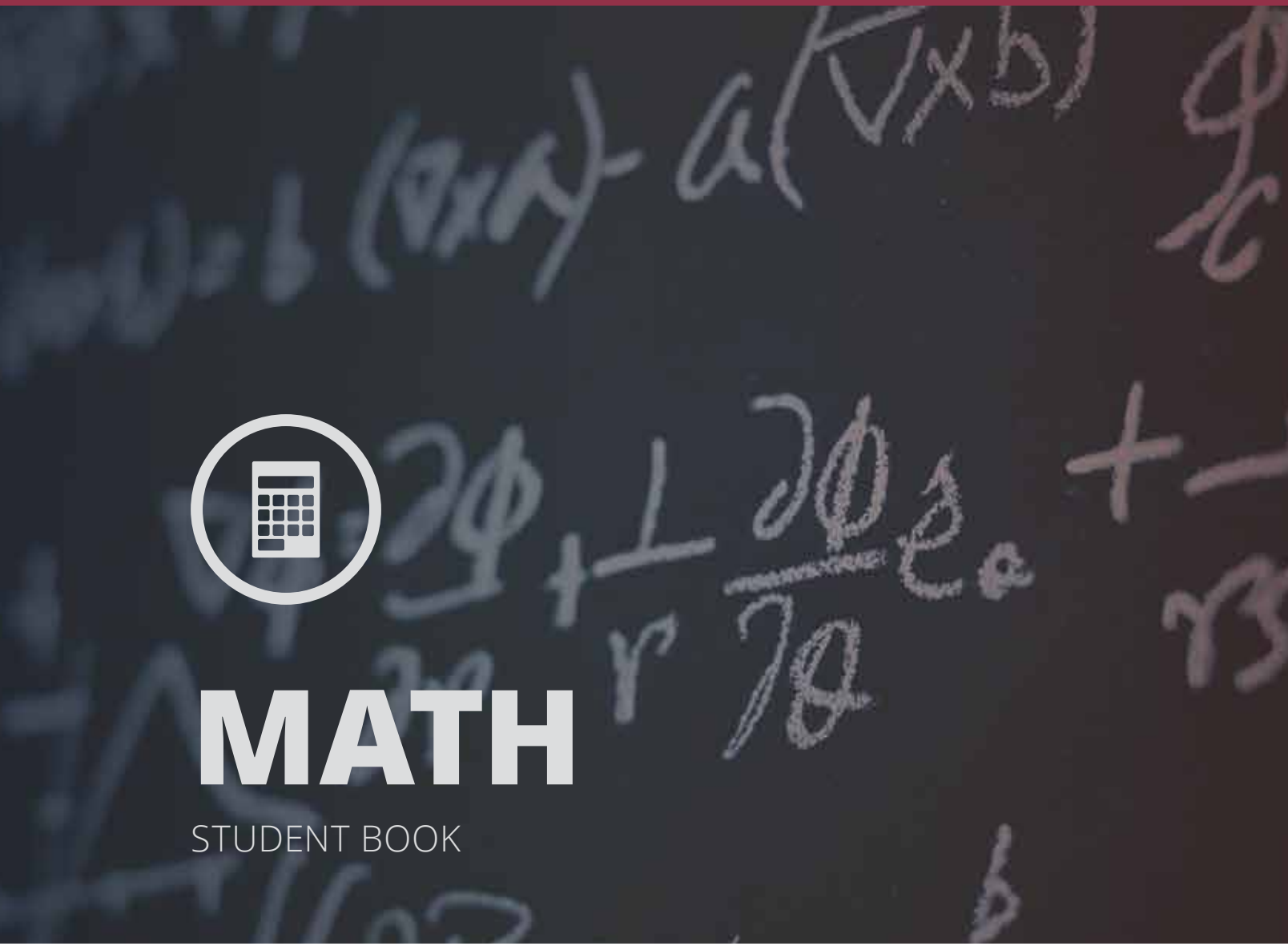
LIFEPAC®



MATH

STUDENT BOOK

9th Grade | Unit 9

Alpha Omega
PUBLICATIONS

Math 909

Systems

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Systems

INTRODUCTION

In this LIFEPAC®, you will continue your study in *algebra* by learning to find any common solutions to groups of open sentences called *systems*—first graphically using the techniques of the preceding LIFEPAC, then algebraically using several different methods. Finally, you will see how systems can be set up to solve verbal problems of various types.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Identify the equations of systems as consistent, equivalent, or inconsistent.
2. Solve systems of linear equations and inequalities by graphing
3. Solve systems of linear equations by the methods of opposite coefficients, comparison, substitution, and determinants.
4. Solve problems using systems of linear equations.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. GRAPHICAL SOLUTIONS

You are already familiar with the procedures for drawing the graph of a linear equation or a linear inequality on the real-number plane. In this section, we shall graph more than one such open sentence on the same grid and then determine whether ordered pairs exist that satisfy the system. You will need to draw your graphs as accurately as possible.

OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Identify the equations of systems as consistent, equivalent, or inconsistent.
2. Solve systems of linear equations and inequalities by graphing.

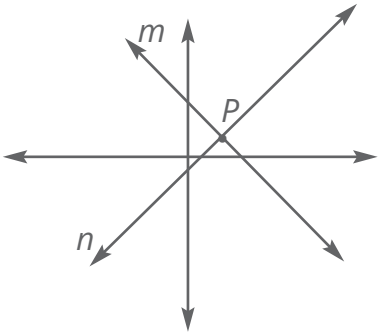
EQUATIONS

We shall begin by looking at systems made up of two-variable linear equations. You need to learn several terms and then learn the procedures for solving these systems graphically.

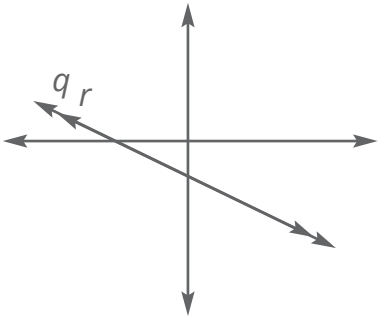
TERMINOLOGY

A system of two linear equations is classified by the number of ordered pairs that satisfy both equations. Since the graph of each linear equation is a line, three possible situations can occur. These cases are shown in the following models.

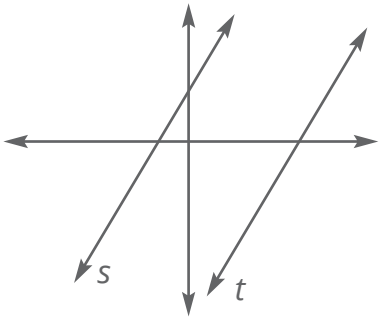
Model 1: Lines m and n *intersect* at one common point P.



Model 2: Lines *q* and *r* *coincide*, having all (infinitely many) common points.



