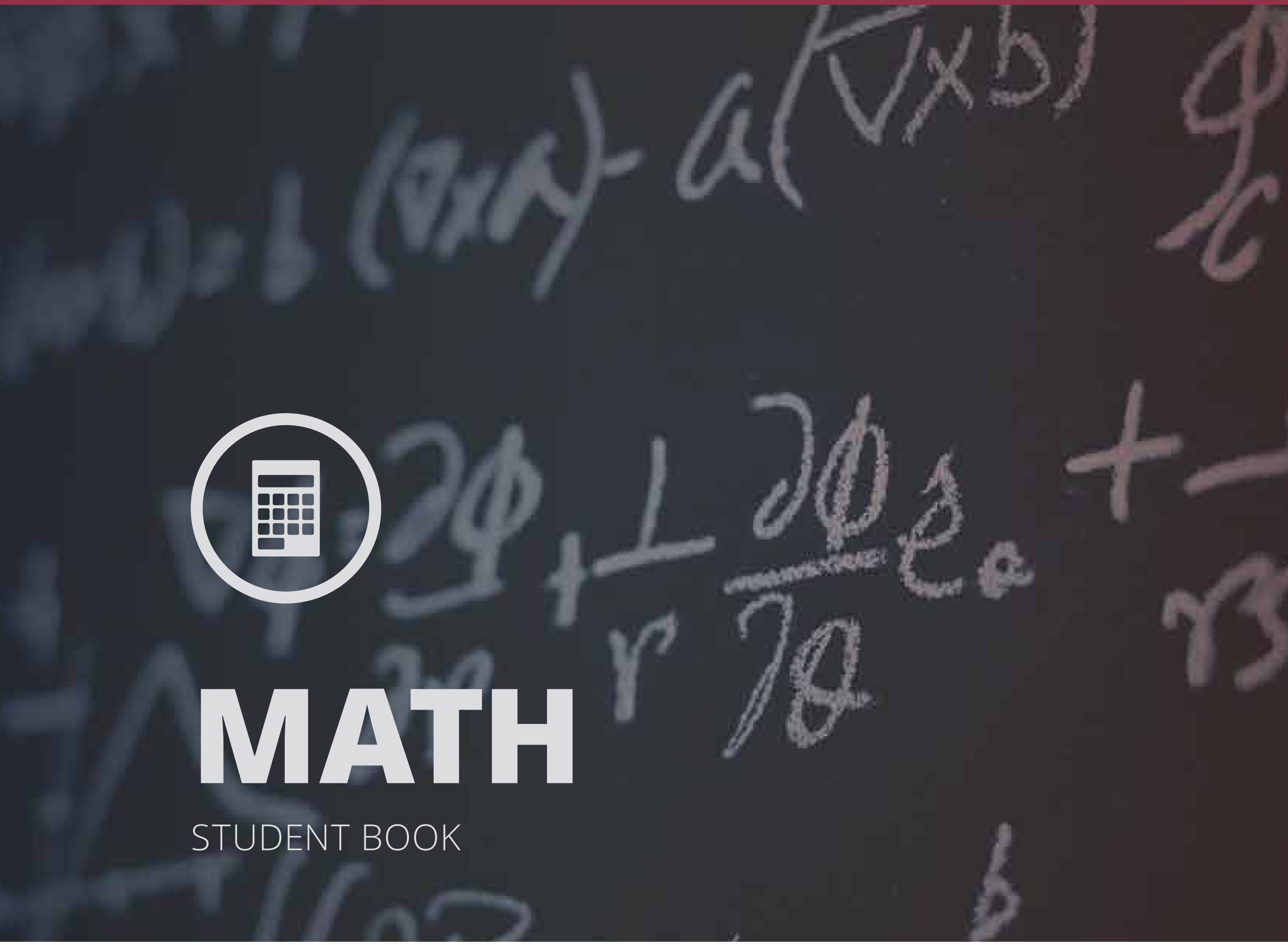
LIFEPAC®



MATH

STUDENT BOOK

9th Grade | Unit 8

Alpha Omega PUBLICATIONS

Math 908

Graphing

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Graphing

INTRODUCTION

In this LIFEPAC® you will continue your study in the mathematical system of algebra by learning about graphing. After seeing how two variables are used, you will learn the various techniques for showing the solutions to open sentences on the real-number plane. Finally, you will learn to write the equations of lines drawn in this plane.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Find ordered-pair solutions for two-variable equations.
2. Locate points on the real-number plane.
3. Translate verbal statements to equations.
4. Draw the graphs for linear equations, linear inequalities, and open sentences involving absolute values.
5. Find the equations of lines from given information.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. USING TWO VARIABLES

In this first section you will learn the introductory concepts and definitions needed for graphing: solving two-variable equations, plotting points on the real-number plane, and translating verbal sentences to equations.

OBJECTIVES

1. Find ordered-pair solutions for two-variable equations.
2. Locate points on the real-number plane.
3. Translate verbal statements to equations.

EQUATIONS

You have already learned to find numerical answers for equations having one variable, such as:
$x + 2 = 5$, $3m + 2 = 7$, and $4 - |t| = 1.2$
For example, you know that the equation, $x + 2 = 5$, has exactly one integral solution, $x = 3$; but what about the equation, $x + y = 5$? Is $x = 3$ still a solution? Is 3 the only value for x that will give a solution? Let us investigate further.

