



# MATH

STUDENT BOOK

▶ **9th Grade** | Unit 10

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# Math 910

## Quadratic Equations and a Review of Algebra

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# Quadratic Equations and a Review of Algebra

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## INTRODUCTION


This LIFEPAC® is the final LIFEPAC in the first-year study of the mathematical system known as algebra. In this LIFEPAC you will learn how to solve equations that involve second-degree polynomials; these equations are called *quadratic equations*. Then you will review some representative exercises and problems from each of the LIFEPACs in this course of study.

## Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Identify quadratic equations.
2. Write quadratic equations in general form.
3. Solve quadratic equations by completing the square, by the quadratic formula, and by factoring.
4. Work representative problems of the first-year algebra course.

Survey the LIFE PAC. Ask yourself some questions about this study and write your questions here.

A large rectangular area with horizontal red lines for writing. The lines are evenly spaced and extend across the width of the box, providing a template for handwritten notes or questions.

# 1. QUADRATIC EQUATIONS

In this section you will need to apply many skills that you have acquired in previous LIFE PACs, while you learn about a new type of equation. After an introduction to

quadratic equations, you will learn three methods for solving them. Finally, you will learn to solve verbal problems that require the use of quadratic equations.

## OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Identify quadratic equations.
2. Write quadratic equations in general form.
3. Solve quadratic equations by completing the square, by the quadratic formula, and by factoring.

## IDENTIFYING QUADRATIC EQUATIONS

First you must be able to recognize quadratic equations. Learn these two basic definitions.

### DEFINITIONS

**quadratic equation**—an equation that can be written as  $Ax^2 + Bx + C = 0$ , where  $A$  is not zero.

In this LIFE PAC, you will consider an equation to be in **general form** when  $A$  is positive and when  $A$ ,  $B$ , and  $C$  are integers whose greatest common factor is 1.

**Model 1:**  $2x^2 + 3x - 4 = 0$  is a quadratic equation in general form, where  $A$  is 2,  $B$  is 3, and  $C$  is -4.

**Model 2:**  $x^2 + 7 = 0$  is a quadratic equation in general form, where  $A$  is 1,  $B$  is 0, and  $C$  is 7.

**Model 3:**  $-5x^2 + x = 0$  is a quadratic equation. Its general form can be found by multiplying both sides of the equation by negative one.

$$-1[-5x^2 + x] = -1[0]$$

$$5x^2 - x = 0$$

Then  $A$  is 5,  $B$  is -1, and  $C$  is 0.

**Model 4:**  $3x - 11 = 0$  is not a quadratic equation since it does not contain a second-degree term.

**Model 5:**  $x^3 - 2x^2 + 1 = 0$  is not a quadratic equation since it contains a third-degree term.

**Model 6:**  $\frac{1}{3}(x + 2)(x - 7) = 5$  is a quadratic equation that can be rewritten.

$$3\left[\frac{1}{3}(x + 2)(x - 7)\right] = 3[5]$$

$$(x + 2)(x - 7) = 15$$

$$x^2 - 5x - 14 = 15$$

$$x^2 - 5x - 29 = 0$$

Then  $A$  is 1,  $B$  is -5, and  $C$  is -29.

**Model 7:**  $0.7x^2 = 1$  is a quadratic equation. Its general form can be found by multiplying both sides of the equation by 10.

$$10[0.7x^2] = 10[1]$$

$$7x^2 = 10$$

$$7x^2 - 10 = 0$$

Then  $A$  is 7,  $B$  is 0, and  $C$  is -10.

**Model 8:**  $2x^2 - 4x + 6 = 0$  is a quadratic equation. Its general form can be found by dividing all the terms by 2.

$$\frac{2x^2}{2} - \frac{4x}{2} + \frac{6}{2} = \frac{0}{2}$$

$$x^2 - 2x + 3 = 0$$

Then  $A$  is 1,  $B$  is -2, and  $C$  is 3.



Indicate (by yes or no) whether each of the following equations is quadratic. If so, give the values of  $A$ ,  $B$ , and  $C$  from each equation's general form; if not, tell why.