Model 3: Lines *s* and *t* are *parallel*, having no common point.

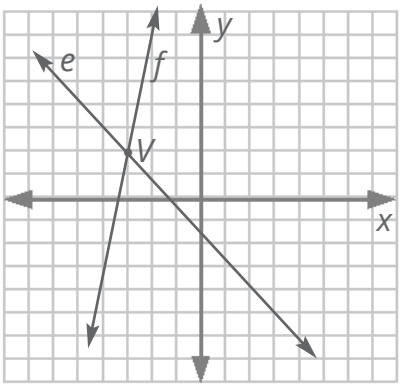


The equations of each of these three systems are identified as *consistent*, *equivalent*, and *inconsistent*, respectively. The set of the ordered pair(s) corresponding to any common point(s) is written as the *solution set* for each system.

VOCABULARY

consistent—In a system, equations having a common solution are consistent.

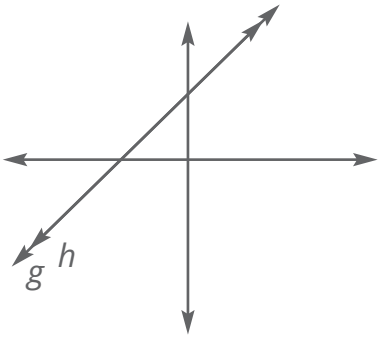
Model: The equations for lines *e* and *f* are consistent. The solution set for this system is {(-3, 2)}, since the common point is *V*.



VOCABULARY

equivalent—In a system, equations having all common solutions are equivalent.

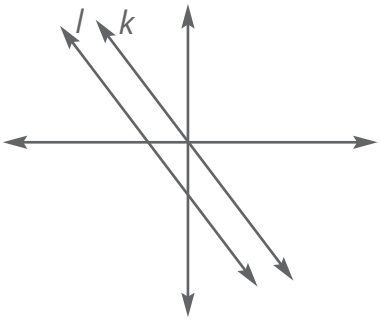
Model: The equations for lines *g* and *h* are equivalent. The solution set for this system is the infinite set of ordered pairs for all points on the line.



VOCABULARY

inconsistent— In a system, equations having no common solutions are inconsistent.

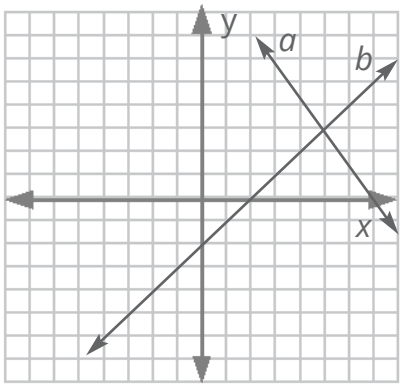
Model: The equations for lines *k* and *l* are inconsistent. The solution set for this system is ∅ (the empty set).



For each of the following pairs of lines, write a. the type of equations (consistent, equivalent, or inconsistent) and b. the solution set of each system.