The equation, $x + y = 5$, indicates that the sum of x and y is five. If x is 3, then the sum of 3 and y must be five; therefore y must be 2. Thus, $x = 3$ is a solution only when $y = 2$. Now 3 and 2 are certainly not the only two numbers having a sum of five; 1 and 4, 5 and 0, and -23 and 28 are just three examples of other pairs of integers with sums of five. Certain pairs of rational numbers (such as $3\frac{1}{3}$ and $1.\overline{6}$) and irrational numbers (such as $\sqrt{3} - 8$ and $13 - \sqrt{3}$) also have a sum of five. In fact, infinitely many real number solutions exist to the equation $x + y = 5$.

When an equation contains two variables, you must look for a relationship between those variables rather than just for the value(s) of a single variable. The solutions for such equations will be pairs of numbers that make true sentences. These solutions are called *ordered pairs* since the numbers are written in the alphabetical order of the two variables.

Model 1: Find several ordered–pair solutions for $x + y = 5$

Solution: The ordered pairs are written as (x, y) since x is before y in the alphabet:

$(3, 2)$, $(1, 4)$, $(5, 0)$, $(-23, 28)$, $(3\frac{1}{3}, 1.\overline{6})$, $(\sqrt{3} - 8, 13 - \sqrt{3})$.

You should notice that if the numbers are reversed in the solutions of Model 1, the resulting ordered pairs will also be solutions of $x + y = 5$. This situation is not always true.

Model 2: Find three solutions for $2a - b = 1$.

Solution: You may use any real values you wish for one of the variables (usually the one nearer the beginning of the alphabet). Substitute each of the chosen values in the equation and solve for the remaining variable.

Suppose we choose 5, $\frac{1}{2}$, and 0 for a:

$$a = 5: \quad \begin{aligned} 2 \cdot 5 - b &= 1 \\ 10 - b &= 1 \\ b &= 9 \end{aligned}$$

$$a = \frac{1}{2}: \quad \begin{aligned} 2 \cdot \frac{1}{2} - b &= 1 \\ 1 - b &= 1 \\ b &= 0 \end{aligned}$$

$$a = 0: \quad \begin{aligned} 2 \cdot 0 - b &= 1 \\ 0 - b &= 1 \\ b &= -1 \end{aligned}$$

$\therefore$ Three ordered-pair solutions of $2a - b = 1$ are (5, 9), ($\frac{1}{2}$, 0), and (0, -1). They are in the order (a, b).

These ordered pairs cannot be reversed and still be solutions for $2a - b = 1$.

(9, 5) is not a solution since $2 \cdot 9 - 5 \neq 1$.

(0, $\frac{1}{2}$) is not a solution since $2 \cdot 0 - \frac{1}{2} \neq 1$.

(-1, 0) is not a solution since $2(-1) - 0 \neq 1$.

You must be very careful to put your pairs of numbers in the correct order when writing solutions to two-variable equations. The ordered pairs that make an equation true are said to *satisfy* that equation.

Complete the following activities.

Find the value of y for the given value of x in each of the following equations. Then write the ordered pairs.

1.1 $x + y = 10$

x	y	(x, y)
0		
2		
4		

1.2 $x - y = 8$

x	y	(x, y)
10		
8		
6		

Complete the ordered–pair solutions for each of the following equations.

1.3 $2x + y = 6$

$A =$ {(1, ____), (0, ____), (-1, ____)... }

1.4 $\frac{x}{3} + y = 15$

$B =$ {(0, ____), (3, ____), (6, ____)... }

1.5 $y = 2x - 3$

$C =$ {(-2, ____), (0, ____), (2, ____)... }

Find three ordered–pair solutions for each of the following equations.

1.6 $x - y = 1$ a. ____________ b. ____________ c. ____________

1.7 $x + y = -1$ a. ____________ b. ____________ c. ____________

1.8 $2x - y = 7$ a. ____________ b. ____________ c. ____________

1.9 $x + 2y = 0$ a. ____________ b. ____________ c. ____________

1.10 $y - 3x + 1 = 0$ a. ____________ b. ____________ c. ____________

Sometimes you may wish to find solutions by first changing the form of the original equation so that the variable nearer the end of the alphabet is written alone on one side of the equation.

Model 1: Solve $y - 2x = 7$ for y.

Solution: $y - 2x = 7$

$y = 7 + 2x$ or $y = 2x + 7$

Model 2: Solve $a + 2b = 5$ for b.

Solution:

$$a + 2b = 5$$
$$2b = 5 - a$$
$$b = \frac{5 - a}{2} \text{ or } b = \frac{-a + 5}{2}$$

Model 3: Solve $m - \frac{n}{2} = 1$ for n.

Solution:

$$2\left[m - \frac{n}{2}\right] = 2[\,1\,]$$
$$2m - n = 2$$
$$-n = -2m + 2$$
$$-1[\,-n\,] = -1[\,-2m + 2\,]$$
$$n = 2m - 2$$

Solve each of the following equations for the variable indicated.

1.11 $3x + y = 1$ for y: ______________________

1.12 $x + 2y = -6$ for y: ______________________

1.13 $3a + 2b = 6$ for b: ______________________

1.14 $\frac{2r}{3} - 3s = 10$ for s: ______________________

1.15 $5x - 2y = 11$ for y: ______________________

VOCABULARY

domain—for a two–variable equation, the *domain* is the set of numbers to be used for the first (alphabetical) variable.

REMEMBER? The elements of a set are listed between braces: { }. The symbol $\in$ means *is an element of the set*. Sets are often named with capital letters.

Model 1: Find the ordered pairs that satisfy the equation $3m + 2n = 7$ when the domain of m is $\{-5, 0, 3\frac{2}{3}\}$.