1.1  $3x^2 + 5x - 7 = 0$  \_\_\_\_\_

1.2  $2x - 1 = 0$  \_\_\_\_\_

1.3  $2x^2 - 1 = 0$  \_\_\_\_\_

1.4  $-4x^2 + 2x - 1 = 0$  \_\_\_\_\_

1.5  $5x^2 + 15x = 0$  \_\_\_\_\_

1.6  $(x - 3)^2 = 0$  \_\_\_\_\_

1.7  $\frac{1}{4}x^2 + 5 = 0$  \_\_\_\_\_

1.8  $2x^3 - x = 0$  \_\_\_\_\_

1.9  $(x + 1)(2x + 3) = 4$  \_\_\_\_\_

1.10  $\frac{2}{3}(x - 4)(x + 5) = 1$  \_\_\_\_\_

1.11  $6x - 1 = 4x + 7$  \_\_\_\_\_

1.12  $6x^2 - 1 = 4x + 7$  \_\_\_\_\_

1.13  $6x^3 - 1 = 4x + 7$  \_\_\_\_\_

1.14  $1.3x^2 + 2.5x - 1 = 0$  \_\_\_\_\_

1.15  $(4x - 1)(3x + 5) = 0$  \_\_\_\_\_

1.16  $-2x^2 - 3x = 5$  \_\_\_\_\_

1.17  $(5 + x)(5 - x) = 7$  \_\_\_\_\_

1.18  $\frac{x^2}{2} = 7x$  \_\_\_\_\_

1.19  $\frac{1}{5}x = \frac{2}{3}x^2 - 2$  \_\_\_\_\_

1.20  $x(x + 1)(x + 2) = 3$  \_\_\_\_\_



## METHODS OF SOLVING QUADRATIC EQUATIONS

We shall now look at three ways to solve quadratic equations. The first two methods may be used with any quadratic equation. The third method may be used only with quadratic equations that have factorable polynomials.

### COMPLETING THE SQUARE

Let us begin by considering the equation  $x^2 = 16$ . You know that  $4^2$  and  $(-4)^2$  equal 16; therefore, 4 or -4 are the two roots for this quadratic equation. These roots can be written in a *solution set* as  $\{4, -4\}$ .

Now consider the equation  $x^2 = 17$ . Since  $(\sqrt{17})^2$  and  $(-\sqrt{17})^2$  equal 17, the solution

set for this quadratic equation is  $\{\sqrt{17}, -\sqrt{17}\}$ . Similarly, the equation  $x^2 = 18$  is satisfied by  $\sqrt{18}$  or  $-\sqrt{18}$ . These radicals can be simplified to  $\sqrt{9}\sqrt{2}$  or  $3\sqrt{2}$ , and  $-\sqrt{9}\sqrt{2}$  or  $-3\sqrt{2}$ .

Therefore, the solution set for this quadratic equation is  $\{3\sqrt{2}, -3\sqrt{2}\}$ .

The equation  $x^2 = -18$  has no real roots since no real number exists whose square is negative.

In general, to solve equations that contain squares of binomials, we use the following property.

### PROPERTY

If  $X^2 = N$  and  $N$  is not negative, then  $X = \sqrt{N}$  or  $X = -\sqrt{N}$ . This property is called the *Square Root Property of Equations*.

Study these models to see how this property can be applied to solve equations that contain squares of binomials.

#### Model 1:

$$\begin{array}{l|l}
 (x + 1)^2 = 16 & \\
 4^2 \text{ is } 16, \text{ so} & (-4)^2 \text{ is } 16, \text{ so} \\
 x + 1 = 4 & x + 1 = -4 \\
 x = 3 & x = -5
 \end{array}$$

The solution set is  $\{3, -5\}$

**Model 2:**

$$(x - 1)^2 = 17$$

$(\sqrt{17})^2$ is 17, so $x - 1 = \sqrt{17}$ $x = 1 + \sqrt{17}$		$(-\sqrt{17})^2$ is 17, so $x - 1 = -\sqrt{17}$ $x = 1 - \sqrt{17}$
-------------------------------------------------------------------------	--	---------------------------------------------------------------------------

The solution set is  $\{1 + \sqrt{17}, 1 - \sqrt{17}\}$

**Model 3:**

$$(3x + 2)^2 = 18$$

$(\sqrt{18})^2$ is 18, so $3x + 2 = \sqrt{18}$ $3x = -2 + \sqrt{18}$ $x = \frac{-2 + \sqrt{18}}{3}$ $x = \frac{-2 + 3\sqrt{2}}{3}$		$(-\sqrt{18})^2$ is 18, so $3x + 2 = -\sqrt{18}$ $3x = -2 - \sqrt{18}$ $x = \frac{-2 - \sqrt{18}}{3}$ $x = \frac{-2 - 3\sqrt{2}}{3}$
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The solution set is  $\left\{\frac{-2 + 3\sqrt{2}}{3}, \frac{-2 - 3\sqrt{2}}{3}\right\}$ .