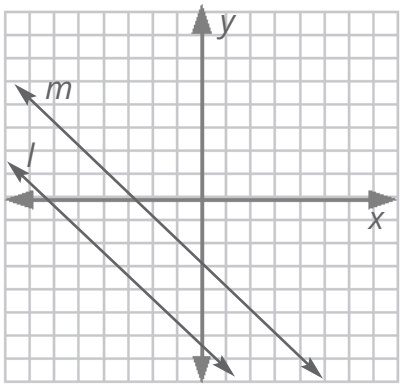
1.1 a._____

b._____



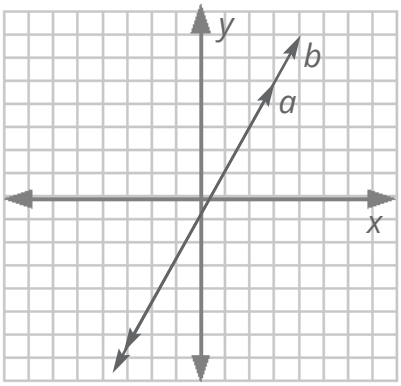
1.2 a._____

b._____



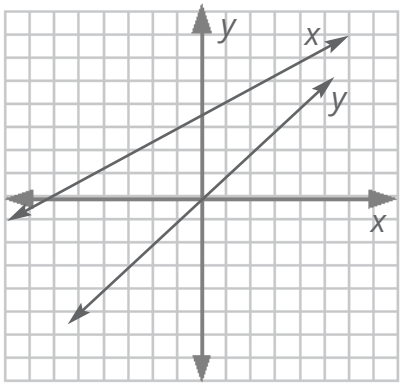
1.3 a._____

b._____



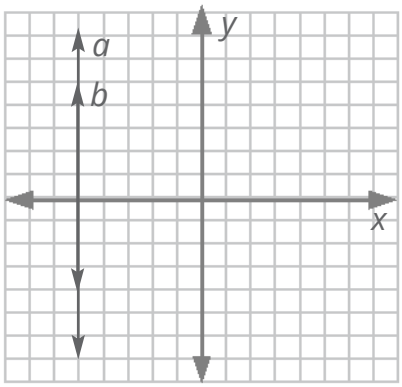
1.4 a._____

b._____



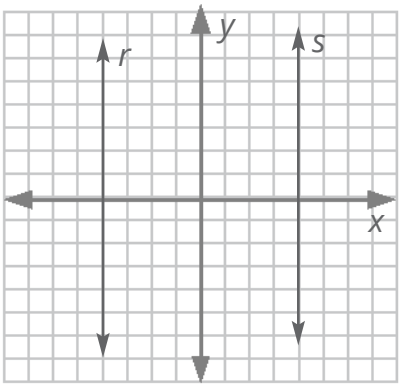
1.5 a._____

b._____



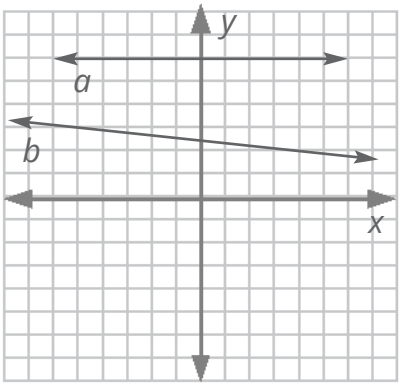
1.6 a._____

b._____



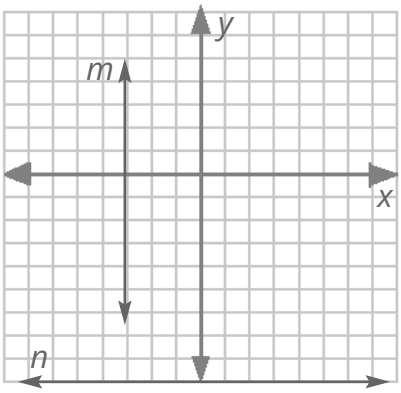
1.7 a._____

b._____



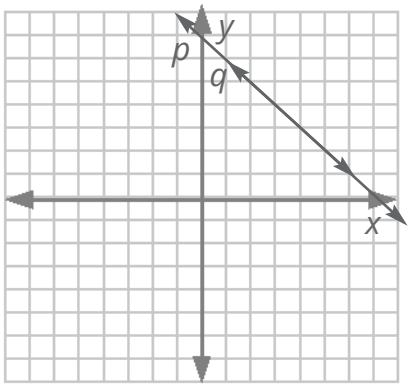
1.8 a._____

b._____



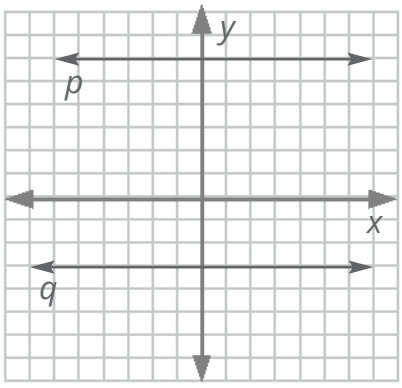
1.9 a._____

b._____



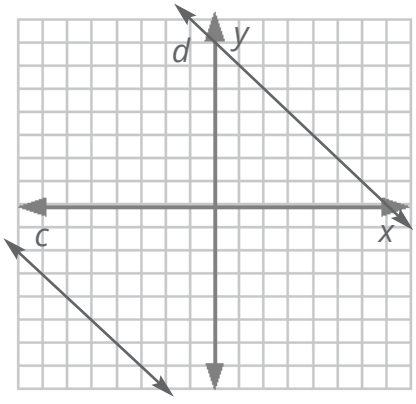
1.10 a._____

b._____



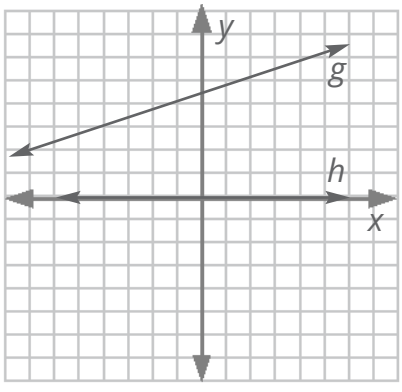
1.11 a._____

b._____



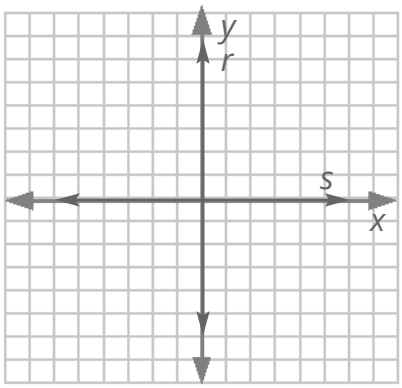
1.12 a._____

b._____



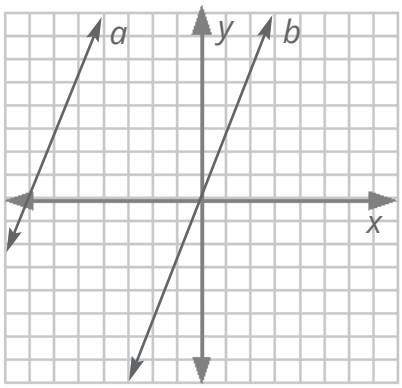
1.13 a._____

b._____



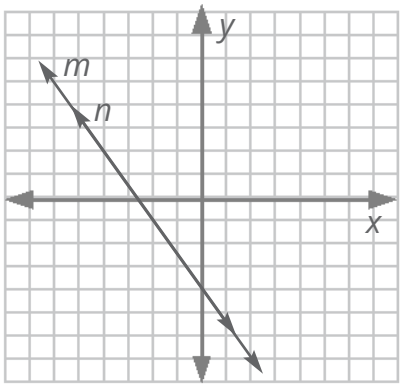
1.14 a._____

b._____



1.15 a._____

b._____



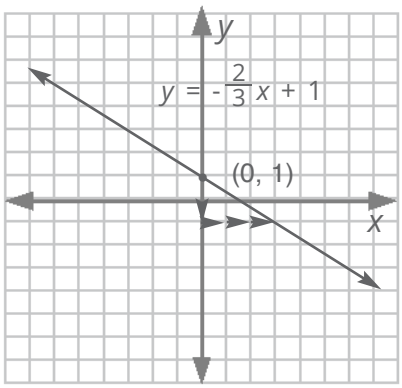
GRAPHS

Now we shall draw the graphs of linear equations to solve a system. This section will be a review of the techniques you learned in Mathematics LIFEPAC 908, except that you will be graphing two lines on the same number-plane grid.

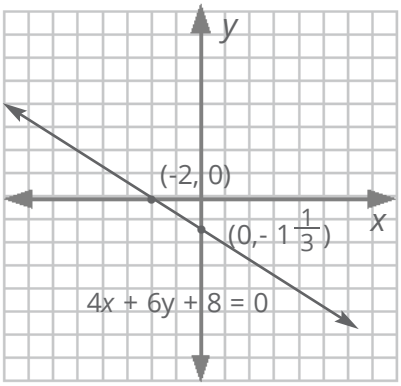
Model 1: Graph and describe the system

$$\begin{cases} y = -\frac{2}{3}x + 1 \\ 4x + 6y + 8 = 0. \end{cases}$$

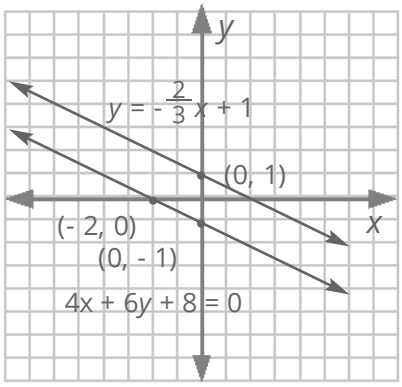
Solution: Step 1. The first equation, $y = -\frac{2}{3}x + 1$, is in slope, y-intercept form. Draw the line through (0, 1) and with a slope of $-\frac{2}{3}$.



Step 2. Use the intercepts method to graph the second equation, $4x + 6y + 8 = 0$. (When $y = 0$, the x-intercept is -2; and when $x = 0$, the y-intercept is $-\frac{4}{3}$ or $-1.\overline{3}$.)