Solution: First solve for n:

$$3m + 2n = 7$$
$$2n = 7 - 3m$$
$$n = \frac{7 - 3m}{2}$$

Then complete the table:

m	-5	0	$3\frac{3}{2}$ or $\frac{11}{3}$
$\frac{7 - 3m}{2}$	$\frac{7 - 3(-5)}{2}$ $\frac{7 + 15}{2}$ $\frac{22}{2}$	$\frac{7 - 3 \cdot 0}{2}$ $\frac{7 - 0}{2}$ $\frac{7}{2}$	$\frac{7 - 3 \cdot \frac{11}{3}}{2}$ $\frac{7 - 11}{2}$ $\frac{-4}{2}$
n	11	$3\frac{1}{2}$	-2

∴ The ordered pairs are $(-5, 11)$, $(0, 3\frac{1}{2})$, and $(3\frac{2}{3}, -2)$.

Model 2: Find the ordered pairs that satisfy the equation $4s - |t| = 1.2$ when $s \in \{-2, 0.3, 1\}$.

Solution: Solve for $|t|$:

$$4s - |t| = 1.2$$
$$-|t| = 1.2 - 4s$$
$$|t| = -1.2 + 4s$$
$$\text{or } |t| = 4s - 1.2$$

Complete the table:

s	-2	0.3	1
$4s - 1.2$	$4(-2) - 1.2$ $-8 - 1.2$	$4(0.3) - 1.2$ $1.2 - 1.2$	$4(1) - 1.2$ $4 - 1.2$
$\|t\|$	-9.2	0	2.8
t	no values (since $\|t\|$ cannot be negative)	0	-2.8 or 2.8

∴ $(0.3, 0)$, $(1, -2.8)$, and $(1, 2.8)$ are solutions of $4s - |t| = 1.2$.

a. Solve each of the following equations for *y*;
b. find the ordered pairs that satisfy the equation for the given domain.

1.16 $x - y = 1$ $x \in \{-1, 0, 1\}$

a. $y =$ ________________ b. (-1, ___), (0, ___), (1, ___)

1.17 $2x + y = 10$ $x \in \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$

a. $y =$ ________________ b. ________________

1.18 $\frac{3x}{5} + 2y = -1$ $x \in \{0, 5, 10\}$

a. $y =$ ________________ b. ________________

1.19 $7x - 2y + 2 = 0$ $x \in \{0.5 \ \ 1.5 \ \ 2.5\}$

a. $y =$ ________________ b. ________________

1.20 $\frac{x}{2} + \frac{y}{3} = \frac{1}{5}$ $x \in \{\frac{1}{3}, \frac{-1}{3}, 0\}$

a. $y =$ ________________ b. ________________

Find three ordered–pair solutions for each of the following equations by selecting three convenient elements of the domain.

1.21 $y = \frac{3x}{2} - 4$ ________________

1.22 $2x + 3y = 5$ ________________

1.23 $\frac{2x}{7} - \frac{y}{2} = 7$ ________________

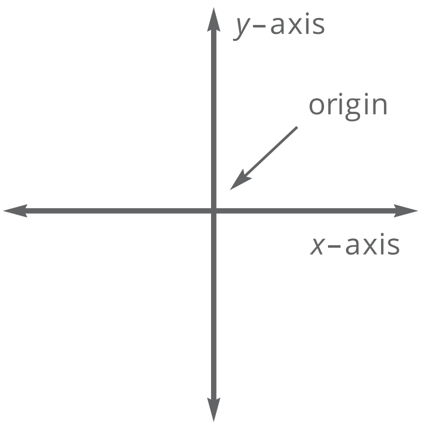
1.24 $|x| + y = 6$ ________________

1.25 $|x| + |y| = 1$ ________________

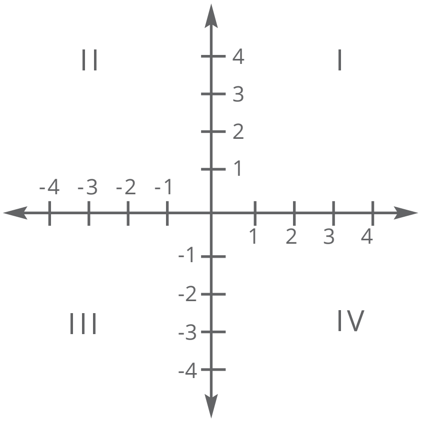
THE REAL–NUMBER PLANE

In Mathematics LIFEPAC 907, you learned to graph the solution points of one-variable equations on the *real-number line*. In this LIFEPAC you will be graphing the solution points of two-variable equations on the *real-number plane*. First, however, you need to learn the terminology and procedures of graphing.

Two reference lines or *axes* are drawn in the plane, one horizontally and one vertically, meeting at a common zero point called the *origin*. Each axis is a number line for one of the two variables. Since *x* and *y* are the letters used most frequently, the horizontal axis is known as the *x-axis* and the vertical axis is known as the *y-axis*.

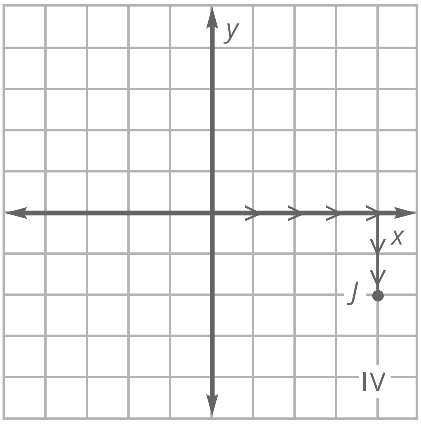


On the *x*-axis, positive numbers are to the right of the origin, and negative numbers are to the left of the origin. On the *y*-axis, positive numbers are above the origin, and negative numbers are below the origin. The axes separate the plane into four regions called *quadrants*, which are labeled with Roman numerals as indicated in the diagram.