**Apply the Square Root Property of Equations to find the solution set for each of the following equations.**

**1.21**      $x^2 = 25$

**1.22**      $x^2 = 26$

**1.23**      $x^2 = 27$

**1.24**      $x^2 - 80 = 0$

**1.25**      $x^2 + 80 = 0$

**1.26**      $(x + 2)^2 = 36$

**1.27**      $(2x - 5)^2 = 11$

**1.28**      $(x - 4)^2 = 12$

**1.29**      $(3x + 1)^2 - 100 = 0$

**1.30**      $(4x - 3)^2 - 50 = 0$

You should know how to find the root for quadratic equations that are written in the form  $X^2 = N$ . In some instances you may be able to write an equation in this form immediately by factoring  $Ax^2 + Bx + C$  to the square of a binomial.

**Model:**  $x^2 + 6x + 9 = 0$

$$(x + 3)(x + 3) = 0$$

$$(x + 3)^2 = 0$$

$$0^2 = 0$$

Therefore,  $x + 3 = 0$

$$x = -3$$

The solution set is  $\{-3\}$ .

*Note:* This quadratic equation has only one root since 0 is the only number whose square is 0.

When  $Ax^2 + Bx + C$  does not factor to the square of a binomial, a procedure known as *completing the square* may be used to write a quadratic equation in the form  $X^2 = N$ . The following steps can be used to accomplish this goal.

1. Write the quadratic equation in general form and identify  $A$  and  $B$ .
2. Multiply the terms of the equation by  $4A$ .
3. Isolate the constant term on the right side of the equation.
4. Add  $B^2$  to each side of the equation.
5. Factor the left side of the equation to the square of a binomial.

These steps will be indicated by number in the solutions to the next three models.

**Model 1:** Solve  $x^2 + 7x + 12 = 0$  by completing the square.

1.  $A$  is 1 and  $B$  is 7.
2.  $4A$  is 4:  $4[x^2 + 7x + 12] = 4[0]$   
 $4x^2 + 28x + 48 = 0$
3.  $4x^2 + 28x = -48$
4.  $B^2$  is 49:  $[4x^2 + 28x] + 49 = [-48] + 49$   
 $4x^2 + 28x + 49 = 1$
5.  $(2x + 7)(2x + 7) = 1$   
 $(2x + 7)^2 = 1$

Now solve:

$1^2$ is 1, so	$(-1)^2$ is 1, so
$2x + 7 = 1$	$2x + 7 = -1$
$2x = -6$	$2x = -8$
$x = -3$	$x = -4$

The solution set is  $\{-3, -4\}$ .

**Model 2:** Solve  $5x^2 = 3x$  by completing the square.

1.  $5x^2 - 3x = 0$ ;  $A$  is 5 and  $B$  is -3.
2.  $4A$  is 20:  $20[5x^2 - 3x] = 20[0]$   
 $100x^2 - 60x = 0$
3. The equation contains no constant term.
4.  $B^2$  is 9:  $[100x^2 - 60x] + 9 = [0] + 9$   
 $100x^2 - 60x + 9 = 9$
5.  $(10x - 3)(10x - 3) = 9$   
 $(10x - 3)^2 = 9$

Now solve:

$3^2$ is 9, so	$(-3)^2$ is 9, so
$10x - 3 = 3$	$10x - 3 = -3$
$10x = 6$	$10x = 0$
$x = 0.6$	$x = 0$

The solution set is  $\{0.6, 0\}$ .

**Model 3:** Solve  $4x(x - 3) = 2$  by completing the square.

1.  $4x^2 - 12x = 2$   
 $4x^2 - 12x - 2 = 0$   
 $2x^2 - 6x - 1 = 0$ ;  $A$  is 2 and  $B$  is -6.
2.  $4A$  is 8:  $8[2x^2 - 6x - 1] = 8[0]$   
 $16x^2 - 48x - 8 = 0$
3.  $16x^2 - 48x = 8$
4.  $B^2$  is 36:  $[16x^2 - 48x] + 36 = [8] + 36$   
 $16x^2 - 48x + 36 = 44$

$$5. \quad \begin{aligned} (4x - 6)(4x - 6) &= 44 \\ (4x - 6)^2 &= 44 \end{aligned}$$

Now solve:  $(\sqrt{44})^2$  is 44 and  $(-\sqrt{44})^2$  is 44, so

$$\begin{aligned} 4x - 6 &= \pm \sqrt{44} \\ 4x &= 6 \pm \sqrt{44} \\ 4x &= 6 \pm 2\sqrt{11} \\ x &= \frac{6 \pm 2\sqrt{11}}{4} \end{aligned}$$

The solution set is  $\left\{\frac{3 + \sqrt{11}}{2}, \frac{3 - \sqrt{11}}{2}\right\}$ .