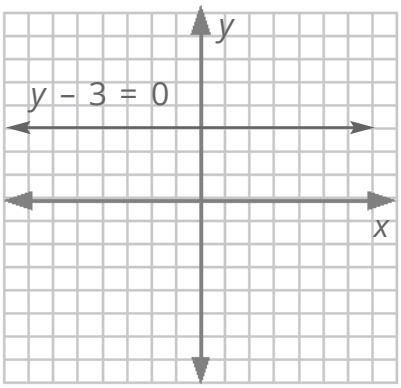
Step 3. Since the lines are parallel, the equations are inconsistent and the solution set of this system is $\varnothing$.



Model 2: Graph and describe the system

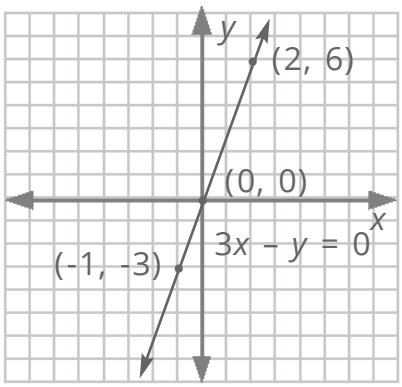
$$\begin{cases} y - 3 = 0 \\ 3x - y = 0. \end{cases}$$

Solution: Step 1. The first equation, $y - 3 = 0$, (or $y = 3$) gives the horizontal line 3 units above the x-axis.

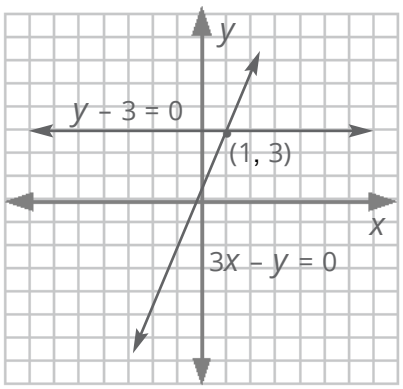


Step 2. You can set up a table of values to find points on the line for the second equation, $3x - y = 0$ (or $3x = y$).

x	2	-1	0
y	6	-3	0



Step 3. Since the lines intersect in one point, the equations are consistent and the solution set of this system is {(1, 3)}.



Checks:

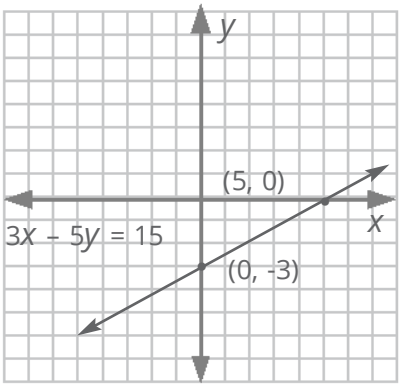
$y - 3 = 0$
$3 - 3 \stackrel{?}{} 0$
$0 = 0$ √

$3x - y = 0$
$3 \cdot 1 - 3 \stackrel{?}{} 0$
$3 - 3 \stackrel{?}{} 0$
$0 = 0$ √

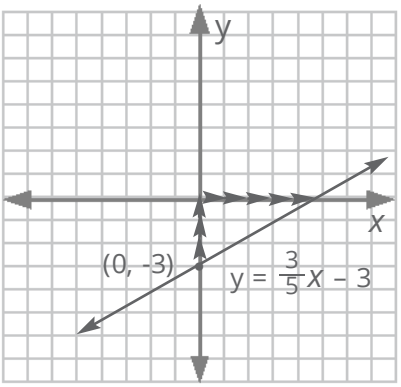
Model 3: Graph and describe the system

$$\begin{cases} 3x - 5y = 15 \\ y = \frac{3}{5}x - 3. \end{cases}$$

Solution: Step 1. Use the intercepts method to graph the first equation, $3x - 5y = 15$. (When $y = 0$, the x-intercept is 5; and when $x = 0$, the y-intercept is -3.)

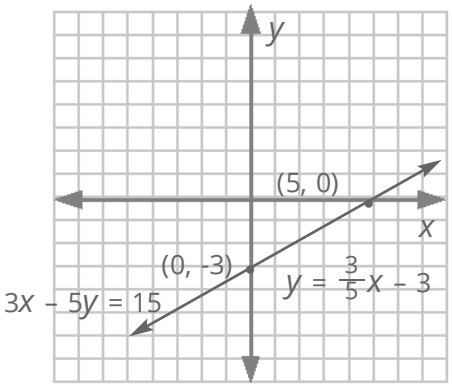


Step 2. The second equation, $y = \frac{3}{5}x - 3$, is in slope, y-intercept form. Draw the line through (0, -3) and with a slope of $\frac{3}{5}$.



Step 3. Since the lines coincide, the equations are equivalent and the solution set for this system is $\{(x, y): 3x - 5y - 15 = 0\}$.

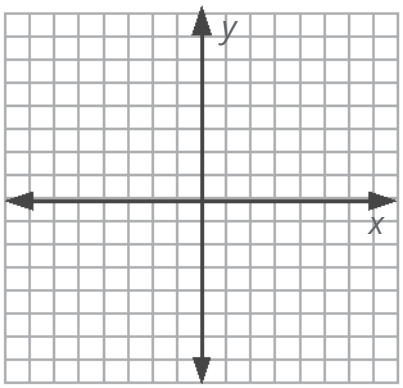
NOTE: The solution set for a system of equivalent equations is written $\{(x, y): Ax + By + C = 0\}$, where $Ax + By + C = 0$ is the general form of the equation for the lines that coincide.



a. Solve the following systems by graphing.
b. Identify the equations by type in each case.