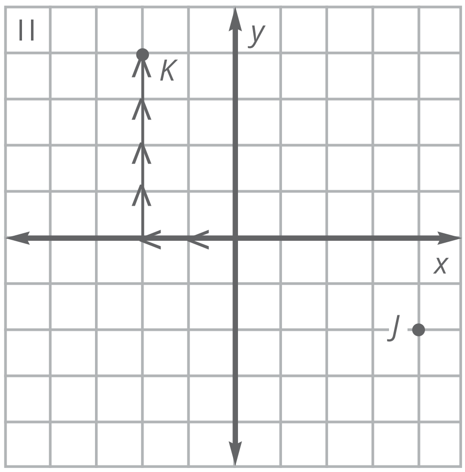


An ordered pair of numbers in the form (*x*, *y*) is used to locate any point in the plane. The value of *x* indicates the horizontal direction and distance of the point from the origin, and the value of *y* indicates the vertical direction and distance of the point from the origin. Of course, the ordered pair (0, 0) represents the origin itself.

Suppose we wish to locate the point corresponding to the ordered pair (4, -2) on the plane at the right. (*NOTE*: A complete grid of intersecting lines is used so that points may be found more easily and accurately.) The first number indicates that the point is four units to the right of the origin; the second number indicates that the point is two units below the origin. Thus, to find the point (4, -2), begin at the origin and move four units right then two units down. You arrive at point *J* in Quadrant IV; a heavy dot is used to show (or *plot*) the point on the plane.



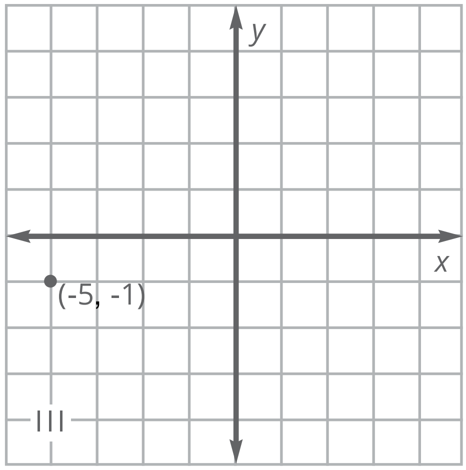
The order of the numbers written in the pair, as well as the order of movements from the origin, is very important. To see this fact, notice that point *K* corresponding to the ordered pair (-2, 4) is in Quadrant II and certainly is not the same point as *J*.



Model 1: Plot the point corresponding to the ordered pair (-5, -1) and describe its location.

Solution: Begin at the origin and move five units left then one unit down.

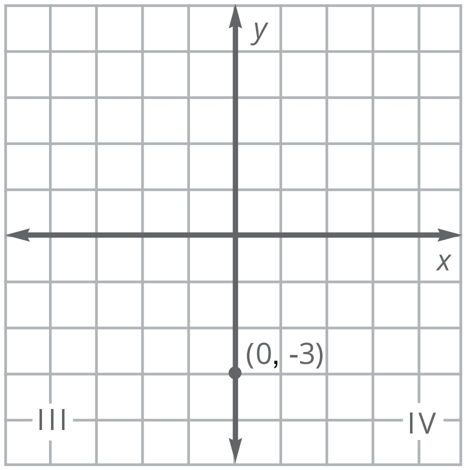
This point is located in Quadrant III.



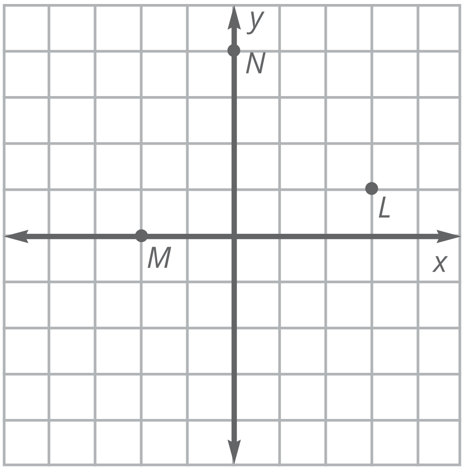
Model 2: Plot the point for (0, -3) and describe its location.

Solution: The first value of 0 indicates that no horizontal movement is to be made. Thus, the point corresponding to (0, -3) is three units below the origin on the *y*-axis.

This point is located between Quadrants III and IV.



Model 3: Describe the locations and name the ordered pairs corresponding to points *L*, *M*, and *N* in the diagram.

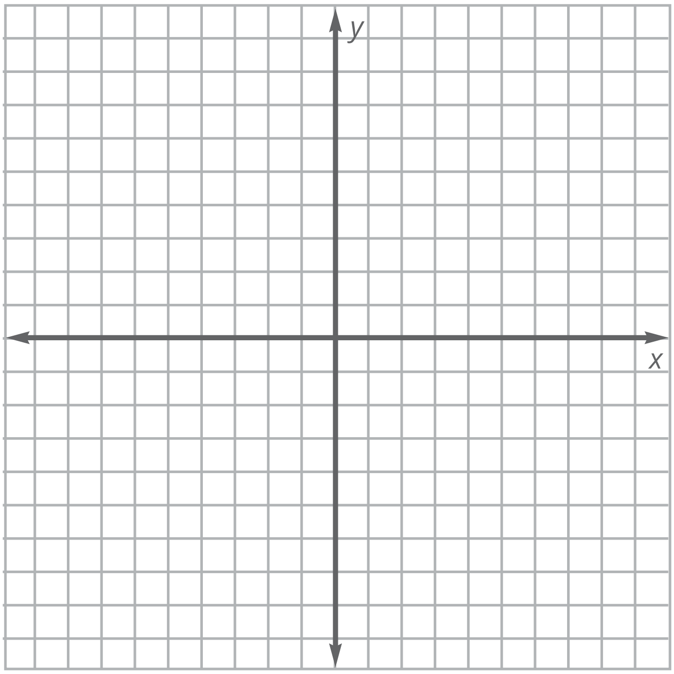


Solution: Point *L*—beginning at the origin, move 3 units right then 1 unit up; thus, the ordered pair is (3, 1).