Note: The  $\pm$  symbol is read “plus or minus.”



**Solve each quadratic equation by completing the square.**

**1.31**      $x^2 + 9x + 8 = 0$

**1.32**      $x^2 - 4x - 7 = 0$

**1.33**      $2x^2 = 7x$

**1.34**      $3x^2 + x = 0$

**1.35**      $x(x + 5) = 3$

**1.36**      $2x(x - 2) + 2 = 0$

**1.37**      $7x - 5x^2 = 2$

**1.38**      $(2x + 5)(x - 1) = 1$

**1.39**      $\frac{1}{2}x^2 + \frac{2}{3}x - \frac{5}{6} = 0$

**1.40**      $0.4x^2 + 1.1x = 2$

**THE QUADRATIC FORMULA**

Another method for solving a quadratic equation is derived from using the

completing-the-square procedure on the general equation  $Ax^2 + Bx + C = 0$ . Study the following steps carefully.

**DERIVATION**

$$Ax^2 + Bx + C = 0$$

Multiply by  $4A$ :

$$\begin{aligned} 4A[Ax^2 + Bx + C] &= 4A[0] \\ 4A^2x^2 + 4ABx + 4AC &= 0 \end{aligned}$$

Isolate the constant:

$$4A^2x^2 + 4ABx = -4AC$$

Add  $B^2$ :

$$4A^2x^2 + 4ABx + B^2 = -4AC + B^2$$

Factor:

$$\begin{aligned} (2Ax + B)(2Ax + B) &= -4AC + B^2 \\ (2Ax + B)^2 &= -4AC + B^2 \end{aligned}$$

Solve for  $x$ :

$$2Ax + B = \pm \sqrt{-4AC + B^2}$$

$$2Ax = -B \pm \sqrt{-4AC + B^2}$$

$$x = \frac{-B \pm \sqrt{-4AC + B^2}}{2A}$$

The result is known as the **quadratic formula** and is usually written as

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

*Note:*  $-B$  is read “the opposite of  $B$ .”

Notice that two possible values of  $x$  are indicated by the quadratic formula—one value is  $\frac{-B + \sqrt{B^2 - 4AC}}{2A}$  and the other value is  $\frac{-B - \sqrt{B^2 - 4AC}}{2A}$ . You should memorize this formula, since it will be used often. To use the formula, replace  $A$ ,  $B$ , and  $C$  by their numerical values in the equation to be solved and simplify.



**Model 1:** Solve  $x^2 + 7x + 12 = 0$  by using the quadratic formula.

$A$  is 1,  $B$  is 7, and  $C$  is 12.

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1}$$

$$x = \frac{-7 \pm \sqrt{49 - 48}}{2}$$

$$x = \frac{-7 \pm \sqrt{1}}{2}, \text{ and since } \pm \sqrt{1} = \pm 1,$$

$$x = \frac{-7 + 1}{2} \quad \Bigg| \quad x = \frac{-7 - 1}{2}$$

$$x = \frac{-6}{2} \quad \Bigg| \quad x = \frac{-8}{2}$$

$$x = -3 \quad \Bigg| \quad x = -4$$

The solution set is  $\{-3, -4\}$ .

**Model 2:** Solve  $4x^2 = 2x + 1$  by using the quadratic formula.

The equation rewritten in general form is  $4x^2 - 2x - 1 = 0$ ;  $A$  is 4,  $B$  is -2, and  $C$  is -1.

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 4(-1)}}{2 \cdot 4}$$

$$x = \frac{2 \pm \sqrt{4 + 16}}{8}$$

$$x = \frac{2 \pm \sqrt{20}}{8}, \text{ and since } \pm \sqrt{20} = \pm 2\sqrt{5},$$

$$x = \frac{2 \pm 2\sqrt{5}}{8}$$

$$x = \frac{\cancel{2}(1 \pm \sqrt{5})}{\cancel{2} \cdot 4}$$

The solution set is  $\left\{ \frac{1 + \sqrt{5}}{4}, \frac{1 - \sqrt{5}}{4} \right\}$ .