1.16 $\begin{cases} x + y = 4 \\ x - y = 6 \end{cases}$

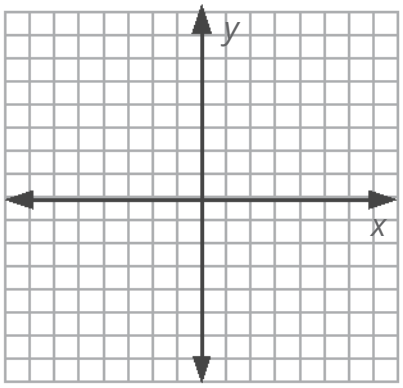
a. _____



b. _____

1.17 $\begin{cases} y = 2x - 3 \\ x + y = 3 \end{cases}$

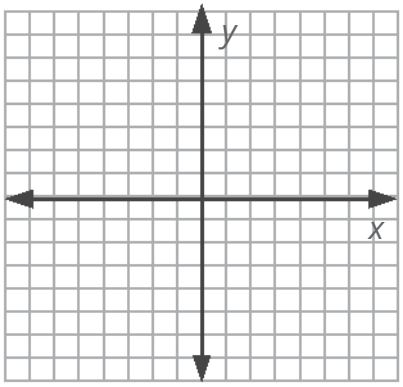
a. _____



b. _____

1.18 $\begin{cases} x + y = 6 \\ y = 3 - x \end{cases}$

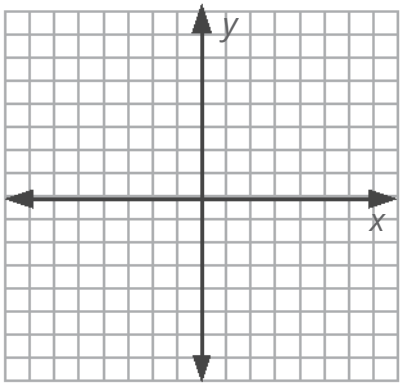
a. _____



b. _____

1.19 $\begin{cases} x + y - 4 = 0 \\ x - y = 0 \end{cases}$

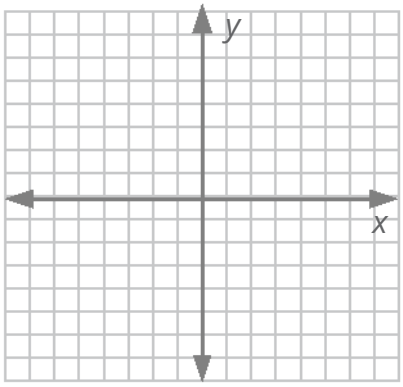
a. _____



b. _____

1.20 $\begin{cases} y = -x + 4 \\ 2x + 2y = 8 \end{cases}$

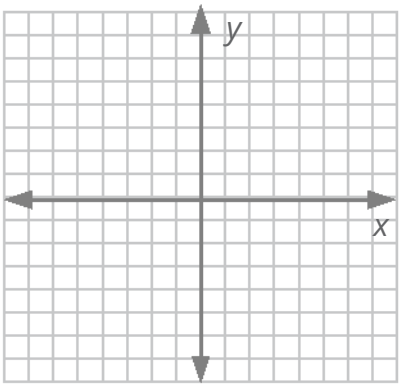
a. _____



b. _____

1.21 $\begin{cases} 2x + y = 2 \\ x - y = 1 \end{cases}$

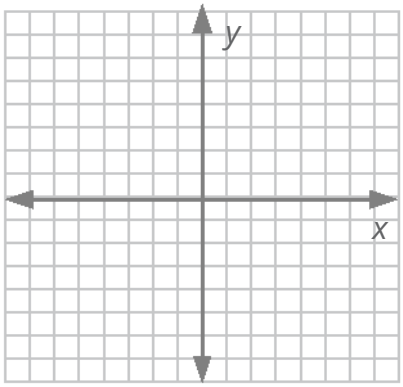
a. _____



b. _____

1.22 $\begin{cases} x + y = 2 \\ x = 1 \end{cases}$

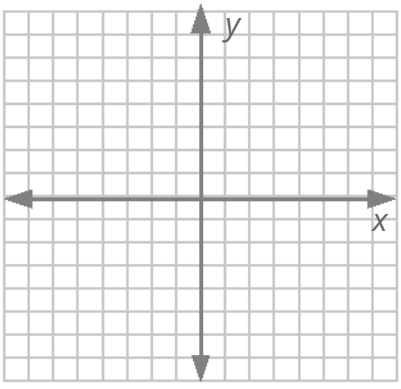
a. _____



b. _____

1.23 $\begin{cases} y - 4 = 0 \\ 2x - y - 2 = 0 \end{cases}$

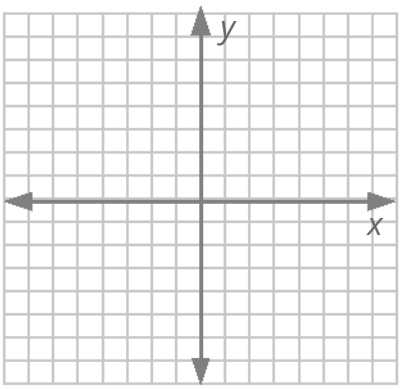
a. _____



b. _____

1.24 $\begin{cases} x = -5 \\ y = -6 \end{cases}$

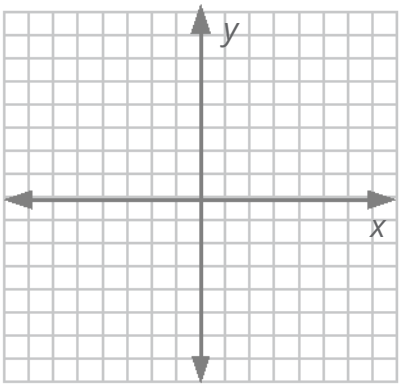
a._____



b._____

1.25 $\begin{cases} x + y = 0 \\ x - y + 2 = 0 \end{cases}$

a._____

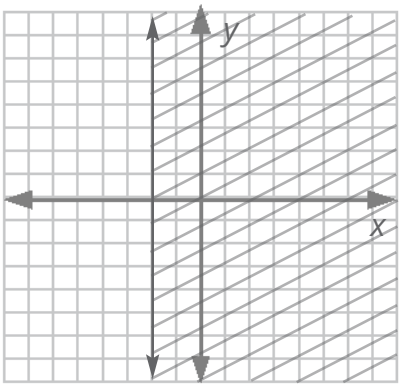


b._____

INEQUALITIES

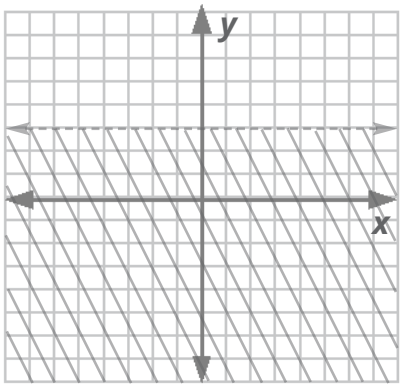
Systems made up of two (or more) linear inequalities can also be solved graphically by sketching the graphs on the same number-place grid. Study the following models carefully.

Model 1: Graph $\begin{cases} x \geq -2 \\ y < 3. \end{cases}$

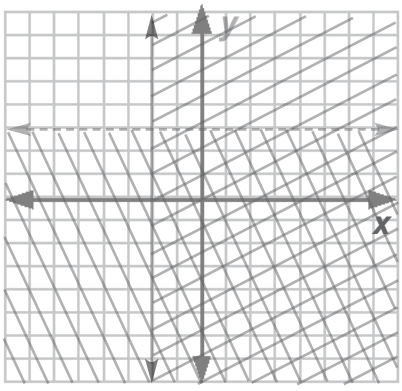


Solution: Step 1. Graph $x \geq -2$ by drawing the solid boundary ($x = -2$) and shading the half-plane to its right.