Point *M*—beginning at the origin, move 2 units left then 0 units vertically; thus, the ordered pair is (-2, 0).

Point *N*—beginning at the origin, move 0 units horizontally then 4 units up; thus, the ordered pair is (0, 4).

Plot and label the point corresponding to each given ordered pair; then describe its location.



1.26 (4, 3) ____________________

1.27 (3, 4) ____________________

1.28 (-2, 5) ____________________

1.29 (-5, 3) ____________________

1.30 (-2, -1) ____________________

1.31 (-6, -3) ____________________

1.32 (1, -3) ____________________

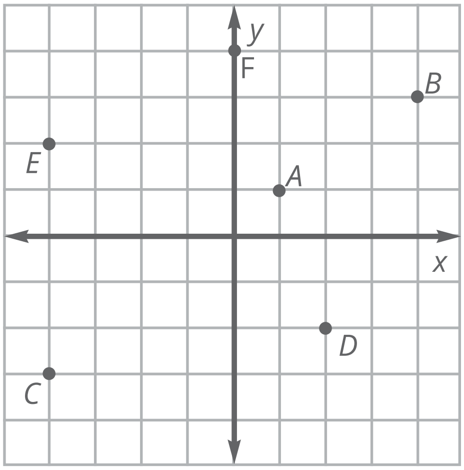
1.33 (3, -3) ____________________

1.34 (0, 4) ____________________

1.35 (4, 0) ____________________

1.36 (0, -2) ____________________

a. Name the ordered pair corresponding to each point given on the graph;
b. describe its location.



1.37 a. Point *A* ______________________

b. ______________________

1.38 a. Point *B* ______________________

b. ______________________

1.39 a. Point *C* ______________________

b. ______________________

1.40 a. Point *D* ______________________

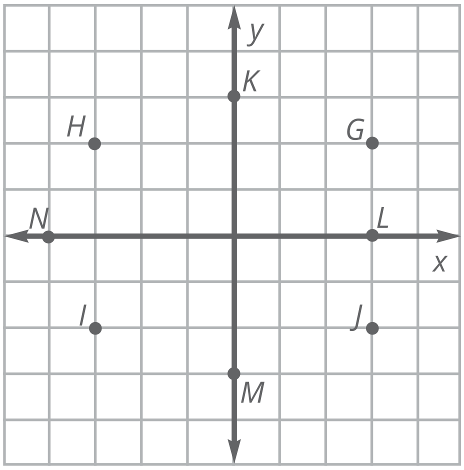
b. ______________________

1.41 a. Point *E* ______________________

b. ______________________

1.42 a. Point *F* ______________________

b. ______________________



1.43 a. Point *G* ______________________

b. ______________________

1.44 a. Point *H* ______________________

b. ______________________

1.45 a. Point *I* ______________________

b. ______________________

1.46 a. Point *J* ______________________

b. ______________________

1.47 a. Point *K* ______________________

b. ______________________

1.48 a. Point *L* ______________________

b. ______________________

1.49 a. Point *M* ______________________

b. ______________________

1.50 a. Point *N* ______________________

b. ______________________

TRANSLATIONS

Now let us see how verbal statements can be translated to two-variable equations and then how points corresponding to their ordered-pair solutions can be graphed. First you need two definitions.

VOCABULARY

abscissa—the first number (or *x*-value) of an ordered pair.

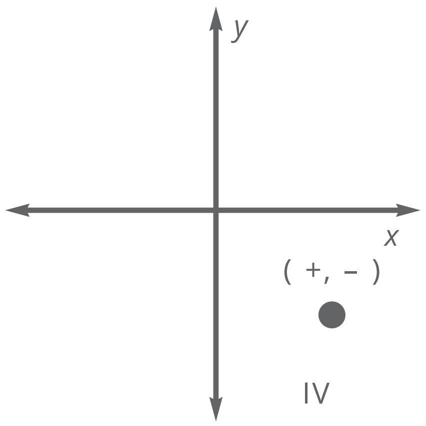
ordinate—the second number (or *y*-value) of an ordered pair.

coordinates—the abscissa and the ordinate together.

Model 1: Points having positive abscissas and negative ordinates are all found in which quadrant?

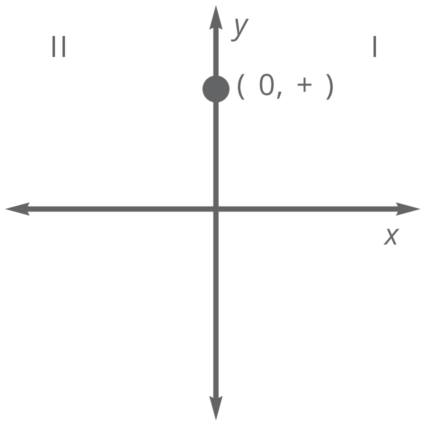
Solution: A positive abscissa indicates a positive *x*-value, and a negative ordinate indicates a negative *y*-value. Thus, the ordered pairs of these points are of the form (+, -). Beginning at the origin, move to the right, then downward.