These values in the solution set are the exact irrational values of  $x$ . You can also find approximate rational values of  $x$  by using a decimal representation for  $\sqrt{5}$  and then rounding to the desired accuracy.

$$\frac{1 + 2.236}{4} \quad \left| \quad \frac{1 - 2.236}{4}$$

$$\frac{3.236}{4} \quad \left| \quad \frac{-1.236}{4}$$

The values of the solution set rounded to the nearest tenth are  $\{0.8, -0.3\}$ .



**Solve each quadratic equation by using the quadratic formula. If the roots are irrational, give both the exact value and the rational approximation to the nearest tenth.**

**1.41**      $x^2 + 5x + 4 = 0$

**1.42**      $x^2 + 5x - 4 = 0$

**1.43**      $2x^2 - 3x - 1 = 0$

**1.44**      $6x^2 - 5x - 6 = 0$

**1.45**      $2x^2 = 8x - 3$

**1.46**      $\frac{x^2}{3} + \frac{7}{12}x + \frac{1}{4} = 0$

**1.47**      $2x(x + 5) = 4$

**1.48**      $\frac{5x}{3} = x^2 + \frac{1}{2}$

**1.49**      $x(x + 8) = 9$

**1.50**      $\frac{x^2}{6} = \frac{x}{2}$

**FACTORING**

A quadratic equation can also be solved using another method when  $Ax^2 + Bx + C$  is factorable. This method requires the use of a simple, yet important property.

**PROPERTY**

$M \cdot N = 0$  means that  $M = 0$  or  $N = 0$  (or both).

This property is called the *Zero-Product Property*.

**Model 1:**  $x(x - 2) = 0$  means that the first factor  $x$  equals 0, or the second factor  $x - 2$  equals 0, or both factors equal zero.

$$\begin{array}{l|l} x = 0 & x - 2 = 0 \\ & x = 2 \end{array}$$

The solution set is  $\{0, 2\}$ .

**Model 2:**  $(x + 1)(2x + 3) = 0$  means that

$$\begin{array}{l|l} x + 1 = 0 & 2x + 3 = 0 \\ x = -1 & 2x = -3 \\ & x = -1.5 \end{array}$$

The solution set is  $\{-1, -1.5\}$ .

The following steps can be used to solve a quadratic equation by the factoring method.

1. Write the quadratic equation in general form.
2. Find the factors of  $Ax^2 + Bx + C$ .
3. Apply the Zero-Product Property.
4. Solve for  $x$ .

These steps will be indicated by number in the solutions to the next three models.

**Model 1:** Solve  $11x = 4 - 3x^2$  by factoring.

$$\begin{array}{l}
 1. \quad 3x^2 + 11x - 4 = 0 \\
 2. \quad (3x - 1)(x + 4) = 0 \\
 3. \quad \begin{array}{l|l} 3x - 1 = 0 & x + 4 = 0 \\ \hline & \end{array} \\
 4. \quad \begin{array}{l|l} 3x = 1 & x = -4 \\ x = \frac{1}{3} & \end{array}
 \end{array}$$

The solution set is  $\{\frac{1}{3}, -4\}$ .

**Model 2:** Solve  $5x^2 - 20 = 0$  by factoring.

$$\begin{array}{l}
 1. \quad \begin{array}{l} \frac{5x^2}{5} - \frac{20}{5} = \frac{0}{5} \\ x^2 - 4 = 0 \end{array} \\
 2. \quad (x + 2)(x - 2) = 0 \\
 3. \quad \begin{array}{l|l} x + 2 = 0 & x - 2 = 0 \\ \hline & \end{array} \\
 4. \quad \begin{array}{l|l} x = -2 & x = 2 \\ \hline & \end{array}
 \end{array}$$

The solution set is  $\{-2, 2\}$ .