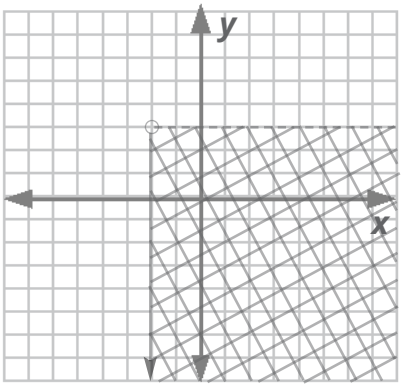
Step 2. Graph $y < 3$ by drawing the dashed boundary ($y = 3$) and shading the half-plane below it.



Step 3. Locate all points that are common to both inequalities — the cross-shaded region.

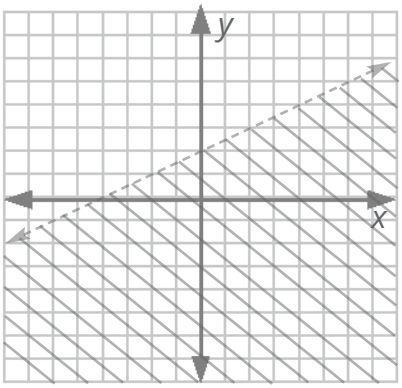


Step 4. Erase the shading of all noncommon points. Note: The point of intersection of the two boundary lines is indicated by a small circle in this case since it is not included in $y < 3$.

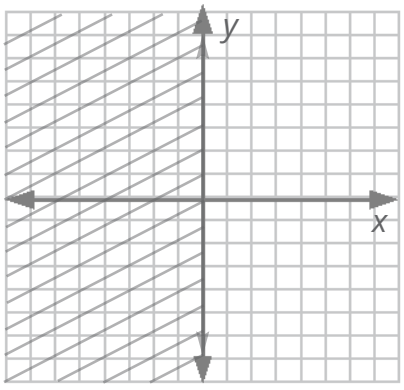


Model 2: Graph $\begin{cases} y < \frac{1}{2}x + 2 \\ x \leq 0 \\ x + y \geq -4. \end{cases}$

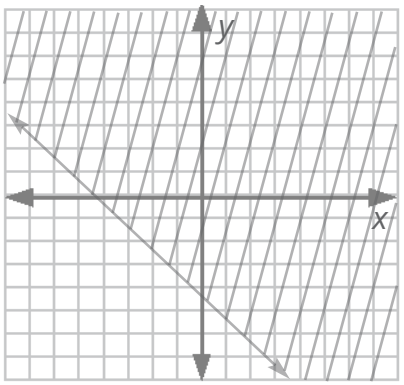
Solution: Step 1. Graph $y < \frac{1}{2}x + 2$ by using the slope, y-intercept form for the dashed boundary ($y = \frac{1}{2}x + 2$) and by testing points to determine the correct half-plane to shade.



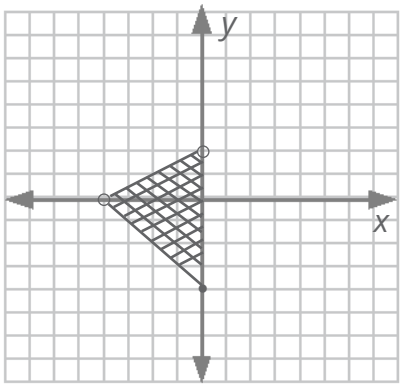
Step 2. Graph $x \leq 0$ by drawing the solid boundary ($x = 0$ is the y-axis) and shading the correct half-plane.



Step 3. Graph $x + y \geq -4$ by using the intercept method to draw the solid boundary ($x + y = -4$) and by testing points to determine the correct half-plane to shade.

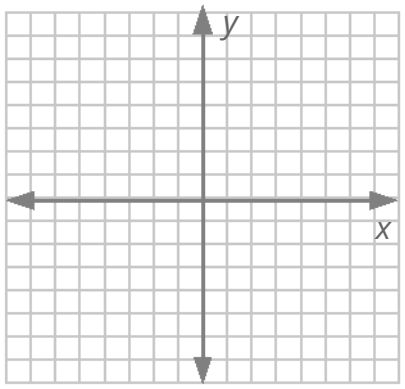


Step 4. Find the common points and erase the shading of all noncommon points. Note: One of the points of intersection of the boundary lines is common; the other two are not.

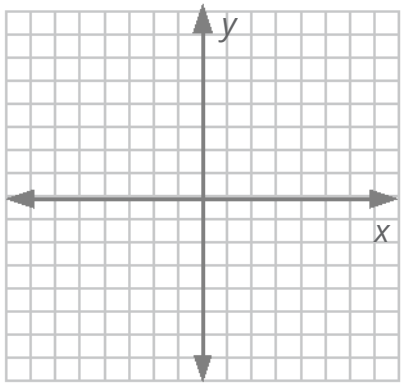


Graph the solution for each of the following linear inequality systems. Show only the final result for each system.

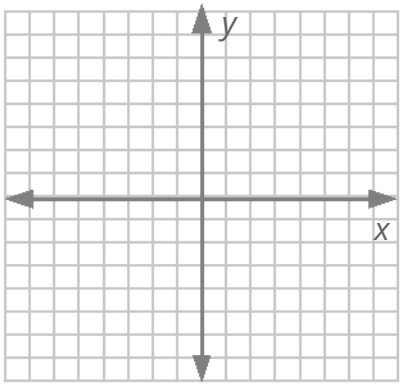
1.26 $\begin{cases} y > x \\ y < -x \end{cases}$



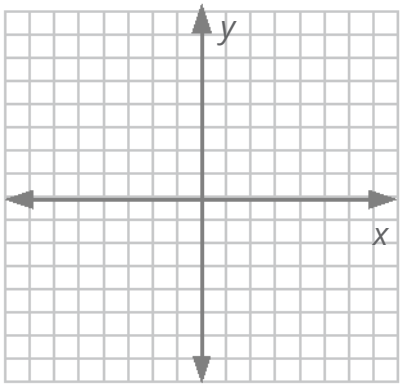
1.27 $\begin{cases} y > 5 \\ y \geq x \end{cases}$