∴ All (+, -) points are in Quadrant IV.



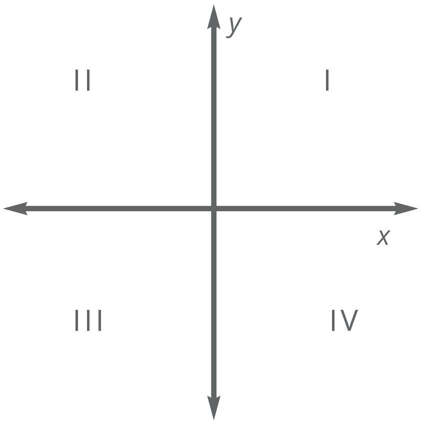
Model 2: Describe the location of all points having an abscissa of zero and a positive ordinate.

Solution: The ordered pairs of these points are of the form (0, +) and are found between Quadrants I and II.



Describe the location of all points having the given coordinates.

Quadrants:



Model: Positive abscissa, nonzero ordinate

Solution: Points are (+, +), (+, –) **Quadrants I & IV**

1.51 Positive abscissa, positive ordinate ____________________

1.52 Negative abscissa, negative ordinate ____________________

1.53 Zero abscissa, negative ordinate ____________________

1.54 Positive abscissa, zero ordinate ____________________

1.55 Negative abscissa, positive ordinate ____________________

1.56 Zero abscissa, zero ordinate ____________________

1.57 Negative abscissa, zero ordinate ____________________

1.58 Nonzero abscissa, positive ordinate ____________________

1.59 Nonzero abscissa, negative ordinate ____________________

1.60 Negative abscissa, nonzero ordinate ____________________

1.61 Abscissa and ordinate have the same sign ____________________

1.62 Abscissa and ordinate have opposite signs ____________________

Any time the word *abscissa* appears in a verbal statement, it may be translated to the letter x. Any time the word *ordinate* appears in a verbal statement, it may be translated to the letter y. Study the following models to see how a relationship expressed verbally can be translated to an equation.

Model 1: The ordinate is four less than the abscissa.

Solution: THE ORDINATE IS FOUR LESS THAN THE ABSCISSA

y $=$

$x - 4$ is the translation.

Model 2: The sum of twice the abscissa and the ordinate is 13.

Solution: $2x + y = 13$ is the translation.

For each of the following sentences, write a translation using *x* and *y*.

1.63 The ordinate is two more than the abscissa. ______

1.64 The ordinate is three less than twice the abscissa. ______

1.65 The sum of the abscissa and the ordinate is six. ______

1.66 The difference between the ordinate and the abscissa is two. ______

1.67 Twice the abscissa increased by three times the ordinate is ten. ______

1.68 The ordinate exceeds half the abscissa by two. ______

Once a verbal statement has been translated to a two-variable equation, ordered-pair solutions can be found, as shown previously in this LIFEPAC. The points corresponding to these ordered pairs can then be graphed on the real-number plane.

Model 1: Graph three points for "the ordinate is four less than the abscissa."

Solution: The translation is $y = x - 4$. First choose three values for x; then substitute each value into the equation and solve for y.

If you select $x = 3$: $y = 3 - 4 = -1$

If you select $x = 0$: $y = 0 - 4 = -4$

If you select $x = 5$: $y = 5 - 4 = 1$

Checks:

(3, -1)

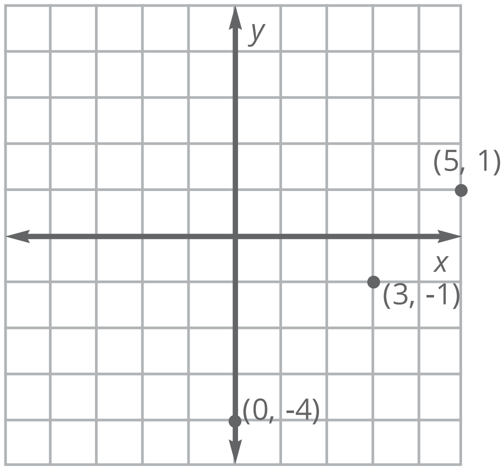
$$y = x - 4$$
$$-1 \stackrel{?}{=} 3 - 4$$
$$-1 \ \checkmark$$

(0, -4)

$$y = x - 4$$
$$-4 \stackrel{?}{=} 0 - 4$$
$$-4 \ \checkmark$$

(5, 1)

$$y = x - 4$$
$$1 \stackrel{?}{=} 5 - 4$$
$$1 \ \checkmark$$



Finally, plot the points for (3, -1), (0, -4), and (5, 1).

As shown in Model 1, when the domain is not given, then you may use any real values for x that you desire; you may wish to choose values that will result in ordered pairs whose points are easy to plot. If the domain of x is given, then you must use those specified values.

Model 2: For $x \in \{-3, 0, 5\}$, graph the points whose ordered pairs correspond to "the difference of the abscissa and twice the ordinate is seven."