**Model 3:** Solve  $(x + 4)(x + 5) = 8$  by factoring.

$$\begin{array}{l}
 1. \quad \begin{array}{l} x^2 + 9x + 20 = 8 \\ x^2 + 9x + 12 = 0 \end{array} \\
 2. \quad \text{The trinomial } x^2 + 9x + 12 \text{ is prime, so you cannot use the} \\
 \quad \text{factoring method.}
 \end{array}$$

Find the value of  $x$  by using the quadratic formula, where  $A$  is 1,  $B$  is 9, and  $C$  is 12.

$$x = \frac{-9 \pm \sqrt{9^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1}$$

$$x = \frac{-9 \pm \sqrt{81 - 48}}{2}$$

$$x = \frac{-9 \pm \sqrt{33}}{2}$$

The exact roots are  $\frac{-9 + \sqrt{33}}{2}$ ,  $\frac{-9 - \sqrt{33}}{2}$ .

The roots to the nearest hundredth are -1.63, -7.37.

Note: You could also have used the completing-the square method to solve  $x^2 + 9x + 12 = 0$ .



Solve each quadratic equation by factoring.

**1.51**      $x^2 + 3x - 4 = 0$

**1.52**      $2x^2 + x = 0$

**1.53**      $4x^2 - 11x + 6 = 0$

**1.54**      $7x^2 - 63 = 0$

**1.55**      $6x^2 = 13x + 5$

**1.56**      $9x^2 = 7x$

1.57  $\frac{4}{5}x^2 = 2x - \frac{4}{5}$

1.58  $(x + 4)(x - 3) = 8$

1.59  $4x^2 - 25 = 0$

1.60  $(x + 5)(x + 3) = 3$



Solve each quadratic equation. Factor when possible; otherwise complete the square or use the quadratic formula. For irrational roots give both the exact forms and the rational approximations to the nearest hundredth.

1.61  $3x^2 + 5x - 2 = 0$

1.62  $3x^2 - 5x - 2 = 0$

**1.63**  $3x^2 - 5x + 2 = 0$

**1.64**  $x^2 - 49 = 0$

**1.65**  $x^2 - 50 = 0$

**1.66**  $2x^2 + 7x + 4 = 0$

**1.67**  $2x^2 + 7x - 4 = 0$

**1.68**  $\frac{x^2}{4} - \frac{x}{3} = \frac{1}{2}$



1.69  $\frac{x^2}{4} - \frac{x}{3} = 0$

1.70  $(2x + 1)(3x + 2) = 1$

## VERBAL PROBLEMS

Several types of verbal problems can be solved using quadratic equations. Study the set-ups and solutions of the following models carefully. Be sure that each result is reasonable for the conditions of the original problem before you give the final answer(s).

**Model 1:** The sum of the squares of two consecutive negative integers is eighty-five. Find the numbers.

Let  $n$  and  $n + 1$  be the consecutive negative integers.

Number	Square	Sum
$n$	$n^2$	$n^2 + n^2 + 2n + 1$
$n + 1$	$(n + 1)^2$ or $n^2 + 2n + 1$	

Thus, the quadratic equation for this problem is

$$2n^2 + 2n + 1 = 85.$$

Solve:  $2n^2 + 2n - 84 = 0$

$$\frac{2n^2}{2} + \frac{2n}{2} - \frac{84}{2} = 0$$

$$n^2 + n - 42 = 0$$

$$(n + 7)(n - 6) = 0$$

$$n + 7 = 0 \quad \left| \quad n - 6 = 0$$

$$n = -7 \quad \left| \quad n = 6$$

$$\text{and } n + 1 = -6 \quad \left| \quad \text{and } n + 1 = 7$$

Since the conditions of the problem indicate that the integers are to be negative, we must reject the pair of numbers 6 and 7.

$\therefore$  The desired integers are -7 and -6.

**Model 2:** The length of a rectangle is two centimeters more than four times its width, and its area is one hundred ten square centimeters. Find the dimensions of this rectangle.



Since the area of a rectangle is found by multiplying its width by its length, the quadratic equation for this problem is  $w(4w + 2) = 110$ .

Solve:

$$\begin{aligned} 4w^2 + 2w &= 110 \\ \frac{4w^2}{2} + \frac{2w}{2} &= \frac{110}{2} \\ 2w^2 + w &= 55 \\ 2w^2 + w - 55 &= 0 \\ (2w + 11)(w - 5) &= 0 \end{aligned}$$

$$\begin{array}{l|l} 2w + 11 = 0 & w - 5 = 0 \\ 2w = -11 & w = 5 \\ w = -\frac{11}{2} & \text{and } 4w + 2 = 22 \\ \text{and } 4w + 2 = -20 & \end{array}$$

Since the length and width of a rectangle must be positive, we must reject the dimensions  $-\frac{11}{2}$  by  $-20$ .