1.28 $\begin{cases} y < 2x \\ 2x + y < -5 \end{cases}$

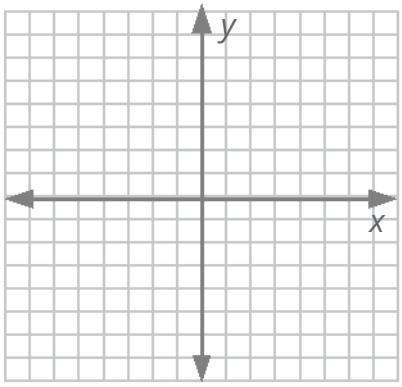


1.29 $\begin{cases} x + y \geq 6 \\ x \geq 4 \end{cases}$



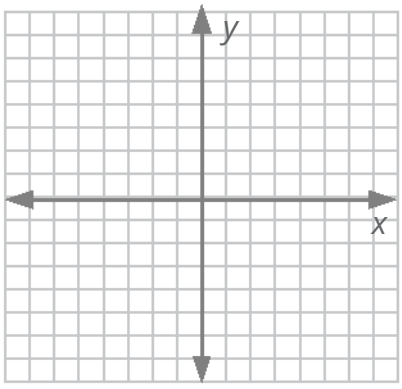
1.30

$$\begin{cases} y < 8 \\ y \geq 3 \end{cases}$$



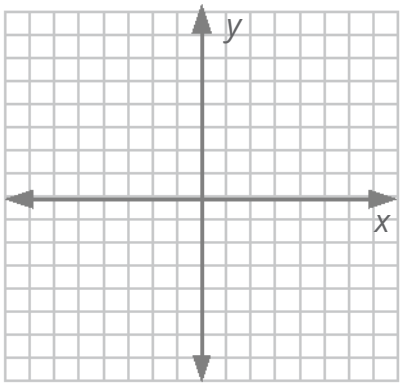
1.31

$$\begin{cases} y \geq 2x - 4 \\ 2x - y \geq -4 \end{cases}$$



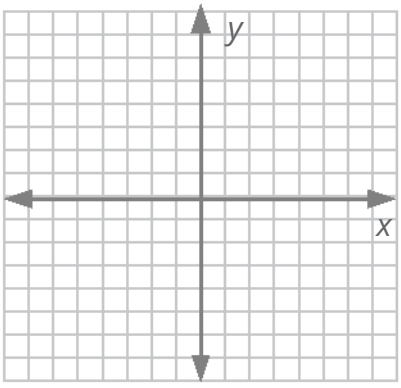
1.32

$$\begin{cases} x + y > 0 \\ x + y + 5 < 0 \end{cases}$$



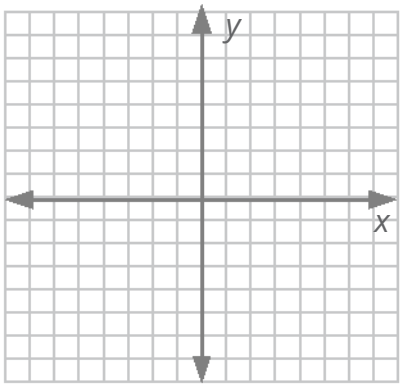
1.33

$$\begin{cases} x + 3 \leq 0 \\ x - 2 \geq 0 \end{cases}$$



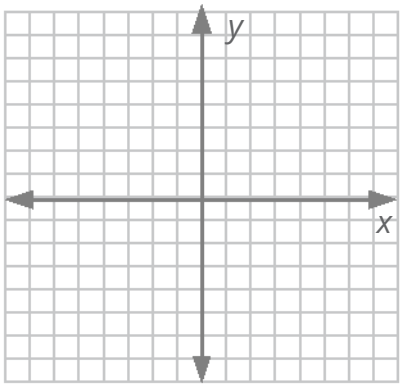
1.34

$$\begin{cases} y \geq 0 \\ y < x \\ x + y < 6 \end{cases}$$



1.35

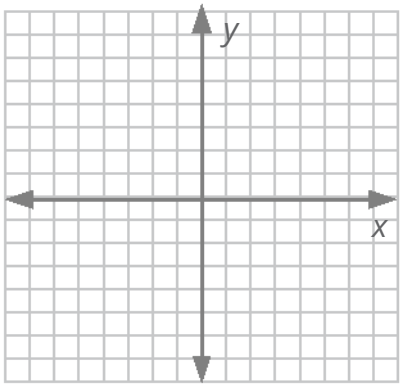
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y - 10 \leq 0 \end{cases}$$



Hint: Let each grid interval equal two units rather than one unit.

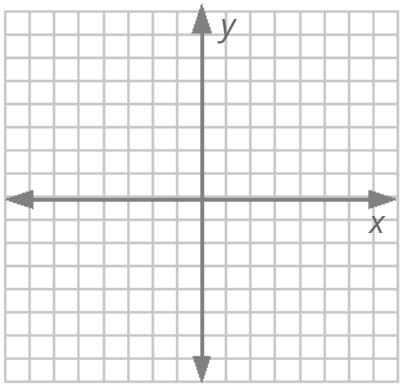
1.36

$$\begin{cases} x \geq -2 \\ x \leq 3 \\ y \leq 3 \\ y \geq -2 \end{cases}$$



1.37

$$\begin{cases} x + 5 \geq 0 \\ y - 4 \leq 0 \\ y + 4 \geq 0 \\ x + y \leq 7 \end{cases}$$



Hint: Let each grid interval equal two units rather than one unit.

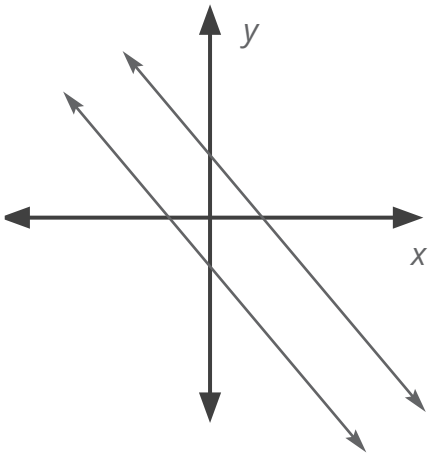
Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific area where restudy is needed for mastery.

SELF TEST 1

For each system of lines, a. write the type and b. find the solution set (each answer, 2 points).