Solution: The translation is $x - 2y = 7$. First solve this equation for y:

$$x - 2y = 7$$
$$-2y = 7 - x$$
$$y = \frac{7 - x}{-2}$$

Then, using the given domain, find the three ordered pairs:

x	-3	0	5
$\frac{7 - x}{-2}$	$\frac{7 - (-3)}{-2}$ $\frac{10}{-2}$	$\frac{7 - 0}{-2}$ $\frac{7}{-2}$	$\frac{7 - 5}{-2}$ $\frac{2}{-2}$
y	-5	-3.5	-1
(x, y)	(-3, -5)	(0, -3.5)	(5, -1)

Checks: (-3, -5)

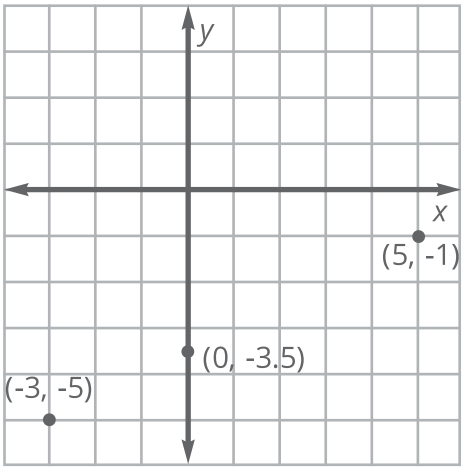
$$x - 2y = 7$$
$$-3 - 2(-5) \stackrel{?}{=} 7$$
$$-3 + 10$$
$$7 \surd$$

(0, -3,5)

$$x - 2y = 7$$
$$0 - 2(-3{,}5) \stackrel{?}{=} 7$$
$$0 + 7$$
$$7 \surd$$

(5, -1)

$$x - 2y = 7$$
$$5 - 2(-1) \stackrel{?}{=} 7$$
$$5 + 2$$
$$7 \surd$$

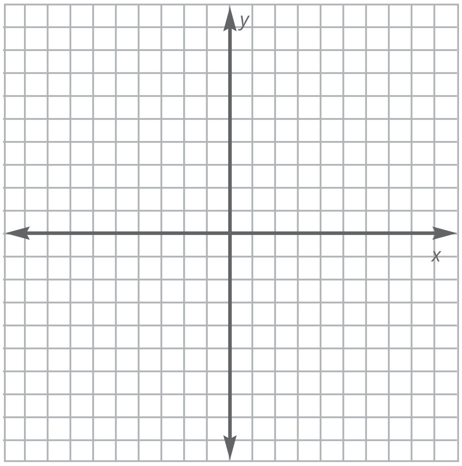


Finally, plot the points.

For each of the following problems, find three ordered pairs and graph them.

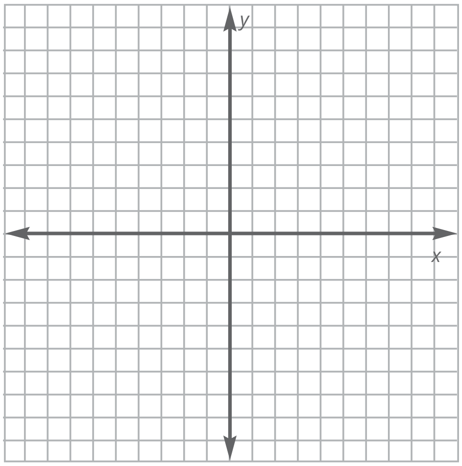
1.69 $x + y = 6$ and $x \in \{0, 1, -1\}$

______________ ______________ ______________



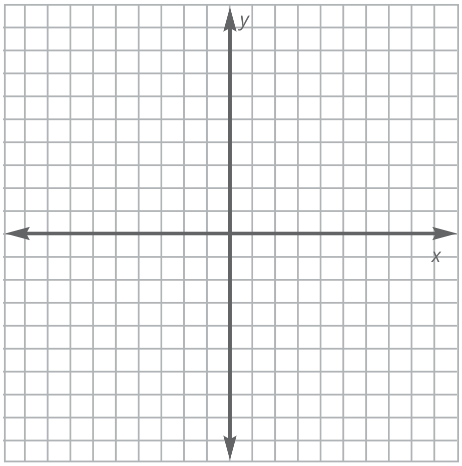
1.70 $y = 2x - 3$ and $x \in \{0, -1, 2\}$

______________ ______________ ______________



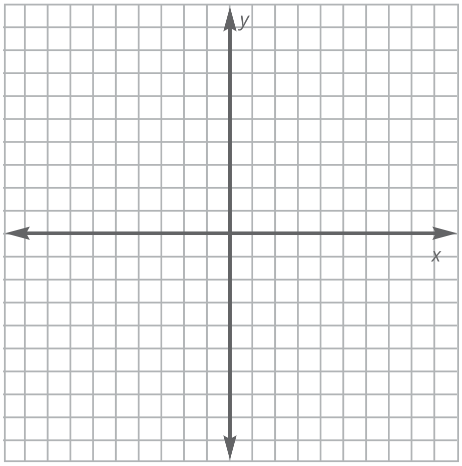
1.71 The ordinate equals the abscissa, and $x \in \{-3, 0, 3\}$.

______________ ______________ ______________



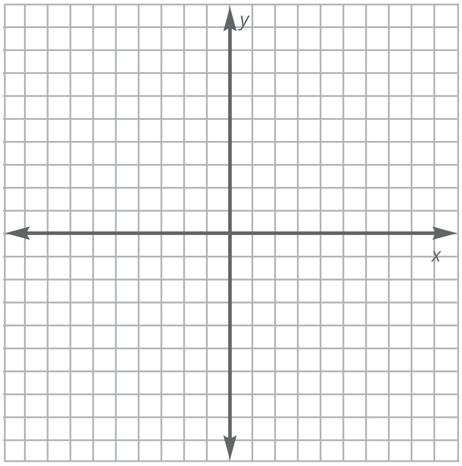
1.72 The ordinate is three more than twice the abscissa.