$\therefore$  The dimensions are 5 cm by 22 cm.



**Solve each verbal problem with a quadratic equation using your choice of methods for solving each equation.**

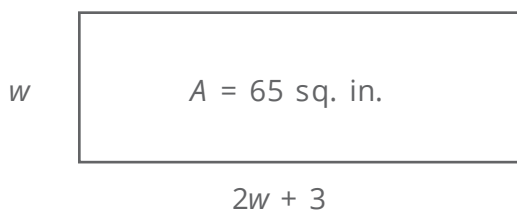
**1.71** The sum of the squares of two consecutive negative integers is forty-one. Find the numbers.

**1.72** The sum of the squares of three consecutive positive even integers is one hundred sixteen. Find the numbers.

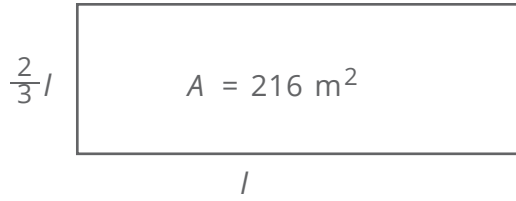
- 1.73** The square of a number is equal to seven times that number less ten. Find the number.  
(Hint: Two numbers will satisfy this condition; include both numbers in your final answer.)

- 1.74** The square of a number exceeds that number by twelve. Find the number (two answers).

- 1.75** The length of a rectangle is three inches more than twice its width, and its area is sixty-five square inches. Find the dimensions.



- 1.76** The width of a rectangle is two-thirds of its length, and its area is two hundred sixteen square meters. Find the dimensions.



- 1.77** The width and length of a rectangle are consecutive odd integers. If the length is increased by five feet, the area of the resulting rectangle is sixty square feet. Find the dimensions and the area of the original rectangle.

- 1.78** Find two negative factors of ninety-six such that one factor is four less than the other.

- 1.79** Find two factors of negative thirty-six such that one factor is eleven less than half of the other factor. (Hint: You should have two pairs of factors.)
- 1.80** The square of a certain positive irrational number is equal to two more than half of that number. Find the exact value of the number and find the value of its approximation to the nearest thousandth.



**Review the material in this section in preparation for the Self Test.** The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific area where restudy is needed for mastery.

# SELF TEST 1

Indicate (by yes or no) whether each of the following equations is quadratic. If so, give the values of  $A$ ,  $B$ , and  $C$  from the general form of each equation; if not, tell why (each part, 3 points).

1.01  $2x^2 - 4x + 1 = 0$  \_\_\_\_\_ ; \_\_\_\_\_

1.02  $x(x^2 + 1) = 0$  \_\_\_\_\_ ; \_\_\_\_\_

1.03  $5(4x + 2) = 3$  \_\_\_\_\_ ; \_\_\_\_\_

1.04  $(x + 3)(x + 4) = 5$  \_\_\_\_\_ ; \_\_\_\_\_

1.05  $-\frac{3}{4}x^2 + 2 = 0$  \_\_\_\_\_ ; \_\_\_\_\_

Apply the Square Roots Property of Equations to find each solution set (each answer, 3 points).

1.06  $x^2 = 9$

1.07  $(3x - 1)^2 = 5$

1.08  $x^2 + 5x + 1 = 0$

1.09  $2x(x - 1) = 3$

Solve each quadratic equation by using the quadratic formula. If the roots are irrational, give both the exact value and the rational approximations to the nearest tenth (each problem, 4 points).

1.010  $3x^2 + 4x - 2 = 0$

1.011  $\frac{x^2}{3} + \frac{1}{2} = \frac{5}{6}x$

Solve each quadratic equation by factoring (each answer, 3 points).

1.012  $7x^2 + 3x = 0$

1.013  $(x - 2)(x - 3) = 2$



**Solve each verbal problem with a quadratic equation using your choice of methods for solving the equation** (each answer, 5 points).

**1.014** The square of a certain negative number is equal to five more than one-half of that number. Find the number.

**1.015** The width and the length of a rectangle are consecutive even integers. If the width is decreased by three inches, then the area of the resulting rectangle is twenty four square inches. Find the dimensions of the original rectangle.

	<b>SCORE</b> _____	<b>TEACHER</b> _____	_____	_____
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