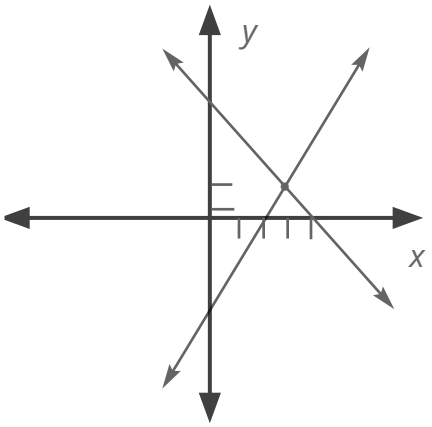
1.01 a._____

b._____



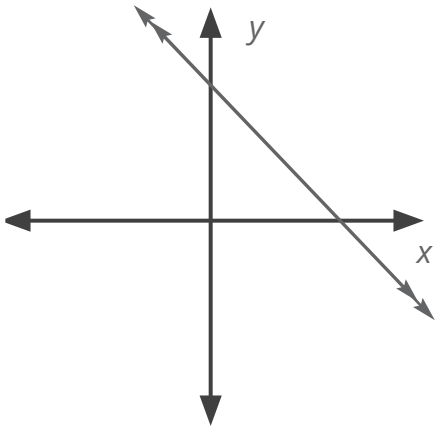
1.02 a._____

b._____



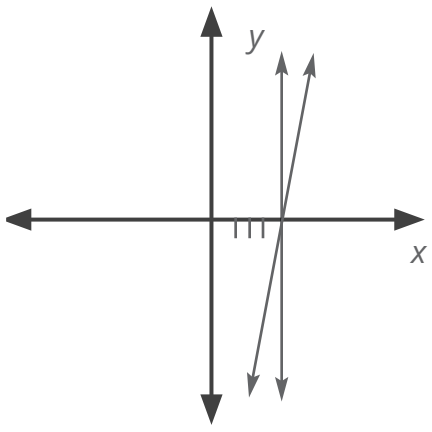
1.03 a._____

b._____



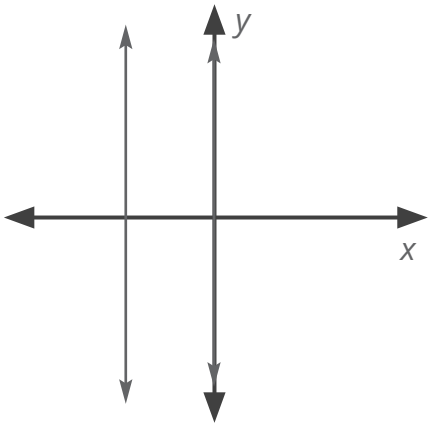
1.04 a._____

b._____.



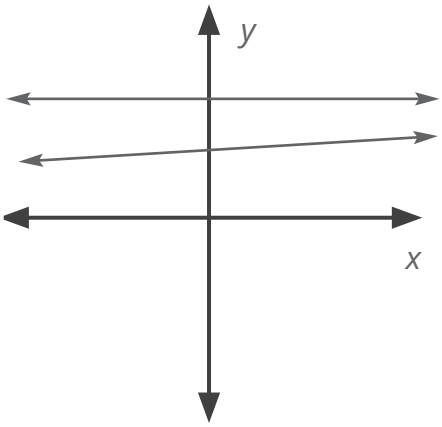
1.05 a._____

b._____



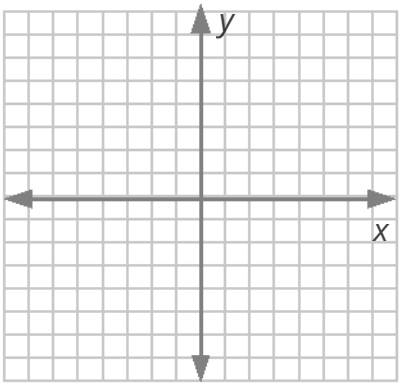
1.06 a._____

b._____

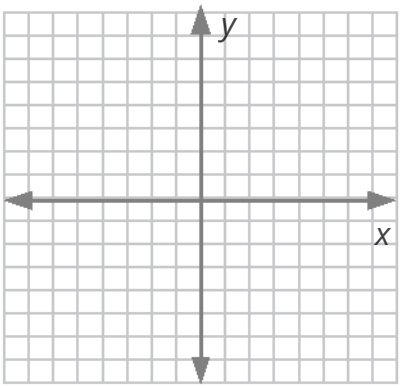


Find the solution set by graphing (each graph, 4 points).

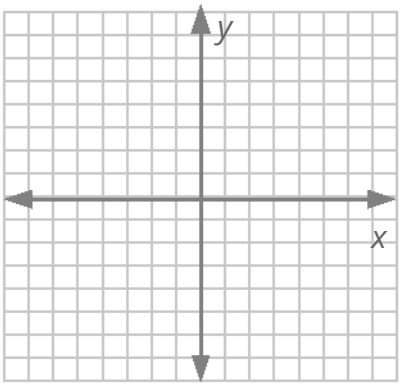
1.07 $\begin{cases} x - y = 4 \\ x + y = 2 \end{cases}$



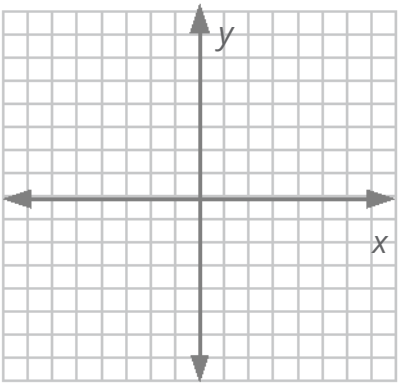
1.08 $\begin{cases} y = 2x \\ x + 2y = 2 \end{cases}$



1.09 $\begin{cases} 2x + 3y - 6 = 0 \\ x - y = 0 \end{cases}$



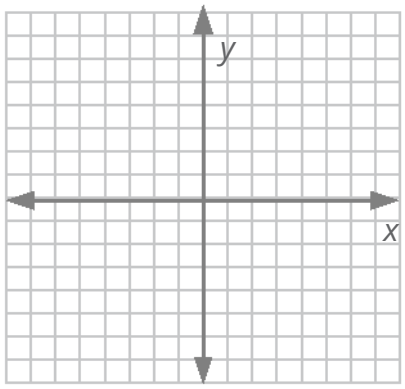
1.010 $\begin{cases} y = -3 \\ x - y = 8 \end{cases}$



Graph the solution for each of the following pairs of inequalities. Show only the final result for each system (each graph, 4 points).

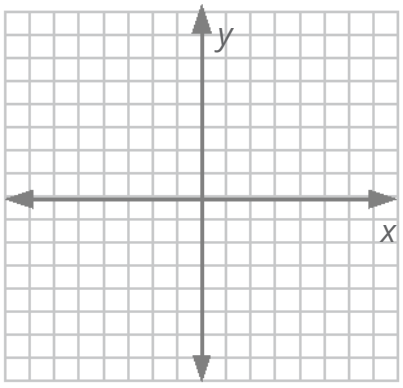
1.011

$$\begin{cases} y \le x \\ x + y \ge 1 \end{cases}$$



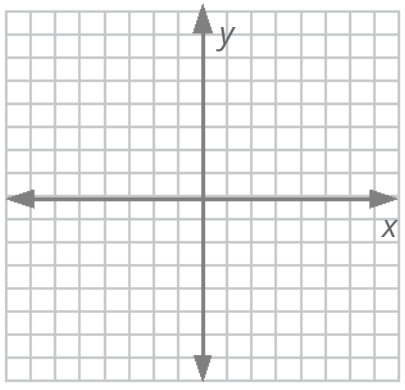
1.012

$$\begin{cases} x + y > 3 \\ x + y < -4 \end{cases}$$



1.013

$$\begin{cases} 2x + y < 1 \\ y \ge -4 - 2x \end{cases}$$



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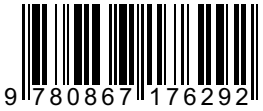

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