____________ ____________ ____________



1.73 Half the ordinate increased by one-third the abscissa equals six.

____________ ____________ ____________



Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific area where restudy is needed for mastery.

SELF TEST 1

For each of the following points, describe its location on a grid (each answer, 3 points).

1.01 (6, 2) ______________________________

1.02 (-3, 5) ______________________________

1.03 (0, 1) ______________________________

1.04 (0, 0) ______________________________

1.05 (-6, -6) ______________________________

Name the ordered pair corresponding to each point on the graph (each answer, 3 points).

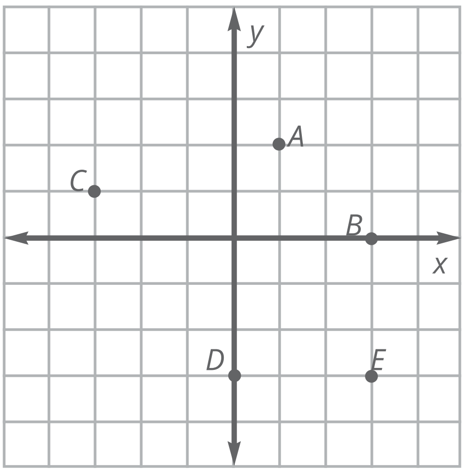
1.06 Point *A* ______________

1.07 Point *B* ______________

1.08 Point *C* ______________

1.09 Point *D* ______________

1.010 Point *E* ______________



For each of the following sentences, write a translation using *x* **and** *y* (each answer, 3 points).

1.011 The ordinate is one-half the abscissa. ______________________________

1.012 The abscissa less the ordinate is one. ______________________________

1.013 The product of the abscissa and ordinate is ten. ______________________________

For each of the following equations, solve for *y* (each answer, 3 points).

1.014 $x + y = 6$ ____________________

1.015 $\frac{x}{2} - y + 6 = 0$ ____________________

1.016 $2x + 3y = 10$ ____________________

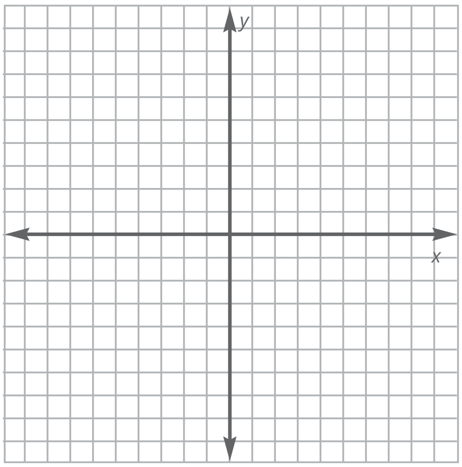
For each of the following equations,
a. solve for *y***;**
b. find three ordered pairs; and
c. graph the points (a. 3 points; b. 3 points; c. 4 points).

1.017 The ordinate is twice the abscissa.

a. ______________________________

b. __________ __________ __________

c.

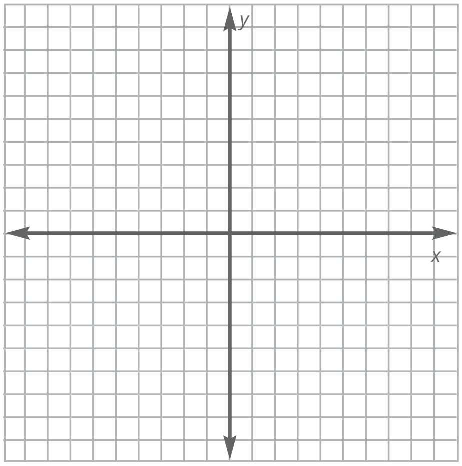


1.018 $x + y = 0$

a. ______________________________

b. __________ __________ __________

c.

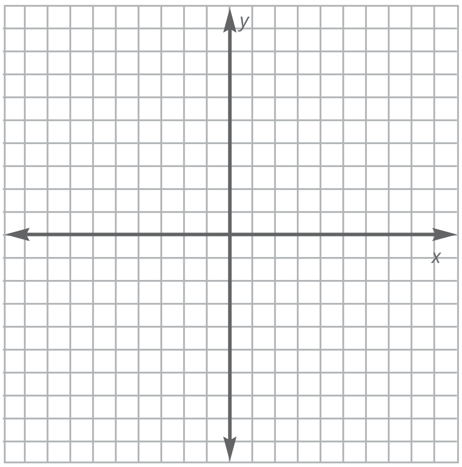


1.019 $x + \frac{y}{2} = 1$ and $x \in \{-2, 0, 2\}$

a. ____

b. ____________ ____________ ____________

c.

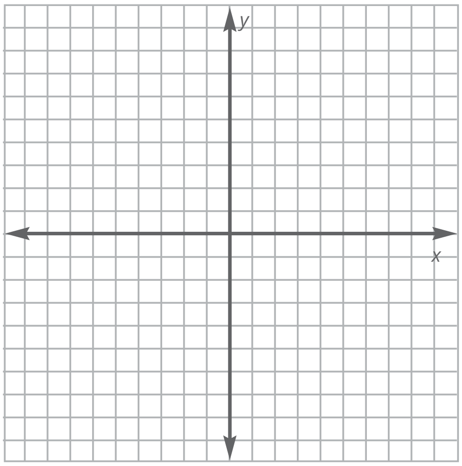


1.020 $3x - 2y = -1$

a. ____

b. ____________ ____________ ____________

c.



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